Ultra Scientist Vol. 26(1)B, 19-28 (2014).

Pulsatile Flow of Blood through a Stenosed Artery with Body Acceleration

[1]VARUN KUMAR TANWAR and [2]N. K. VARSHNEY

[1, 2]Department of Mathematics, S. V. College, Aligarh (INDIA)

Postal Address: U-655, Jalvayu Vihar, Builders Area, Sector-p4, Greater Noida, Pin Code-201308, Email-varuniit2000@gmail.com

(Acceptance Date 7th February, 2014)

Abstract

A mathematical model has been developed for the studying of pulsatile flow of blood through a stenosed artery with periodic body acceleration. Herschel – Bulkley fluid has been taken to represent the non-Newtonian character of blood. Perturbation method is used to solve the resulting system of non-linear partial differential equation with appropriate boundary condition to analyze the flow. Analytical expressions are obtained for axial velocity, volumetric flow rate and wall shear stress. Their variations with different flow parameters are plotted in figures. It has been observed that the axial velocity increases as body acceleration increases but the volumetric flow rate increases with the increase of body acceleration with time.

Key words: Pulsatile flow, Body acceleration, Herschel-Bulkley fluid, stenosed artery, slip velocity.

I. Introduction

One of the most serious consequences of the stenosis in the arteries is the increased resistance and the associated reduction of the blood flow to the particular vascular bad supplied by the artery. Due to the presence of stenosis in an artery, normal flow of blood is disturbed appreciably. It is important to analyze the effect of periodic body acceleration on different part of body. Prolonged exposure of a healthy human body to external acceleration may cause serious health problem like headache, loss of vision, abdominal pain, and increase pulse rate. Due to physiological importance of body acceleration, many researchers have been proposed for blood flow with body acceleration. Agarwal and varshney[21] considered MHD pulsatile flow of couple stress fluid through an inclined circular tube with periodic body acceleration. Biswas and Chakraborty[20] considered pulsatile blood flow through through a catheterized artery with an axially non symmetrical stenosis. Biswas and Laskar[22] investigated study flow of blood through a stenosed artery: A non-Newtonian fluid model. Chaturani and Palanisami[6] investigated pulsatile flow of power law fluid model for blood flow under periodic body acceleration. Chaturani and Palanisamy[7] considered pulsatile flow of blood with periodic body acceleration. Chaturani and Samy[5] studied pulsatile flow of Casson's fluid through stenosed arteries with applications to blood flow. Chien[4] discussed hemorheology in clinical medicine, Recent Advances in Cardio-vascular Diseases. El-Shehed[11] considered pulsatile flow of blood through a stenosed porous medium under periodic body acceleration. Elshehawey *et.al.*[9] discussed pulsatile flow of blood through a porous medium under periodic body acceleration. Mandal *et.al.*[16] considered effect of body acceleration on unsteady pulsatile flow of non-Newtonian fluid through a stenosed artery. Maruthi Prasad and Radhakrishnamacharya[15] studied effect of multiple stenoses on Herschel-Bulkley fluid through a tube with non-uniform cross-section. Maruthi Prasad and Radhakrishnamacharya[17] studied flow of Herschel-Bulkley fluid through an inclined tube of non-uniform cross-section with multiple stenoses. Merrill *et.al.*[1] considered pressure flow relations of human blood in hollow fibre at low shear rates. Nagarani and Sarojamma[18] studied effect of body acceleration on pulsatile flow of Casson fluid through a mild stenosed artery. Sankar and Hemalatha[13] investigated pulsatile flow of Herschel-Bulkey fluid through stenosed arteries a mathematical model. Sarojamma and Nagarani[10] considered pulsatile flow of Casson fluid in a homogeneous porous medium subject to external acceleration. Shukla *et.al.*[3] investigated effects of stenosis on non-Newtonian flow through an artery with mild stenosis. Siddiqui and Mishra[14] studied a study of modified Casson's fluid in modeled normal and stenotic capillary–tissue diffusion phenomena. Siddiqui *et.al.*[19] considered Mathematical modelling of pulsatile flow of Casson's fluid in arterial stenosis. Tu and Deville[8] discussed pulsatile flow of non-Newtonian fluids through arterial stenoses. Vajravelu *et.al.*[12] investigated peristaltic transport of a Herschel-Bulkley fluid in an inclined tube. Young[2] discussed effect of time dependent steonosis on flow through a tube.

