Ultra Scientist Vol. 26(1)B, 13-18 (2014).

Theory of Expanding Universe & F-R Space of Constant Curvature

R. K MISHRA[1], ARUNESH PANDEY[2]

[1,2]Department of Mathematics, SLIET Deemed University
Longowal-148106, Sangrur Punjab (INDIA)
Email[1]: ravkmishra@yahoo.co.in
Email[2]: ankpandey11@rediffmail.com

(Acceptance Date 10th February, 2014)

Abstract

Recently Many researchers has given attention on the study of the expanding nature of the universe with variable fundamental constants. It is also believed that the inclusion of vacuum energy term can greatly affect the cosmological theories. The basic purpose of this investigation is to highlight the hidden connection between Finsler Geometry and Riemannian Geometry along with theory of expanding universe.

1-Introduction

In 1929, Edwin Hubble working at the Carnegie observatories in Pasadena, California, measured the red shift of a number of distant galaxies; he also measured their relative distances by measuring the apparent brightness of variable stars called cepherds in each galaxy. When he plotted red shift against relative distance he found that the red shift of distant galaxies increased as a linear function of their distance. The only explanation for this observational experiment is that **the universe was expanding.** Hubble pointed out that the galaxies are receding from us. The speed v of the recession of the galaxies is proportional to its distance 'D' from us. In this way Hubble proposed his famous law as:

$$v = HD \tag{1}$$

where H is the Hubble's constant. The constant 'H' is one of the important parameter in all cosmological theories. Several ways[1-6] have been proposed to measure the Hubble's constant H.

2-Theory of Expanding Universe:

The expanding universe is finite in both time & space. The reason that the universe didn't collapse as Newton's and Einstein's equations said it might, is that it had been expanding from the moment of its creation. The expanding universe is a new idea of modern cosmology. The three possible type of expanding universe are called open, flat and closed universe. If the universe were open, it would expand forever. If the universe were flat it would also expand forever but the expansion rate would be slow to zero after an infinite amount of time. If the universe were closed, it would eventually stop expanding and recollapse on itself, possibly leading to another big bang. In all the above three cases, the expansion and the force that causes the slowing rate is gravity.

3-Relativistic Formulation & Hubble's Law:

Following the above equation and as discuss in the papers[1-2], let us consider a fundamental particle at the origin $r = 0$ and another particle at 'r' then the proper distance 'D' between the two particles at a time 't' is given

$$D = R(t)\int_0^r \frac{dr}{\sqrt{1-kr^2}} = \begin{cases} R\sinh^{-1} r & (k=-1) \\ Rr & (k=0) \\ R\sin^{-1} r & (k=1) \end{cases} \quad (2)$$

Therefore, the proper distance 'D' is proportional to the scale '$R(t)$'. The proper velocity 'v' of the particle at 'r' relative to the particle at the origin is obtained by differentiating 'D' w. r. t. 't' realizing that 'r' remains constant because it is a co-moving coordinate. Thus, we have

$$v = \dot{D} = \dot{R}\int_0^r \frac{dr}{\sqrt{1-kr^2}} = \frac{\dot{R}}{R}D \quad (3)$$

where a dot on the symbol means differentiation with respect to 't'. This tells us that at any time 't' the speed 'v' is proportional to the proper distance 'D'. Comparing this result with the Hubble's law (2),

we get, $$H = \frac{\dot{R}}{R} \quad (4)$$

In fact, we cannot stretch the measuring tape between the particles, therefore, the proper distance 'D' is not a measurable quantity. However, the great advantage of the relativistic formulation is that, it gives relationship between quantities such as red-shifts, apparent magnitudes, number counts etc. which can be measured. The important point is that, they all involve only the scale factor $R(t)$. In a similar way $\dot{R}$ & $\ddot{R}$ are treated as measure of the velocity and the acceleration of the fundamental particles related to the origin respectively.

4. Variable Cosmological Constant :

The cosmological constant is an extra term in the Einstein equation of General theory of Relativity which physically represents the possibility that there is a density and pressure associated with 'empty' space. It is denoted by 'Λ' (Lambda). The inclusion of this vacuum energy term can greatly affect the cosmological theories. Here as pointed out by Lindey that the spontaneous symmetry breaking result is a change in the temperature of the medium which decreases with the increase of the energy and vanishes at a certain critical temperature. This implies a dependence of the cosmological constant on temperature and hence in an expanding hot big bang model. This implies a dependence of the cosmological constant on time as may be seen in many models including Weinberg-Salam model[3]. The General theory of Relativity has two distinct kind of correspondence with the Special theory of Relativity. The first is the limit of vanishing gravitational field locally. It is the demand of the equivalence principle.

