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Hydromagnetic Radiative Convection Flow through Porous Medium Between Two Semi-Infinite Parallel Plates with Periodic Cross Flow Velocity and Periodic Temperature: Analysis With Internal Heat Generation/Absorption

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Abstract

Aim of the proposed paper is to investigate hydromagnetic radiative convection flow of a viscous, conducting, incompressible fluid through a porous medium between two semi-infinite parallel plates with periodic cross-flow velocity and periodic temperature, taking into account the internal heat generation/ absorption.The flow is subjected to optically thin transparent medium and the uniform magnetic field is applied normal to the flow. For a small time dependent perturbation of the fluid velocity and temperature, the nonlinear equations are solved by asymptotic approximation. The resulting velocity and temperature fields are shown graphically for different values of the physical parameters. The skin-friction and the rate of heat transfer in terms of Nusselt number, at the plates are derived, discussed numerically and shown graphically.

Key words: Hydromagnetic, radiative convection flow, porous medium, cross-flow velocity, internal heat generation/ absorption.

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Nomenclature

A : real positive constant,
B_0 : the magnetic field intensity,
C_p : specific heat at constant pressure,
Gr : convection parameter,
g : acceleration due to gravity,
K : permeability parameter,
K_T : thermal conductivity,
M : magnetic parameter,
N^2 : radiation parameter,
Pr : Prandtl number,
p' : pressure,
Q' : heat generation/absorption coefficient,
Q : heat generation/absorption parameter,
q : radiative heat transfer,
Re : cross-flow Reynolds number,
Re_1 : cross-flow parameter,
t' : time,
T_0 : reference temperature,
T' : fluid temperature,
T_∞ : temperature at equilibrium,
U_0 : mean velocity,
u' : velocity component along x' –direction,
v' : velocity component along y' –direction,
v_0 : cross-flow mean velocity ($v_0 > 0$),

Greek symbols

$\alpha\left(<<1\right)$: absorption coefficient,
β : coefficient of volume expansion,
$\varepsilon\left(<<1\right)$: small parameter,
σ : electrical conductivity of the fluid,
ρ : density,
υ : kinematic viscosity,
ω' : frequency.

Introduction

The problem of hydromagnetic radiative convection flow through porous medium between two semi-infinite parallel plates finds numerous and wide ranging applications in many engineering processes. The problem of radiative heat transfer in a vertical channel has been studied in recent times as a model for the re-entry problem. This is due to the significant role of thermal radiation in surface heat transfer, particularly in free convection problems involving absorbing - emitting fluids, MHD power generators, accelerators and pumps. Cess[1] considered absorbing - emitting gray fluids with a black vertical plate. Novotny *et al*[2]. studied the same problem employing the method of local non-similarity transformations and the continuous correlation for the band absorption. Mansour[3] investigated interaction of mixed convection with thermal radiation in laminar boundary layer flow over horizontal, continuous moving sheets with suction and injection. Hossain and Takhar[4] analyzed the effect of radiation using the Rosseland diffusion approximation that leads to non-similar boundary layer equations governing the mixed convection flow of a optically dense viscous incompressible fluid past a heated vertical plate with a uniform free stream velocity and surface temperature. Sattar and Kalim[5] studied the effect of unsteady free convection interaction with thermal radiation in a boundary layer flow. Alabraba *et al*[6]. investigated laminar convection in hydromagnetic flow of a binary mixture with radiative heat transfer under uniform transverse magnetic field. Alagoa *et al*[7]. considered the effect of radiation and free convection on hydromagnetic flow through a porous medium confined between two infinite parallel plates with time dependent suction. Bestman[8] investigated hydromagnetic radiative free convection heat transfer to steady non-Newtonian flow past a vertical porous plate with usual Boussinesq approximation. Singh[9] discussed radiative convection flow of a viscoelastic fluid past a stretching sheet in the presence of heat source/sink and transversely applied uniform magnetic field. Recently, Singh *et al*[10]. studied unsteady hydromagnetic convective flow with radiative heat transfer past an infinite porous vertical plate in rotating system, when the plate is subjected to a variable suction and is embedded in a time dependent porous medium. More recently, Singh *et al*[11]. have discussed effects of thermophoresis on hydromagnetic mixed convection and mass transfer flow past a vertical permeable plate with thermal radiation and variable suction. Singh *et al*[12]. have also discussed three-dimensional free convection laminar flow of viscous fluid past an infinite vertical porous plate embedded in a highly porous medium subjected to periodic suction velocity and heat source/sink.

However, the radiative convection flow through porous medium confined in channels have been given less importance, although such flows have numerous engineering and geophysical applications (see Modest[13]). Recently, Sharma and Mehta[14] have investigated radiative convective flow of a viscous incompressible fluid through a porous medium between two parallel plates with cross-flow velocity and oscillatory temperature following the method suggested by Bestman[8]. The model suggested in the proposed work enhances the application of the work of Sharma and Mehta[14] addition of the internal heat generation/ absorption. The results of the study are discussed by the use of graphs and tables. The proposed model finds practical application in metallurgical processes involving cooling of the continuous stretching strips, which may be made of drawing, annealing and tinning of copper wire. In all these cases, the quality of the final product, to a great extent, depends on the rate of cooling by drawing such strips in an electrically conducting fluid flow through porous medium confined in channel subjected to magnetic field and radiation. The rate of cooling is controlled for the final product of desired characteristics by suitable adjustment of the magnetic parameter, radiation parameter, heat generation/absorption parameter.