In the present investigation an effect has been made to the pulsatile flow of Hershel-Bulkley fluid with stenoses artery with body acceleration subject to a slip velocity condition at the constricted wall. Analytical expressions for axial velocity and volumetic flow rate have been derived and the effects of various parameters on these flow variables have been studied.

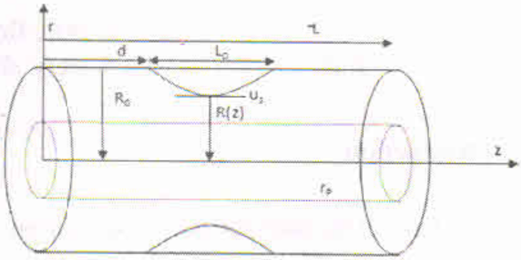

Figure:1 Flow geometry and coordinates system

II. Mathematical Formulation :

It is assumed that the stenoses is

developed in an axially-symmetric manner. The radius of the artery, $R(z)$ in the stenotic region can be taken as (Young, 1968)

$$R(z)=\begin{cases} R_0-\dfrac{\delta}{2}\left(1+\cos\dfrac{2\pi}{L_0}\left(z-d-\dfrac{L_0}{2}\right)\right) & ;\, d\le z\le d+L_0, \\ R_0, & otherwise \end{cases} \quad (1)$$

Where R_0 and $R(z)$ represent the radii of the uniform and constricted regions, L_0 is the length of the stenosis and δ is the maximum height of the stenosis, r and z are the radial and axial co-ordinates.

The pressure gradient and body acceleration are given by:

$$-\frac{\partial p}{\partial z}=A_0+A_1\cos(\omega_p t), \quad (2)$$

$$G(t)=a_0\cos(\omega_b t+\phi) \quad (3)$$

Where A_0 and A_1 are pressure gradient of steady flow and amplitude of oscillatory part respectively, a_0 is the amplitude of body acceleration, $\omega_p=2\pi f_p$, $\omega_b=2\pi f_b$ with f_p is the pulse frequency and f_b is body acceleration frequency, ϕ is the phase angle of body acceleration p is the pressure gradient and t is time.

III. Flow Analysis :

Unsteady laminar flow of blood a red cell suspension through a straight rigid tube of circular cross section, with the formulation of a stenosis at the inner most vessel wall is considered. Flowing fluid blood which is incompressible has been assumed to behave like a Herschel- Bulkely fluid.

The governing equation of motion for laminar and incompressible, fully-developed, one-dimensional flow of blood are given by

$$\rho\frac{\partial u}{\partial t}=-\frac{\partial p}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}(r\tau)+G(t) \quad (4)$$

$$\rho\frac{\partial u}{\partial t}=A_0+A_1\cos(\omega_p t)+\frac{1}{r}\frac{\partial}{\partial r}(r\tau)+a_0\cos(\omega_b t+\phi) \quad (5)$$

$$\frac{\partial p}{\partial r}=0 \quad (6)$$

Where r and z denotes the radial and axial coordinates respectively and ρ denotes density, u axial velocity of blood, and τ is the share stress.

The constitutive equation for Herschel-Bulkley fluid may be written as (Sankar and Hamalatha, 2006)

$$-\frac{\partial u}{\partial r}=\begin{cases}\dfrac{1}{\mu_H}(\tau-\tau_H)^n, & if\,\tau\ge\tau_H \\ 0, & if\,\tau<\tau_H\end{cases} \quad (7)$$

Where u stands for the axial velocity of blood and $\tau_H(\ge 0)$ is the yield shear stress, n is the power law index, μ_H is the coefficient of viscosity for Herschel-Bulkley fluid.

The boundary conditions are:

(i) $u = u_s \quad at \quad r = R(z)$ (8)

(ii) τ is finite at $r = 0$ (9)

Where u_s is the axial slip velocity.

Introducing the non-dimensional variables:

$$\bar{u} = \frac{u}{A_0 R_0^2 / 4\mu_0}, \quad \bar{z} = \frac{z}{R_0}, \quad \bar{t} = \omega_p t,$$

$$\bar{\delta} = \frac{\delta}{R_0}, \quad \bar{\tau} = \frac{\tau}{A_0 R_0 / 2}$$

$$\bar{\tau}_H = \frac{\tau_H}{A_0 R_0 / 2}, \quad \bar{R}(\bar{z}) = \frac{R(z)}{R_0},$$

$$\bar{r} = \frac{r}{R_0}, \quad e = \frac{A_1}{A_0} \quad B = \frac{a}{A_0}$$

$$\omega = \frac{\omega_b}{\omega_p}, \; z_0 = \frac{z_0}{R_0} \; \mu_0 = \bar{\mu}_H \left(\frac{2}{R_0 A_0}\right)^{\left(\frac{1}{n}-1\right)} \quad (10)$$

The non-dimensional momentum equation (4) becomes after dropping bars as

$$\alpha^2 \frac{\partial u}{\partial t} = 4(1 + e\cos t) + 4B\cos(\omega t + \phi) + \frac{2}{r}\frac{\partial}{\partial r}(r\tau) \quad (11)$$

Where $\alpha^2 = \dfrac{\omega_p R_0^2}{\mu/\rho}$, α is Womersley frequency parameter.

