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LRS Bianchi Type –III Massive String Cosmological Model With Electromagnetic Field

VARUN HUMAD[1], SAPNA SHRIMALI[2] and GAJENDRA PAL SINGH[3]

[1]Research Scholar, Mewar University, Chittorgarh (Rajasthan), (India)
[2]Department of Mathematics, Pacific Academy of Higher Education & Research University,Udaipur (Rajasthan), (India)
[3]Department of Mathematics, Geetanjali Institute of Technical Studies, Udaipur (Rajasthan) (India)

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Abstract

We have investigated LRS Bianchi Type -III massive string cosmological model with electromagnetic field. For the complete determination of the model, we assume a relation between the rest energy density (ρ) and the string tension density (λ) as ρ = λ. Some physical and geometrical features of the model are also discussed.

Key words: Bianchi Type-III, Massive String, Electromagnetic Field, Cosmology.

1. Introduction

It is still a challenging problem before us to know the exact physical situation at very early stages of the formation of our universe. The string theory is a useful concept before the creation of the particle in the universe. The string are nothing but the important topological stable defects due to the phase transition that occurs as the temperature lower below some critical temperature at the very early stages of the universe. The present day configuration of the universe are not contradicted by the large scale network of strings in the early universe. Moreover, the galaxy formation can be explained by the density fluctuations of the vacuum strings.

The general relativistic treatment of strings was obtained by Letelier[1,2] and Stachel[3]. Letelier[1] has obtained the solution to Einstein's field equations for a cloud of strings with spherical, plane and cylindrical symmetry. Then, in 1983, he solved Einstein's field equations for a cloud of massive strings and obtained cosmological models in Bianchi Type I and Kantowski-Sachs space-times. Bali *et al*.[4-8] have obtained Bianchi type IX, type V and type I string cosmological models in general relativity. Exact solutions of string cosmology for Bianchi type II, VI_0, VIII and IX space-times have been obtained by Krori *et al.*[9] and Wang[10]. Singh *et al.*[11] investigated Bianchi type III cosmological models with gravitational constant G and the cosmological constant Λ. Recently Kaluza-Klein cosmological solutions are obtained by Yilmaz[12] for quark matter coupled to the string cloud in the context of general relativity. Yavuz[13] have obtained charged strange quark matter attached to the string cloud in the spherical symmetric space-time admitting one-parameter group of conformal motion. Tikekar and patel[14] investigated some exact solutions of massive string of Bianchi type III space time in presence of magnetic field. They have also discussed some string solutions of Bianchi typt III space time in absence of magnetic field. Recently Bali and Pradhan[15] have obtained a formalism for studying the new integrability of Bianchi type III massive strings cosmological models in general relativity. Recently Rathore *et al.*[16] investigated Bianchi type III string cosmological models with Bulk viscosity and Electromagnetic field. Singh and Tyagi[17-19] investigated various Bianchi Type cosmological models with variable cosmological and gravitational constant in presence and absence of magnetic field. The perfect and bulk viscous fluids are considered as source of matter.

Motivated by aforesaid, We have investigated LRS Bianchi Type-III massive string cosmological model with electromagnetic field. For the complete determination of the model, we assume a relation between the rest energy density (ρ) and the string tension density (λ) as ρ = λ. Some physical and geometrical features of the model are also discussed.

2. Metric and Field Equation :

We consider an LRS Bianchi Type - III metric of the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2(e^{2x}dy^2 + dz^2) \quad (1)$$

Where A and B are function of time t. The energy momentum tensor (T_i^j) for a cloud of massive string is given by Letelier[1] in the presence of electromagnetic field is taken in the form

$$T_i^j = \rho v_i v^j - \lambda x_i x^j + E_i^j \quad (2)$$

With $\rho = \rho_p + \lambda$ (3)

Here ρ is the proper energy density for a cloud of strings with particles attached to them, ρ_p is the density of the particles, λ is the string tension density, v^i the four velocity of the particles and x^i is a unit space-like vector representing the direction of string satisfying

$$v_i v^i = -x_i x^i = -1, v^i x_i = 0$$
$$x_1 \neq 0, x_2 = x_3 = x_4 \quad (4)$$

In a comoving coordinate system, we have

$$v^i = (0,0,0,1) \text{ and } x^i = \left(\frac{1}{A}, 0,0,0\right) \quad (5)$$

The electromagnetic field E_i^j is given by Lichnerowicz[20] as

$$E_i^j = \bar{\mu}\left[|h|^2\left(v_i v^j + \frac{1}{2} g_i^j\right) - h_i h^j\right] \quad (6)$$

With

$$h_i = \frac{\sqrt{-g}}{2\bar{\mu}} \in_{ijkl} F^{kl} v^j \quad (7)$$

Where h_i is the magnetic flux vector, $\in_{ijkl}$ the Levi-Civita tensor, F^{kl} the electromagnetic field tensor, $\bar{\mu}$ the magnetic permeability and $|h|^2 = h_l h^l$, g_{ij} the matric tensor.