Formulation of the Problem :

We consider unsteady flow of an incompressible, viscous, electrically conducting fluid through porous medium between semi-infinite vertical parallel plates with internal heat generation/absorption, radiative heat transfer and transverse magnetic field. In two-dimensional coordinate system (x', y'), let x' –axis be chosen along the plate ($y = 0$) in the direction of flow and y' –axis normal to it. A uniform magnetic field of small strength is applied normal to the plates i.e. along the y' –axis. It is assumed that the magnetic Reynolds number is small, so that the induced magnetic field is neglected in comparison to the applied magnetic field (Ferraro & Plumpton[15]). Also, the parallel plates are long enough so that all the physical quantities are functions of y' and t' only. Therefore, following Sharma and Mehta[14], the equations governing the flow under usual Boussinque approximation are:

$$\frac{\partial v'}{\partial y'} = 0 \quad . \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = g\beta\left(T' - T_\infty\right) + \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\upsilon}{K}u' - \frac{\sigma B_0^2}{\rho}u' \,. \tag{2}$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{K_T}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho C_p}\frac{\partial q'}{\partial y'} - \frac{Q'\left(T' - T_\infty\right)}{\rho C_p} \,. \tag{3}$$

$$\frac{\partial^2 q'}{\partial y^2} - 3\alpha^2 q' - 16\sigma\alpha T'^3\frac{\partial T'}{\partial y'} = 0 \,. \tag{4}$$

Equation (1) implies that v' is either constant or a function of time. Hence, we assume cross-flow velocity in the following form:

$$v' = v_0\left(1 + \varepsilon A e^{i\omega' t'}\right). \tag{5}$$

Present investigation is limited to the condition of 'optically thin environment', where the fluid is assumed to be of low density. Hence, the absorption coefficient α is less than unity (i.e. $\alpha << 1$) so that in the spirit of Cess[1], the radiative heat flux shown in equation (4) on integrating within the limits T_∞ to T' yields:

$$\frac{\partial q'}{\partial y'} = 4\sigma\alpha\left(T'^4 - T_\infty^4\right). \tag{6}$$

Substituting (5) and (6) into equation (2) and (3), we get:

$$\frac{\partial u'}{\partial t'} + v_0\left(1 + \varepsilon A e^{i\omega' t'}\right) = g\beta\left(T' - T_\infty\right)$$
$$+\upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\upsilon}{K}u' - \frac{\sigma B_0^2}{\rho}u' \,. \tag{7}$$

$$\frac{\partial T'}{\partial t'} + v_0\left(1 + \varepsilon A e^{i\omega' t'}\right)\frac{\partial T'}{\partial y'} = \frac{K_T}{\rho C_p}\frac{\partial^2 T'}{\partial y'^2}$$
$$-\frac{4\sigma\alpha}{\rho C_p}\left(T'^4 - T_\infty^4\right) - \frac{Q'\left(T' - T_\infty\right)}{\rho C_p} \tag{8}$$

The boundary conditions relevant to the proposed problem (Sharma and Mehta[14]) are:

$$u' = 0, \quad T' = T_0\left(1 + \varepsilon e^{i\omega' t'}\right) \text{ at } \quad y' = 0,$$
$$u' = U_0\left(1 + \varepsilon e^{i\omega' t'}\right), \; T' = T_\infty \text{ at } y' = \frac{h}{Re}. \tag{9}$$

Here, $Re = \frac{v_0 h}{\upsilon}$ is cross-flow Reynolds number.

Now, we introduce the following non-dimensional quantities:

$$y' = \frac{h}{Re}\eta, \; t' = \frac{h}{v_0 Re}t, \; \omega' = \frac{v_0 Re}{h}\omega, \; u' = uU_0,$$
$$v' = U_0 v, \quad T' = \theta T_\infty, \quad T_0 = \theta_w T_\infty .$$

Introducing these non-dimensional quantities in equations (7) and (8), we obtain:

$$\frac{\partial u}{\partial t}+\left(1+\varepsilon Ae^{i\omega t}\right)\frac{\partial u}{\partial \eta}=\frac{\partial^2 u}{\partial \eta^2}-\left(Re_1^2+M^2\right)u+Gr\left(\theta-1\right). \qquad (10)$$

$$Pr\frac{\partial \theta}{\partial t}+Pr\left(1+\varepsilon Ae^{i\omega t}\right)\frac{\partial \theta}{\partial \eta}=\frac{\partial^2 \theta}{\partial \eta^2}-N^2\left(\theta^4-1\right)-Q\left(\theta-1\right). \qquad (11)$$

where $Pr=\frac{\mu C_p}{K_T}$ (Prandtl number),

$N^2=\frac{4\sigma\alpha T^3_\infty h^2}{K_T Re^2}$ (radiation parameter),

$Re_1=\frac{h}{\sqrt{K Re}}$ (cross-flow parameter), $Gr=\frac{g\beta h T_\infty}{v_0 U_0 Re}$ (convection parameter), $M=\frac{\sqrt{\sigma\mu}B_0}{\rho v_0}$ (magnetic parameter) and $Q'=\frac{QK_T Re^2}{h^2}$ (heat generation/ absorption parameter).

The boundary conditions (9) in non-dimensional form are:

$$u=0, \quad \theta=\theta_w\left(1+\varepsilon e^{i\omega t}\right) \quad \text{at} \quad \eta=0$$
$$u=1+\varepsilon e^{i\omega t}, \quad \theta=1 \quad \text{at} \quad \eta=1. \qquad (12)$$

Method of Solution :

Equations (10) and (11) show that the flow variables are highly nonlinear and coupled. Therefore, one of the global approach to find the solutions of such highly nonlinear coupled differential equations is the regular perturbation technique taking ε as the small parameter, such that $\varepsilon \ll 1$. Therefore, the velocity (u) and temperature (θ) can be expanded in the power of ε, in the following form:

$$u\left(\eta,t\right)=u_0\left(\eta\right)+\varepsilon u_1\left(\eta\right)e^{i\omega t}.$$
$$\theta\left(\eta,t\right)=\theta_0\left(\eta\right)+\varepsilon\theta_1\left(\eta\right)e^{i\omega t}. \qquad (13)$$

On substituting (10) and (11) in (13), we obtain:

Zeroth-order equations:

$$\frac{d^2u_0}{d\eta^2}-\frac{du_0}{d\eta}-\left(Re_1^2+M^2\right)u_0=-Gr\left(\theta_0-1\right). \qquad (14)$$

$$\frac{d^2\theta_0}{d\eta^2}-Pr\frac{d\theta_0}{d\eta}-N^2\left(\theta_0^4-1\right)-Q\left(\theta_0-1\right)=0. \qquad (15)$$

First-order equations:

$$\frac{d^2u_1}{d\eta^2}-\frac{du_1}{d\eta}-i\omega u_1-\left(Re_1^2+M^2\right)u_1=A\frac{du_0}{d\eta}-Gr\theta_1. \qquad (16)$$

$$\frac{d^2\theta_1}{d\eta^2}-Pr\frac{d\theta_1}{d\eta}-i\omega Pr\theta_1-4N^2\theta_0^2\theta_1-Q\theta_1=APr\frac{d\theta_0}{d\eta}. \qquad (17)$$

Introducing (13) into the non-dimensional boundary conditions (12), we obtain:

$$u_0=0, \quad \theta_0=\theta_w, \quad \theta_1=\theta_w \quad \text{at} \quad \eta=0$$
$$u_1=1, \quad \theta_0=1, \quad \theta_1=0 \quad \text{at} \quad \eta=1. \qquad (18)$$

Solutions of the equations (14)-(17) for the flow variables can be obtained, if we start with the equation (15). Assuming that the temperature difference between the plate at $\eta=0$ and its nearest adjacent layer is small, the temperature θ_0 can be expanded (see Bestman[8]), in the following form:

$$\theta_0\left(\eta\right)=\theta_w+\psi\left(\eta\right), \qquad (19)$$

where $\psi\left(\eta\right)$ is the small correction factor and θ_w is the constant temperature at the plates such that $O(\theta_w) < \psi(\eta) < O(1)$.

Introducing (19) into the equation (15), neglecting squares and higher powers of the correction quantity $\psi(\eta)$, the equation (15) reduces to:

$$\frac{d^2\psi}{d\eta^2} - Pr\frac{d\psi}{d\eta} - \left(4N^2\theta_w^3 + Q\right)\psi = N^2\left(\theta_w^4 - 1\right) + Q\left(\theta_w - 1\right). \quad (20)$$

The boundary conditions (18), corresponding to $\psi(\eta)$ reduce to:

$$\psi = 0 \text{ at } \eta = 0 \text{ and } \psi = 1 - \theta_w \text{ at } \eta = 1. \quad (21)$$

The solution of equation (20) under the boundary conditions (21) is given by:

$$\psi(\eta) = C_1 e^{m_1\eta} + C_2 e^{m_2\eta} - K_1. \quad (22)$$

Introducing $\psi(\eta)$ in (19), the solution $\theta_0(\eta)$ is given by:

$$\theta_0(\eta) = C_1 e^{m_1\eta} + C_2 e^{m_2\eta} + \left(\theta_w - K_1\right). \quad (23)$$

The solutions of the equations (14), (16) and (17) under corresponding boundary conditions are as follows:

$$u_0(\eta) = C_3 e^{m_3\eta} + C_4 e^{m_4\eta} - b_1 e^{m_1\eta} - b_2 e^{m_2\eta} - b_3. \quad (24)$$

$$\theta_1(\eta) = C_5 e^{m_5\eta} + C_6 e^{m_6\eta} + \left(b_4 + ib_5\right)e^{m_1\eta} + \left(b_6 + ib_7\right)e^{m_2\eta}. \quad (25)$$

$$u_1(\eta) = C_7 e^{m_7\eta} + C_8 e^{m_8\eta} + \left(K_{12} + iK_{13}\right)e^{m_3\eta} + \left(K_{14} + iK_{15}\right)e^{m_4\eta} - \left[\left(K_{16} + K_{20}\right) + i\left(K_{17} + K_{21}\right)\right]e^{m_1\eta}$$
$$- \left[\left(K_{18} + K_{22}\right) + i\left(K_{19} + K_{23}\right)\right]e^{m_2\eta} - \frac{Gr}{P_5^2 + P_6^2}e^{\alpha_1\eta}$$
$$\left[\left\{P_5\left(K_8\cos\beta_1\eta - K_9\sin\beta_1\eta\right) + P_6\left(K_9\cos\beta_1\eta + K_8\sin\beta_1\eta\right)\right\}\right.$$
$$\left. + i\left\{P_5\left(K_9\cos\beta_1\eta + K_8\sin\beta_1\eta\right) - P_6\left(K_8\cos\beta_1\eta - K_9\sin\beta_1\eta\right)\right\}\right]$$
$$- \frac{Gr}{P_5^2 + P_7^2}e^{\alpha_1\eta}\left[\left\{P_5\left(K_{10}\cos\beta_1\eta + K_{11}\sin\beta_1\eta\right)\right.\right.$$
$$\left. + P_7\left(K_{11}\cos\beta_1\eta - K_{10}\sin\beta_1\eta\right)\right\} + i\left\{P_5\left(K_{11}\cos\beta_1\eta - K_{10}\sin\beta_1\eta\right)\right.$$
$$\left.\left. - P_7\left(K_{10}\cos\beta_1\eta + K_{11}\sin\beta_1\eta\right)\right\}\right]. \quad (26)$$