Equation (7) can be written as

$$(\tau - \tau_H)^n = \left(-\frac{1}{2}\frac{\partial u}{\partial r}\right) \quad if\, \tau \ge \tau_H \quad (12)$$

$$\frac{\partial u}{\partial r} = 0 \quad \text{if} \quad \tau < \tau_H \quad (13)$$

The boundary conditions (equations 8 and 9) reduce to

(i) $u = u_s$ at $r = R(z)$ (14)

(ii) τ is finite at $r = 0$ (15)

The geometry of the stenosis in non-dimensional form is given by

$$R(z) = \begin{cases} 1 - \dfrac{\delta}{2}\left(1 + \cos\dfrac{2\pi}{L_0}\left(z - d - \dfrac{L_0}{2}\right)\right); & d \le z \le d + L_0, \\ 1; & otherwise \end{cases} \quad (16)$$

IV. Method of Solution :

On using perturbation method, the velocity u and shear stress τ, plug core velocity u_p, plug core radius R_p are expanded as follows in terms of α^2 (where $\alpha^2 \ll 1$)

$$u(z,r,t) = u_0(z,r,t) + \alpha^2 u_1(z,r,t) + \ldots\ldots\ldots (17)$$

$$\tau(z,r,t) = \tau_0(z,r,t) + \alpha^2 \tau_1(z,r,t) + \ldots\ldots\ldots (18)$$

$$u_p(z,r,t) = u_{0p}(z,r,t) + \alpha^2 u_{1p}(z,r,t) + \ldots\ldots\ldots (19)$$

$$R_p(z,r,t) = R_{0p}(z,r,t) + \alpha^2 R_{1p}(z,r,t) + \ldots\ldots\ldots (20)$$

Substituting (17) and (18) in equation (11) and equating the constant term and α^2 term we get

$$\frac{\partial}{\partial r}(r\tau_0) = -2r\{(1 + e\cos t) + B\cos(\omega t + \phi)\} \quad (21)$$

$$\frac{\partial u_0}{\partial t} = \frac{2}{r}\frac{\partial}{\partial r}(r\tau_1) \quad (22)$$

Integrate equation (21) and using boundary condition (15)

$$\tau_0 = -f(t)r \quad (23)$$

Where $f(t) = 1 + e\cos t + B\cos(\omega t + \phi)$

Substituting (17) and (18) in (12)

$$-\frac{\partial u_0}{\partial r} = 2\tau_0^{n-1}\left(\tau_0 - n\tau_H\right) \quad (24)$$

$$-\frac{\partial u_1}{\partial r} = 2n\tau_0^{n-2}\tau_1\left(\tau_0 - (\mathrm{n}-1)\tau_H\right) \quad (25)$$

Integrating equation (24) using the relation (23) and the boundary condition (15) we obtain

$$u_0 = u_s + A_1\left(R^{n+1} - r^{n+1}\right) + A_2\left(R^n - r^n\right) \quad (26)$$

Where $A_1 = \frac{2(-1)^{n-1} f^n(t)}{n+1}$, $A_2 = 2(-1) f^{n-1}(t)\tau_H$

The plug core velocity u_{0p} can be obtained from equation (26) as

$$u_{0p} = u_s + A_1\left(R^{n+1} - R_{0p}^{n+1}\right) + A_2\left(R^n - R_{0p}^n\right) \quad (27)$$

Neglecting the terms of O (α^2) and higher powers of α in equation (20) R_{0p} can be obtained from (23) as

$$R_{0p} = \frac{\tau_H}{f(t)} \quad (28)$$

Using equation (22) we get the solution for τ_1 as,

$$\tau_1 = a_6 r - a_5 r^{n+1} - a_3 r^{n+2} \quad (29)$$

Where

$a_1 = (-1)^{n-1} \mathrm{f}^{n-1}(\mathrm{t})\mathrm{f}'(\mathrm{t})$, $a_2 = \frac{n a_1 R^{n+1}}{2(\mathrm{n}+1)}$,