We assume that magnetic field is due to an electric current produced along z- axis, so that

$$h_1 = h_2 = h_4 = 0 \text{ and } h_3 \neq 0 \quad (8)$$

Thus F_{12} is the only non-vanishing component of electromagnetic field tensor F_{ij}. This leads to $F_{13} = F_{23} = 0$ by virtue of (7)

Then the set of Maxwell's equations

$$F_{ij;k} + F_{jk;i} + F_{ki;j} = 0 \text{ and } F^{ij}_{;j} = 0 \quad (9)$$

are satisfied by

$$F_{12} = constant = H \quad (10)$$

We also find that $F_{14} = F_{24} = F_{34} = 0$ due to the assumption of infinite electrical conductivity[21]

The non-vanishing components of h_i is given as

$$h_3 = \frac{H}{\bar{\mu} A} \quad (11)$$

and $|h|^2 = h_l h^l = h_3 h^3 = g^{33}(h_3)^2 = \frac{H^2}{\bar{\mu}^2 A^2 B^2}$ (12)

Now, from equation (6) and (12), we have

$$E_1^1 = E_2^2 = -E_3^3 = -E_4^4 = \frac{H^2}{2\bar{\mu} A^2 B^2} \text{ and}$$

$$E_i^j = 0 \text{ for } i \neq j \quad (13)$$

From (2) and (13), we get

$$T_1^1 = T_2^2 = \frac{H^2}{2\bar{\mu} A^2 B^2} \quad (14)$$

$$T_3^3 = -\lambda - \frac{H^2}{2\bar{\mu} A^2 B^2} \quad (15)$$

$$T_4^4 = -\rho - \frac{H^2}{2\bar{\mu} A^2 B^2} \quad (16)$$

$$T_i^j = 0 \text{ for } i \neq j \quad (17)$$

The Einstein's field equation (in gravitational units c = 1, 8πG = 1) for a system of string

$$R_i^j - \frac{1}{2} R g_i^j = -T_i^j \quad (18)$$

For the metric (1) the Einstein's field equations (18) takes the form

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = -\frac{H^2}{2\bar{\mu} A^2 B^2} \quad (19)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = -\frac{H^2}{2\bar{\mu} A^2 B^2} \quad (20)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = \lambda + \frac{H^2}{2\bar{\mu} A^2 B^2} \quad (21)$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = \rho + \frac{H^2}{2\bar{\mu} A^2 B^2} \quad (22)$$

and $\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0$ (23)

Where an over dot stands for the first and double dot for the second derivative with respect to cosmic time t.

From equation (23), we have

$$A = m B \quad (24)$$

Where m is the constant of integration without loss of generality we can take m = 1, equation (24) leads to

$$A = B \quad (25)$$

For the metric (1), the expression for scalar of expansion θ and shear scalar σ are

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{2\dot{B}}{B} \quad (26)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left(\frac{\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2} - \frac{2\dot{A}\dot{B}}{AB}\right) \quad (27)$$

3. Solution of the Field Equations :

The field equations (19) to (22) are four equations in four unknowns A, B, λ and ρ. In order to obtain a determinate solution, we assume a relation between the rest energy density (ρ) and the string tension density (λ),

$$\rho = \lambda \quad (28)$$

The equation (28), (25), (21) and (22) together leads to

$$\frac{\ddot{A}}{\dot{A}} = \frac{\dot{A}}{A} \quad (29)$$

Equation (29) leads to

$$A = le^{kt} \quad (30)$$

Where *l* and *k* are constant of integration.

Equatio (25) and (30) leads to

$$B = le^{kt} \quad (31)$$

Hence the metric (1) takes the form

$$ds^2 = -dt^2 + l^2e^{2kt}(dx^2 + e^{2x}dy^2 + dz^2) \quad (32)$$

After using a suitable transformation of coordinates the model (32) reduce to

$$dS^2 = -\frac{dT^2}{4k^2} + l^2e^T(dX^2 + e^{2X}dY^2 + dZ^2) \quad (33)$$

Where 2kt, x = X, y = Y, z = Z

4. Physical and Geometrical aspects of the model in presence of Electromagnetic Field:

For the model of equation (33), the other physical and geometrical parameters can be easily obtained. The rest energy density (ρ), the string tension density (λ), the scalar of expansion (θ), the shear scalar (σ) and deceleration parameter (q) are respectively given by

$$\rho = 3k^2 - \frac{1}{l^2e^T} - \frac{H^2}{2\bar{\mu}l^4e^{2T}} \quad (34)$$

$$\lambda = 3k^2 - \frac{1}{l^2 e^T} - \frac{H^2}{2\bar{\mu} l^4 e^{2T}} \qquad (35)$$

$$\theta = 3k \qquad (36)$$

$$\sigma = 0 \qquad (37)$$

$$\frac{\sigma}{\theta} = 0 \qquad (38)$$

$$q = -1 < 0 \qquad (39)$$

5. Physical and Geometrical aspects of the model in absence of Electromagnetic Field:

$$\rho = 3k^2 - \frac{1}{l^2 e^T} \qquad (40)$$

$$\lambda = 3k^2 - \frac{1}{l^2 e^T} \qquad (41)$$

$$\theta = 3k \qquad (42)$$

$$\sigma = 0 \qquad (43)$$

$$\frac{\sigma}{\theta} = 0 \qquad (44)$$

$$q = -1 < 0 \qquad (45)$$

6. Conclusions

In presence and absence of electromagnetic field, the rest energy density (ρ) and the string tension density (λ) increse as time T increase. Also ρ and λ are constant, as $T \to \infty$. Further expansion of the model (33) is constant througout evolution of the universe. Since $\frac{\sigma}{\theta} = 0$, hence the mode (33) represent an isotropic universe. The deceleration perameter $q < 0$ for the model (33), hence the model represent an accelerated universe.

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