Finally, the expressions for $\theta(\eta, t)$ and $u(\eta, t)$ are obtained in the following form:

$$\theta(\eta, t) = \theta_0(\eta) + \varepsilon\left[\left(M_r\cos\omega t - M_i\sin\omega t\right) + i\left(M_i\cos\omega t + M_r\sin\omega t\right)\right]. \quad (27)$$

$$u(\eta, t) = u_0(\eta) + \varepsilon\left[\left(N_r\cos\omega t - N_i\sin\omega t\right) + i\left(N_i\cos\omega t + N_r\sin\omega t\right)\right]. \quad (28)$$

where

$$M_r(\eta) = e^{\alpha_1\eta}\left[\left(K_8 + K_{10}\right)\cos\beta_1\eta + \left(K_{11} - K_9\right)\sin\beta_1\eta\right] + b_4 e^{m_1\eta} + b_6 e^{m_2\eta},$$

$$M_i(\eta) = e^{\alpha_1\eta}\left[\left(K_9 + K_{11}\right)\cos\beta_1\eta + \left(K_8 - K_{10}\right)\sin\beta_1\eta\right] + b_5 e^{m_1\eta} + b_7 e^{m_2\eta},$$

$$N_r(\eta) = \frac{e^{\alpha_2\eta}}{2e^{\alpha_2}\sin\beta_2}\left[\left(K_{28} + K_{30}\right)\cos\beta_2\eta + \left(K_{29} - K_{31}\right)\sin\beta_2\eta\right]$$
$$- \frac{Gr}{P_5^2 + P_6^2}e^{\alpha_1\eta}\left[P_5\left(K_8\cos\beta_1\eta - K_9\sin\beta_1\eta\right) + P_6\left(K_9\cos\beta_1\eta + K_8\sin\beta_1\eta\right)\right]$$
$$- \frac{Gr}{P_5^2 + P_7^2}e^{\alpha_1\eta}\left[P_5\left(K_{10}\cos\beta_1\eta + K_{11}\sin\beta_1\eta\right) + P_7\left(K_{11}\cos\beta_1\eta - K_{10}\sin\beta_1\eta\right)\right]$$

$$+e^{\alpha_1\eta}\left[\left(K_{12}+K_{14}\right)\cos\beta_1\eta+\left(K_{15}-K_{13}\right)\sin\beta_1\eta\right]$$

$$-\left(K_{16}+K_{20}\right)e^{m_1\eta}-\left(K_{18}+K_{22}\right)e^{m_2\eta},$$

$$N_i\left(\eta\right)=\frac{e^{\alpha_2\eta}}{2e^{\alpha_2}\sin\beta_2}\left[\left(K_{28}-K_{30}\right)\sin\beta_2\eta-\left(K_{29}+K_{31}\right)\cos\beta_2\eta\right]$$

$$-\frac{Gr}{P_5^2+P_6^2}e^{\alpha_1\eta}\left[P_5\left(K_9\cos\beta_1\eta+K_8\sin\beta_1\eta\right)-P_6\left(K_8\cos\beta_1\eta-K_9\sin\beta_1\eta\right)\right]$$

$$-\frac{Gr}{P_5^2+P_7^2}e^{\alpha_1\eta}\left[P_5\left(K_{11}\cos\beta_1\eta-K_{10}\sin\beta_1\eta\right)-P_7\left(K_{10}\cos\beta_1\eta+K_{11}\sin\beta_1\eta\right)\right]$$

$$+e^{\alpha_1\eta}\left[\left(K_{13}+K_{15}\right)\cos\beta_1\eta+\left(K_{12}-K_{14}\right)\sin\beta_1\eta\right]$$

$$-\left(K_{17}+K_{21}\right)e^{m_1\eta}-\left(K_{19}+K_{23}\right)e^{m_2\eta}. \qquad (29)$$

Since, only the real parts of the complex quantities have physical significance in the problem, the velocity and the temperature distribution can be expressed in fluctuating parts as follows:

$$\theta\left(\eta,t\right)=\theta_0\left(\eta\right)+\varepsilon\left[\left(M_r\left(\eta\right)\cos\omega t-M_i\left(\eta\right)\sin\omega t\right)\right] \qquad (30)$$

$$u\left(\eta,t\right)=u_0\left(\eta\right)+\varepsilon\left[\left(N_r\left(\eta\right)\cos\omega t-N_i\left(\eta\right)\sin\omega t\right)\right]. \qquad (31)$$

Therefore, the expressions for transient velocity and temperature field at $\omega t=\frac{\pi}{2}$ can be deduced from (30) and (31) in the following form:

$$\theta\left(\eta,\frac{\pi}{2\omega}\right)=\theta_0\left(\eta\right)-\varepsilon M_i\left(\eta\right). \qquad (32)$$

$$u\left(\eta,\frac{\pi}{2\omega}\right)=u_0\left(\eta\right)-\varepsilon N_i\left(\eta\right). \qquad (33)$$

Skin-Friction and Rate of Heat Transfer:

The skin-frictions τ_0 at the plate $\eta=0$ is:

$$\tau_0=-\left.\frac{du_0}{d\eta}\right|_{\eta=0}-\varepsilon\left.\frac{du_1}{d\eta}\right|_{\eta=0}e^{i\omega t}=Q_1+\varepsilon Q_2 e^{i\omega t}. \qquad (34)$$

The skin-frictions τ_0 in terms of amplitude (R) and phase (α) at the plate $\eta=0$ is:

$$\tau_0=-\left(\frac{du_0}{d\eta}\right)_{\eta=0}+\left|R\right|\cos\left(\omega t+\alpha\right), \qquad (35)$$

where $R=N_r+N_i$, $\left|R\right|=\sqrt{N_r^2+N_i^2}$ and $\tan\alpha=\frac{N_i}{N_r}$.

The rate of heat transfer in terms of Nusselt number (Nu) at the plate $\eta=0$ is:

$$\left(Nu\right)_0=-\left.\frac{d\theta_0}{d\eta}\right|_{\eta=0}-\varepsilon\left.\frac{d\theta_1}{d\eta}\right|_{\eta=0}e^{i\omega t}=Q_3+\varepsilon Q_4 e^{i\omega t}. \qquad (36)$$

The rate of heat transfer in terms of amplitude (S) and phase (β) at the plate $\eta=0$ is:

$$Nu_0=-\left(\frac{d\theta_0}{d\eta}\right)_{\eta=0}+\left|S\right|\cos\left(\omega t+\beta\right), \qquad (37)$$

where $S=M_r+M_i$, $\left|S\right|=\sqrt{M_r^2+M_i^2}$ and $\tan\beta=\frac{M_i}{M_r}$.

Verification of the Problem :

When the heat generation/absorption parameter (Q) is not considered in the problem, the results of the present study are exactly the same as obtained by Sharma and Mehta[14] except notations.