According to which the metric tensor g_{ij} of a gravitational field satisfies the conditions that

$$g_{ij} = \eta_{ij} \tag{5}$$

and $\frac{\partial g_{ij}}{\partial x^k} = 0$, $(i, j, k = 1,2,3,4)$ in a local inertial frame. (6)

Therefore the L.H.S. of the Einstein field equation is:

$$R_{ij} - \frac{1}{2} g_{ij} = -\frac{8\pi G}{c^4} T_{ij}, \tag{7}$$

Here $(i, j = 1,2,3,4)$ do not vanish in the local inertial frame. The result obtained from the equation may be physically interpreted in a local inertial frame. In fact the space-time of a local inertial frame is not a flat space time, unless

$$\frac{\partial^2 g_{ij}}{\partial x^k \partial x^n} = 0 \quad (i, j, k, n = 1,2,3,4) \tag{8}$$

Therefore the geometrical object characterizing the gravity does not vanish in a local inertial frame. If the Einstein tensor vanishes in a local inertial frame, it must vanish everywhere and in every coordinate system. This is the case of second kind of Special Relativity limit of the General theory of Relativity.

Cosmological Term Λg_{ij} In The Einstein Field Equation.

If we introduce the cosmological term in the Einstein field equation. *i.e.*

$$R_{ij} - \frac{1}{2} R g_{ij} + \Lambda g_{ij} = -\left(\frac{8\pi G}{c^4}\right) T_{ij} \tag{9}$$

$(i, j, k, n = 1,2,3,4)$ Since the space-time under consideration is flat, the quantity $R_{ij} - \frac{1}{2} R g_{ij}$ vanish identically in the space-time. On substituting the values from the equation

$$T_{ij} = \left(\frac{p}{c^2} + \rho\right) u^i u^j - p g_{ij} \tag{10}$$

into (9), and after simplification we will get:

$$\Lambda = -\frac{8\pi G}{c^4}\left[\left(\frac{p}{c^2} + \rho\right) u^i u^j - p\right],$$
$$(i, j = 1,2,3; \quad i \neq j) \tag{11}$$

Here we may write $u^i = 0 \ (i = 1,2,3)$,

which in combination with the equation

$$u^i u_i = c^2, \ (i = 1,2,3,4) \tag{13}$$

obtained from the metric of the space-time, we get $u_4 = 0$ (14)

From above equations we may write

$$p = \frac{\Lambda c^4}{8\pi G}, \ \rho = -\frac{\Lambda c^2}{8\pi G} \tag{15}$$

which shows that the density and pressure of perfect fluid are of opposite sign and hence, the result is either physically in admissible or trivial. In an attempt to construct a model of the universe Guth[3] has suggested that the early universe had gone through a period of rapid expansion. For increasing rate of expansion one need anti gravitational force in the model, this may be produced by considering the cosmological term Λg_{ij} in to the Einstein field equations. Many cosmological models have been constructed with suitable assumptions of variable cosmological constants along with other remarkable results[4-9].

Finsler Metric & Riemannian Metric:

Finsler metric on a manifold is a family of minkowski norms on the tangent spaces, and as we know that a Minkowski norms on a vector space V is non-negative function $F: V \to [0, \omega)$ with the following properties:

- F is positively y–homogeneous of degree one i.e. $F(ky) = kF(y), \forall\, y \epsilon V \;\&\; any\ k > 0$.
- F is C^{ω} on and for any tangent vector.

Let <M,F> be a Finsler manifold. Let $< x^i, y^i >$ be standard local coordinate system in TM. It means's y^i's may be determined by

$y = y^i \left(\partial/_{\partial x^i}\right)_{|x}$ Let $g_{ij}(x, y) = \frac{1}{2}[F^2]y^i y^j(x, y)$ then the induced inner product g_y is given by

$g_y(u, v) = g_{ij}(x, y)u^i v^j$ Where $u = u^i \left(\partial/_{\partial x^i}\right)_{|x}$ & $v = v^i \left(\partial/_{\partial x^i}\right)_{|x}$

By the homogeneity of F

$$F(x, y) = \sqrt{g_y(y, y)} \tag{20}$$

$$F(x, y) = \sqrt{g_{ij}(x, y)y^i y^j} \tag{21}$$

Therefore A Finsler metric $F = F(x, y)$ is called a Riemannian metric if $g_{ij} = g_{ij}(x)$ are function of $x \epsilon M$ only.