$a_3 = \frac{n a_1}{(\mathrm{n}+1)(\mathrm{n}+3)}$, $a_4 = \frac{(\mathrm{n}-1)\mathrm{a}_1 \tau_H f(\mathrm{t})\mathrm{R}^n}{2}$,

$a_5 = \frac{(\mathrm{n}-1)\mathrm{a}_1 \tau_H f(\mathrm{t})}{(\mathrm{n}+2)}$, $a_6 = a_2 + a_4$

Similarly using equations (25) and (29) we can obtain the solution for u_1 as

$$u_1 = \frac{a_9}{n+1}(\mathrm{R}^{n+1} - \mathrm{r}^{n+1}) + \frac{a_{10}}{2n+2}(\mathrm{R}^{2n+2} - \mathrm{r}^{2n+2}) + \frac{a_{15}}{2n+1}(\mathrm{R}^{2n+1} - \mathrm{r}^{2n+1}) + \frac{a_{12}}{n}(\mathrm{R}^n - \mathrm{r}^n) + \frac{a_{14}}{2n}(\mathrm{R}^{2n} - \mathrm{r}^{2n}) \quad (30)$$

$$u_{1p} = \frac{a_9}{n+1}(\mathrm{R}^{n+1} - R_{0p}^{n+1}) + \frac{a_{10}}{2n+2}(\mathrm{R}^{2n+2} - R_{0p}^{2n+2}) + \frac{a_{15}}{2n+1}(\mathrm{R}^{2n+1} - R_{0p}^{2n+1}) + \frac{a_{12}}{n}(\mathrm{R}^n - R_{0p}^n) + \frac{a_{14}}{2n}(\mathrm{R}^{2n} - R_{0p}^n) \quad (31)$$

Where

$$a_7 = -2n(-1)^{n-2} f^{n-1}(\mathrm{t}), \quad \mathrm{a}_8 = -2n(-1)^{n-2} f^{n-2}(\mathrm{t})(\mathrm{n}-1)\tau_H,$$

$$a_9 = a_7 a_6, \quad a_{10} = -a_7 a_3, \quad a_{11} = -a_7 a_5,$$

$$a_{12} = a_8 a_6, \quad a_{13} = -a_8 a_3, \quad a_{14} = -a_8 a_5,$$

$$\mathrm{a}_{15} = a_{11} + a_{13} \quad (32)$$

Using equations (17) and (18) the total velocity distribution and shear stress can be written as

$$u = u_s + A_1\left(R^{n+1} - r^{n+1}\right) + A_2\left(R^n - r^n\right) + \alpha^2\left[\frac{a_9}{n+1}(R^{n+1} - r^{n+1}) + \frac{a_{10}}{2n+2}(R^{2n+2} - r^{2n+2}) + \frac{a_{15}}{2n+1}(R^{2n+1} - r^{2n+1}) + \frac{a_{12}}{n}(R^n - r^n) + \frac{a_{14}}{2n}(R^{2n} - r^{2n})\right] \tag{33}$$

$$u_p = u_s + A_1\left(R^{n+1} - R_{0p}^{n+1}\right) + A_2\left(R^n - R_{0p}^n\right) + \alpha^2\left[\frac{a_9}{n+1}(R^{n+1} - R_{0p}^{n+1}) + \frac{a_{10}}{2n+2}(R^{2n+2} - R_{0p}^{2n+2}) + \frac{a_{15}}{2n+1}(R^{2n+1} - R_{0p}^{2n+1}) + \frac{a_{12}}{n}(R^n - R_{op}^n) + \frac{a_{14}}{2n}(R^{2n} - R_{0p}^{2n})\right] \tag{34}$$

$$\tau = \left(\tau_0 + \alpha^2\tau_1\right)_{r=R} = -f(t)R + \alpha^2\left\{a_6 r - a_5 r^{n+1} - a_3 r^{n+2}\right\} \tag{35}$$

The second approximation plug core radius R_{1p} can be obtained by neglecting terms of O (α^4) and higher powers of α in equation (20) as

$$R_{1p} = \frac{\tau_1(R_{0p})}{f(t)} \tag{36}$$

With the help of equations (20), (28) and (35), R_p can be given by

$$R_p = \frac{\tau_H}{f(t)} + \frac{\alpha^2}{f(t)}\left(a_6 R_{0p} - a_5 R_{0p}^{n+1} - a_3 R_{0p}^{n+2}\right) \tag{37}$$