Results and Discussion

Hydromagnetic radiative convection

flow through porous medium confined between two semi-infinite, vertical parallel plates with periodic cross-flow velocity and internal heat generation/absorption are studied subject to transversely applied uniform magnetic field. Employing regular perturbation technique, the zeroth order and first order differential equations for temperature and velocity are obtained and expressed in (14)-(17). The corresponding boundary conditions are expressed in (18). By the use of small correction factor, $\psi(\eta)$, the equation for steady temperature, $\theta_0(\eta)$, is solved under the restriction $O(\theta_w) < \psi(\eta) < O(1)$ and is expressed in (23). The solutions of equations (14)-(17) under the boundary conditions (18) are obtained and expressed in (24)-(26). To be realistic the values of Prandtl number (*Pr*) are chosen to be $Pr = 1.0$ and $Pr = 7.0$, which correspond to electrolyte solutions and water respectively, at 20^0C and one atmospheric pressure. The values of the other parameters are chosen arbitrarily and following Sharma and Mehta[14].

Fig.1 shows the velocity distribution (*u*) versus non-dimensional η for different values of the magnetic parameter (*M*), Prandtl number (*Pr*) and the cross-flow parameter (Re_1), when $N^2 = 1$, $Gr = 2.0$, $Q = 5.0$, $\theta_w = 2.0$, $A = 1.0$, $\varepsilon = 0.01$ and $\omega t = \pi/6$. It is observed that an increase in magnetic parameter (*M*) decreases the velocity. In fact, an increase in the transverse magnetic field produces a restrictive type force (Lorentz force) similar to the drag force, which tends to resist the fluid flow and thus reduces the velocity. Also, an increase in cross-flow parameter decreases the velocity because it resists the main flow (*u*).

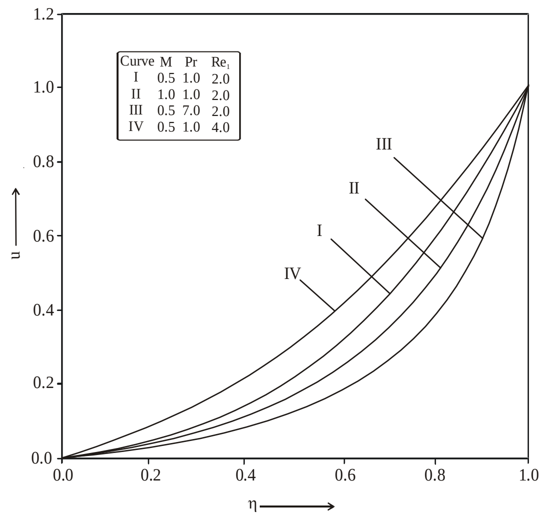

Fig. 1: Velocity distribution versus η for different values of M, Pr and Re_1.
($N^2 = 1$, Gr = 2.0, Q = 5.0, $\theta_w = 2.0$, A = 1.0, ε = 0.01 and ωt = π/6).

Physically, cross-flow parameter is associated with permeability parameter (*K*). An increase in Re_1 results in decrease in permeability parameter (*K*). As such, the bulk porous medium resistance increases, which decreases the momentum of the flow regime, thereby decrease in the velocity. Besides, an increase in Prandtl number (*Pr*) decreases the velocity. The physics behind this phenomenon is hidden in the definition of Prandtl number (*Pr*). Increase in magnitude of the Prandtl number implies decrease in the thermal conductivity of the fluid, which in turn decreases the fluid velocity.

Fig. 2 is intended to elucidate the velocity field (u) against non-dimensional η for different values of radiation parameter (N^2), convection parameter (Gr) and heat generation/ absorption parameter (Q), when $M = 0.5$, $Pr = 1.0$, $Re_1 = 2.0$, $\theta_w = 2.0$, $A = 1.0$, $\varepsilon = 0.01$ and $\omega t = \pi/6$. It is observed that an increase in radiation parameter (N^2) decreases the velocity. Physically in the energy equation, the term of radiative parameter (N^2) occurs with negative sign, which results in decreased velocity. Also, we note that an increase in convection parameter (Gr) increases the velocity. Physically, $Gr>0$ implies heat addition

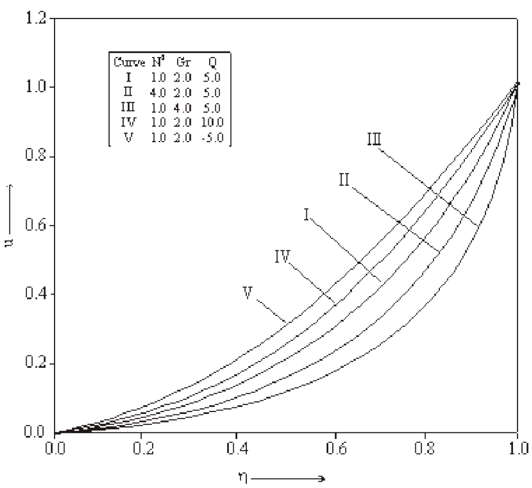

Fig. 2: Velocity distribution versus η for different values of N^2, Gr and Q.

($M = 0.5$, $Pr = 1.0$, $Re_1 = 2.0$, $\theta_w = 2.0$, $A = 1.0$, $\varepsilon = 0.01$ and $\omega t = \pi/6$).

to the surfaces, as such the convection parameter has dominant effect in escalating velocity. Besides, an increase in Q decreases the velocity. Physically, a positive values of Q imply internal heat absorption whereas the negative values of Q imply internal heat generation. Therefore, positive values of Q decreases the velocity and the negative values of Q increase the velocity. When the numerical value of Q is zero, our findings are exactly the same as obtained by Sharma and Mehta[14].