There are three special Riemannian Metrices:

- Euclidean metric
- Spherical metric
- Hyperbolic metric

Non-Riemannian Finsler Metrices are Funk metric & Stern metric or generalized metric

Finsler's Geometry& Reimannian Geometry:

Recently Many researchers has given attention on the study of Finsler's Geometry & as a result interesting development was noticed in the area of Finsler geometry & Riemannian geometry in recent years. If we wish to relate both the geometries then we may say that:

- The modern differential geometry provides the tools to effect a treatment of Riemannian geometry, without the quadratic restriction,
- This may provide a better understanding of the geometry & also may open a new developments of algebraic geometry from quadrics to general algebraic varieties.

Following connections may be observed after investigating both the subject in detail

- The fundamental problem in local Finsler geometry is the equivalence problem so to decide when two Finsler metrics differ by a coordinate transformation but in the Riemannian case this problem was solved by E. B. Christoffel and R. Lipschitz in 1870. In his solution Christoffel introduced the requisite covariant differentiation, which Ricci developed into his tensor analysis, making it a fundamental tool in classical differential geometry.

- The key idea in Finsler geometry is to consider the bundle of line elements of the manifold *M*. The reason is that all geometric quantities constructed from *F* are homogeneous of degree zero in *y* and naturally live on bundle of line elements,

To describe one such quantity,

let $x_i, 1 \leq i <$ in, be local coordinates on *M*.

Express tangent vectors as $y = y_i \frac{\partial}{\partial x_i}$

The function $F(x^1, x^2 \ldots\ldots x^n, y^1, y^2, \ldots\ldots\ldots y^n)$ is linearly homogeneous in the *y*'s.

The fundamental tensor g_{ij} is defined as the *y*-Hessian $(1/2\ F^2)y^i\ y^j$. But in case F^2 is quadratic, then g_{ij} reduce to the usual $g_{ij}(x)$'s of the equation:

$$F^2 = g_{ij}(x)\, dx^i\, dx^j \qquad (22)$$

which then also live on *M*.

In case F^2 is quadratic, these g_{ij} simply reduce to the usual $g_{ij}(x)$'s of the above equation, which then also live on *M*. so the most fundamental invariant should be the Ricci scalar. It is a scalar function on line elements and is defined as : $g_{ij}(l^k R_{kijl}\ l^l)$ its companion[10] is the Ricci tensor $R_{ij} = (1/2\ F^2 R) y^i\ y^j$ (23)

Concluding Remarks

As discussed in this paper that there is a need to investigate the all the possible hidden connections between finsler geometry and reimannian geometry it is our believe that many important result of Reimanian geometry may be improved with the help of Finsler geometry. As we know that an anisotropic direction-dependent expansion of the universe may be present if its geometry is anisotropic. We have already presented the study of the anisotropic gravitational field which is emerged by a Finslerian structure of space-time. The principle of equivalence of Riemannian general relativity may be set in the framework of a generalized metrically relativistic field theory. The anisotropy vector of the microwave background radiation is present in the geodesics. As a consequence, it creates a rotation in the form of the geodesics, which is caused by the direction of a Finslerian anisotropic universe. A correlation of these spaces with perfect fluids gives rise to the form of an anisotropic gravitational influence of matter. The scale factor in a Finslerian anisotropic model, can provide a useful tool in the study of a Finslerian-perturbed Robertson-Walker metric. Finally, the cosmological constant problem of general relativity may be considered and extended for an F-R space of constant curvature[1-10].

References

1. Isham, C. J. Salam, A. and Strathdee, J.

Nature-Phys Sci., 244, pp. 82-84 (1973).
2. Pande H.D, Chandra R.C. Mishra R.K, *Indian Journal of pure. appl. Math; 31(2)*: 161-175, February (2000).
3. Guth, A. H., *Phys. Rev.*, D23, pp. 347-356 (1981).
4. Mishra R.K, Jr. of Mathematics education India, Vol. *38*, no.4, December (2004).
5. Pande H.D, Chandra R.C and Mishra R. K., *J. Nat. Acad. Math.* Vol. *11*, pp. 118-127 (1999).
6. R.K.Mishra and Chanchal Chawla. *International Journal of Theoretical physics* (IJTP) Vol.52, Number 4, (2013).
7. Chanchal Chawla, Mishra R.K, Pradhan Anirudh, *Eur. Phys. J. Plus* 127-137 (2012).
8. Chawla, Chanchal, Mishra R.K, *Romanian Journal of Physics* Vol. *58*, Number 1-2, (2013).
9. Arunesh Kumar Pandey, R. K. Mishra, Anirudh Pradhan, *ARPN Journal of Sc. & Tech.* Vol 3, No.9 Sep (2013).
10. Generalized Einstein manifolds, *J. Geom. Phys. 17*, 342–380 (1995).