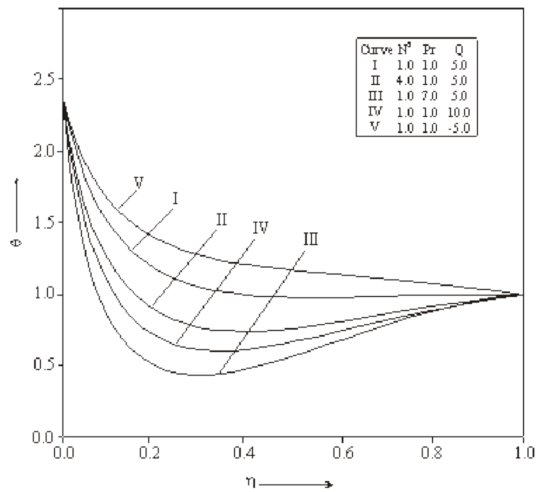

Fig.3: Temperature distribution versus η for different values of N^2, Pr and Q.

($\theta_w = 2.0$, $A = 1.0$, $\varepsilon = 0.01$ and $\omega t = \pi/6$).

Fig.3 illustrates the temperature distribution (θ) with respect to non-dimensional η for different values of radiation parameter (N^2), Prandtl number (Pr) and heat generation/ absorption parameter (Q), when $\theta_w = 2.0$, $A = 1.0$, $\varepsilon = 0.01$ and $\omega t = \pi/6$. It is observed that an increase in radiation parameter (N^2) decreases the temperature. The physics behind this phenomenon lies in the fact that increased values of imply decreased thermal conductivity of the fluid, as such increase in radiation parameter decreases the temperature. It is also observed that an increase in Prandtl number (Pr) decreases the temperature. The reason is that the smaller values of *Pr* are equivalent to increasing of the thermal conductivity. Therefore heat is able to diffuse away from the heated surface more rapidly for higher values of *Pr* compared to lower values of *Pr*. Besides, an increase in *Q* ($Q > 0$) decreases the temperature whereas decreased values of *Q* ($Q < 0$) increase the temperature because positive numerical values of *Q* imply heat absorption and negative numerical values of *Q* imply heat generation.

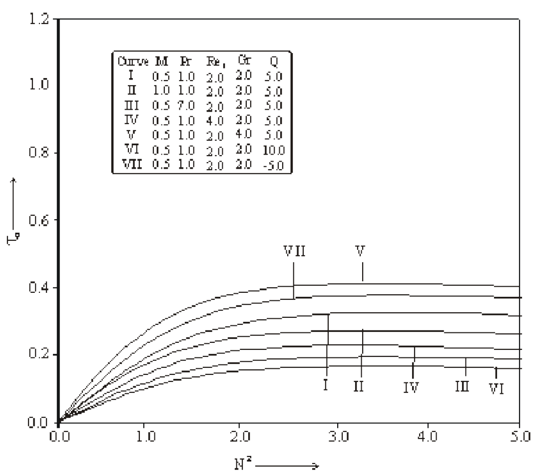

Fig.4: Skin-fraction (τ_0) at the plate $\eta= 0$ versus N^2 for different values of M, Pr, Re_1, Gr andQ. (A = 1.0, ε = 0.01 and $\omega t = \pi/6$).

Fig. 4 depicts the skin-friction (τ_0) versus radiation parameter (N^2) at the plate ($\eta = 0$) for different numerical values magnetic parameter (*M*), Prandtl number (*Pr*), cross-flow parameter (Re_1), convection parameter (*Gr*) and heat generation/absorption parameter (*Q*), when $A = 1.0$, $\varepsilon = 0.01$ and $\omega t = \pi/6$. It is observed that the skin-friction at the plate ($\eta = 0$) decreases due to increase in the Prandtl number, cross-flow parameter or convection parameter, whereas it increases due to increase in the magnetic parameter or radiation parameter, when other parameters fixed.

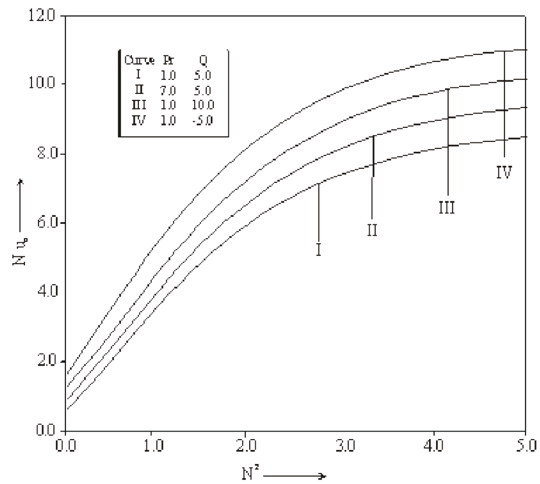

Fig.5: Heat transfer rate (Nu_0) at the plate N^2 for different values of Pr and Q.
(θ_w = 2.0, A = 1.0, ε = 0.01 and $\omega t = \pi/6$).

Fig.5 shows the rate of heat transfer in terms of Nusselt number (*Nu*) at the plate ($\eta = 0$) for different numerical values of Prandtl number (*Pr*), radiation parameter (N^2) and heat generation/absorption parameter (*Q*), when $A = 1.0$, $\varepsilon = 0.01$ and $\omega t = \pi/6$. It is observed that the rate of heat transfer at the plate ($\eta = 0$) decreases with increase in Prandtl number (*Pr*) or heat absorption parameter (*Q*>0), whereas it increases with increase in radiation parameter or heat generation parameter (*Q*<0) for fixed values of the other parameters.

Conclusions

- The fluid velocity decreases with increase in magnetic field intensity and Prandtl number but increases with increase in cross-flow parameter.

- The velocity increases with increase in heat generation parameter but decreases with increase in heat generation parameter, radiation parameter and convection parameter.

- The fluid temperature increases with increase in heat generation parameter, whereas it decreases with increase in Prandtl number, radiation parameter or heat absorption parameter.

- The skin-friction at the lower plate increases due to increase in radiation parameter or heat absorption parameter, whereas it decreases with increase in convection parameter, cross-flow parameter, magnetic parameter, Prandtl number or heat generation parameter.

- The rate of heat transfer at the lower plate increases due to increase in radiation parameter, Prandtl number or heat generation parameter, whereas decreases with increase in heat absorption parameter.

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Appendix

$$C_1 = \frac{1-\theta_w + K_1 - K_1 e^{m_2}}{e^{m_1} - e^{m_2}}, \quad C_2 = \frac{K_1 e^{m_1} + \theta_w - K_1 - 1}{e^{m_1} - e^{m_2}},$$

$$K_1 = \frac{N^2\left(\theta_w^4 - 1\right) + Q\left(\theta_w - 1\right)}{4N^2\theta_w^3 + Q},$$

$$m_1, m_2 = \frac{Pr \pm \sqrt{Pr^2 + 4\left(4N^2\theta_w^3 + Q\right)}}{2},$$

$$m_3, m_4 = \frac{1 \pm \sqrt{1 + 4\left(\mathrm{Re}_1^2 + M^2\right)}}{2}, \quad C_3 = \frac{K_2 - K_3}{e^{m_3} - e^{m_4}},$$

$$C_4 = \frac{K_4 - K_2}{e^{m_3} - e^{m_4}}, \quad K_2 = 1 + b_1 e^{m_1} + b_2 e^{m_2} + b_3,$$

$$K_3 = \left(b_1 + b_2 + b_3\right) e^{m_4}, \quad K_4 = \left(b_1 + b_2 + b_3\right) e^{m_3},$$

$$b_1 = \frac{Gr\, C_1}{m_1^2 - m_1 - \left(Re_1^2 + M^2\right)}, \quad b_2 = \frac{Gr\, C_2}{m_2^2 - m_2 - \left(Re_1^2 + M^2\right)},$$

$$b_3 = \frac{Gr\left(1 + K_1 - \theta_w\right)}{Re_1^2 + M^2}, \quad m_5, m_6 = \alpha_1 \pm \beta_1,$$

$$b_4 = \frac{AC_1 m_1 Pr K_5}{K_5^2 + K_6^2}, \quad b_5 = \frac{AC_1 m_1 Pr K_6}{K_5^2 + K_6^2}, \quad b_6 = \frac{AC_2 m_2 Pr K_7}{K_7^2 + K_6^2},$$

$$b_7 = \frac{AC_2 m_2 Pr K_6}{K_7^2 + K_6^2}, \quad K_5 = m_1^2 - Pr\, m_1 - 4N^2\theta_w^3 - Q, \quad K_6 = \omega Pr,$$

$$K_7 = m_2^2 - Pr\, m_2 - 4N^2\theta_w^3 - Q, \quad C_5 = K_8 + iK_9,$$

$$C_6 = K_{10} + iK_{11}, \quad A_1 = b_4 + b_6 - \theta_w, \quad A_2 = b_4 e^{m_1} + b_6 e^{m_2},$$

$$A_3 = e^{\alpha_1}\left(A_1 \cos\beta_1 + B_1 \sin\beta_1\right), \quad B_1 = b_5 + b_7,$$

$$B_2 = b_5 e^{m_1} + b_7 e^{m_2}, \quad B_3 = e^{\alpha_1}\left(B_1 \cos\beta_1 - A_1 \sin\beta_1\right),$$

$$K_8 = \frac{B_3 - B_2}{2e^{\alpha_1}\sin\beta_1}, \quad K_9 = \frac{A_2 - A_3}{2e^{\alpha_1}\sin\beta_1},$$

$$A_4 = e^{\alpha_1}\left(A_1 \cos\beta_1 - B_1 \sin\beta_1\right), \quad K_{10} = \frac{B_2 - B_4}{2e^{\alpha_1}\sin\beta_1},$$

$$B_4 = e^{\alpha_1}\left(B_1\cos\beta_1 + A_1\sin\beta_1\right),\quad K_{11} = \frac{A_4 - A_2}{2e^{\alpha_1}\sin\beta_1},$$

$$K_{12} = \frac{AP_1 m_3 C_3}{P_1^2 + \omega^2},\quad K_{13} = \frac{A\omega m_3 C_3}{P_1^2 + \omega^2},\quad K_{14} = \frac{AP_2 m_4 C_4}{P_2^2 + \omega^2},$$

$$P_1 = m_3^2 - m_3 - \left(Re_1^2 + M^2\right),\qquad K_{15} = \frac{A\omega m_4 C_4}{P_2^2 + \omega^2},$$

$$P_2 = m_4^2 - m_3 - \left(Re_1^2 + M^2\right),\qquad K_{16} = \frac{AP_3 m_1 b_1}{P_3^2 + \omega^2},$$

$$P_3 = m_1^2 - m_1 - \left(Re_1^2 + M^2\right),\qquad K_{17} = \frac{A\omega m_1 b_1}{P_3^2 + \omega^2},$$

$$K_{18} = \frac{AP_4 m_2 b_2}{P_4^2 + \omega^2},\quad K_{19} = \frac{A\omega m_2 b_2}{P_4^2 + \omega^2},\quad K_{20} = \frac{Gr\left(P_8 b_4 - \omega b_5\right)}{P_8^2 + \omega^2},$$

$$P_4 = m_2^2 - m_2 - \left(Re_1^2 + M^2\right),\quad K_{21} = \frac{Gr\left(\omega b_5 + P_8 b_5\right)}{P_8^2 + \omega^2},$$

$$K_{22} = \frac{Gr\left(P_9 b_6 - \omega b_7\right)}{P_9^2 + \omega^2},\quad K_{23} = \frac{Gr\left(\omega b_6 + P_9 b_7\right)}{P_9^2 + \omega^2},$$

$$P_5 = \alpha_1^2 - \beta_1^2 - \alpha_1 - Re_1^2 - M^2,\quad P_6 = 2\alpha_1\beta_1 - \beta_1 - \omega,$$

$$P_7 = \beta_1 - 2\alpha_1\beta_1 - \omega,\qquad P_8 = m_1^2 - m_1 - Re_1^2 - M^2,$$

$$P_9 = m_2^2 - m_2 - Re_1^2 - M^2,\qquad C_7 = \frac{K_{28} - iK_{29}}{2e^{\alpha_2}\sin\beta_2},$$

$$C_8 = \frac{K_{30} - iK_{31}}{2e^{\alpha_2}\sin\beta_2},\qquad m_7, m_8 = \alpha_2 + i\beta_2,$$

$$Q_1 = m_1 b_1 + m_2 b_2 - m_3 C_3 - m_4 C_4,$$

$$Q_2 = m_1\left[\left(K_{16} + K_{20}\right) + i\left(K_{17} + K_{21}\right)\right] + m_2\left[\left(K_{28} + K_{22}\right) + i\left(K_{19} + K_{23}\right)\right]$$
$$- m_7 C_7 - m_8 C_8 - m_3\left(K_{12} + iK_{13}\right) - m_4\left(K_{14} + iK_{15}\right)$$
$$- \frac{Gr}{P_5^2 + P_6^2}\left[\left(\alpha_1 + i\beta_1\right)\left(P_5 K_8 + P_6 K_9\right) + \left(\beta_1 + i\alpha_1\right)\left(P_5 K_9 - P_6 K_8\right)\right]$$
$$- \frac{Gr}{P_5^2 + P_7^2}\left[\left(\alpha_1 - i\beta_1\right)\left(P_5 K_{10} + P_7 K_{11}\right) + \left(\beta_1 + i\alpha_1\right)\left(P_7 K_{10} - P_5 K_{11}\right)\right],$$

$$Q_3 = -m_1 C_1 - m_2 C_2,$$

$$Q_4 = -\left[m_5 C_5 + m_6 C_6 + m_1\left(b_4 + ib_5\right) + m_2\left(b_6 + ib_7\right)\right],$$

$$K_{24} = K_{22} + K_{20} + K_{18} + K_{16} - K_{14}$$
$$- K_{12} + \frac{Gr\left(P_5 K_8 + P_6 K_9\right)}{P_5^2 + P_6^2} + \frac{Gr\left(P_5 K_{10} + P_7 K_{11}\right)}{P_5^2 + P_7^2},$$

$$K_{25} = K_{23} + K_{21} + K_{19} + K_{17} - K_{15}$$
$$- K_{13} + \frac{Gr\left(P_5 K_9 - P_6 K_8\right)}{P_5^2 + P_6^2} + \frac{Gr\left(P_5 K_{11} - P_7 K_{10}\right)}{P_5^2 + P_7^2},$$

$$\alpha_1 = \frac{Pr}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{\left(Pr^2 + 16N^2\theta_w^3 + 4Q\right)^2 + 16\omega^2 Pr^2} + \left(Pr^2 + 16N^2\theta_w^3 + 4Q\right)\right]^{1/2}$$

$$\beta_1 = \frac{1}{2\sqrt{2}}\left[\sqrt{\left(Pr^2 + 16N^2\theta_w^3 + 4Q\right)^2 + 16\omega^2 Pr^2} - \left(Pr^2 + 16N^2\theta_w^3 + 4Q\right)\right]^{1/2}$$

$$\alpha_2 = \frac{1}{2} + \frac{1}{2\sqrt{2}}\left[\sqrt{\left(1 + 4Re_1^2 + 4M^2\right) + 16\omega^2} + \left(1 + 4Re_1^2 + 4M^2\right)\right]^{1/2}$$

$$\beta_2 = \frac{1}{2\sqrt{2}}\left[\sqrt{\left(1 + 4Re_1^2 + 4M^2\right) + 16\omega^2} - \left(1 + 4Re_1^2 + 4M^2\right)\right]^{1/2}$$

$$K_{26} = 1 - K_{12}e^{m_3} - K_{14}e^{m_4} + \left(K_{16} + K_{20}\right)e^{m_1} + \left(K_{18} + K_{22}\right)e^{m_3}$$

$$+ \frac{Gr}{P_5^2 + P_6^2} e^{\alpha_1}\left[P_5\left(K_8 \cos\beta_1 - K_9 \sin\beta_1\right) + P_6\left(K_9 \cos\beta_1 + K_8 \sin\beta_1\right)\right]$$

$$+ \frac{Gr}{P_5^2 + P_7^2} e^{\alpha_1}\left[P_5\left(K_{10} \cos\beta_1 - K_{11} \sin\beta_1\right) + P_7\left(K_{11} \cos\beta_1 + K_{10} \sin\beta_1\right)\right]$$

$$K_{27} = \left(K_{17} + K_{21}\right)e^{m_1} + \left(K_{19} + K_{23}\right)e^{m_2} - K_{13}e^{m_3} - K_{15}e^{m_4}$$

$$+ \frac{Gr}{P_5^2 + P_6^2} e^{\alpha_1}\left[P_5\left(K_9 \cos\beta_1 + K_8 \sin\beta_1\right) - P_6\left(K_8 \cos\beta_1 - K_9 \sin\beta_1\right)\right]$$

$$+ \frac{Gr}{P_5^2 + P_7^2} e^{\alpha_1}\left[P_5\left(K_{11} \cos\beta_1 - K_{10} \sin\beta_1\right) + P_7\left(K_{10} \cos\beta_1 - K_{11} \sin\beta_1\right)\right]$$

$$K_{28} = K_{27} - e^{\alpha_2}\left(K_{25} \cos\beta_2 - K_{24} \sin\beta_2\right),$$

$$K_{29} = K_{26} - e^{\alpha_2}\left(K_{24} \cos\beta_2 + K_{25} \sin\beta_2\right),$$

$$K_{30} = e^{\alpha_2}\left(K_{25} \cos\beta_2 + K_{24} \sin\beta_2\right) - K_{27},$$

$$K_{31} = e^{\alpha_2}\left(K_{24} \cos\beta_2 - K_{25} \sin\beta_2\right) - K_{26}.$$