The volumetric flow rate Q is given by

$$Q = \int_0^{R(z)} 2\pi r\, u(z,r,t)\, dr$$

$$Q=\pi R^2 u_s+2\pi\left[A_1R^{n+3}\frac{n}{2(n+3)}+A_2R^{n+2}\frac{n}{2(n+2)}+\alpha^2\left\{R^{n+3}\frac{a_9}{2(n+3)}+R^{2n+4}\frac{a_{10}}{2(2n+4)}+R^{2n+3}\frac{a_{15}}{2(2n+3)}+R^{n+2}\frac{a_{12}}{2(n+2)}+R^{2n+2}\frac{a_{14}}{2(2n+2)}\right\}\right] \tag{38}$$

V. Result and Discussion

The velocity profile for the pulsatile flow of Herschel – Bulkley fluid with stenoses artery with periodic body acceleration is computed by using (33) for different values of parameter e, body acceleration parameter B, time t, yield stress τ_H have been shown through figures 2- 5. Figure - 2 shows that the variation of velocity profile for different values of parameter e. It can be noted here that as the parameter e increases, the velocity profile increases. In the presence of body acceleration, velocity increases rapidly. As the body acceleration increases, the plug region shrinks and hence more flow takes place (figure 3). It can easily be seen from figures 4, an increase in the time t, decrease in the velocity profile, It can be noted here that as the parameter τ_H increases the velocity profile decreases (figure 5). the variation of the volumetric flow rate with time for different values of e, body acceleration B, phase angle of body acceleration ϕ have been shown through figures 7- 8. Figure 7 shows that at all instants of time, the volumetric flow rate increases with the increase of parameter e.

It can be noted here that the volumetric flow rate increases with increase of body acceleration B, (Figure 7) and increases of phase angle of body acceleration, the volumetric flow rate is also increases (figure 8).

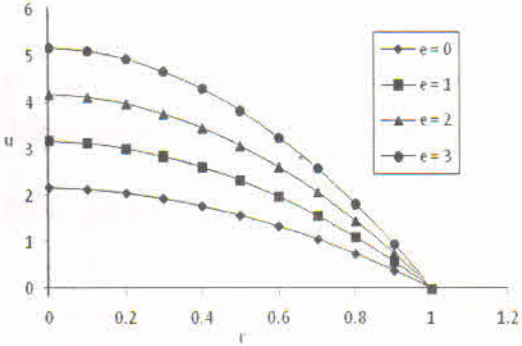

Fig. 2. Variation of axial velocity with radial distance r for
$\tau_H=0.1, B=1, \omega=1, \phi=0.2, \alpha=0.1, u_s=0.09$

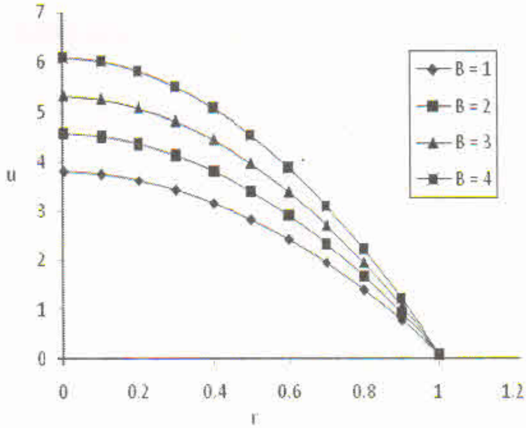

Fig. 3. Variation of axial velocity with radial distance r for
$\tau_H=0.1, e=2, \omega=1, \phi=0.2, \alpha=0.1, u_s=0.09$

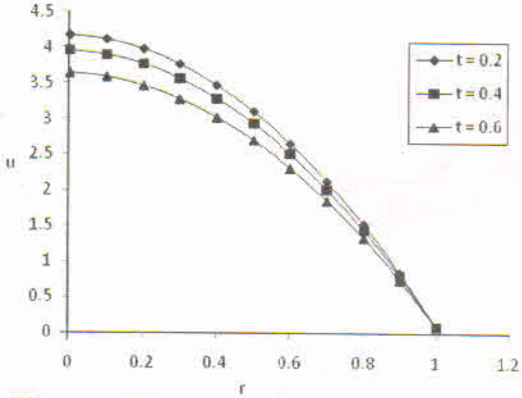

Fig. 4. Variation of axial velocity with radial distance r for

$\tau_H = 0.1,\ B = 1,\ \omega = 1,\ \phi = 0.2,\ \alpha = 0.1, u_s = 0.09$

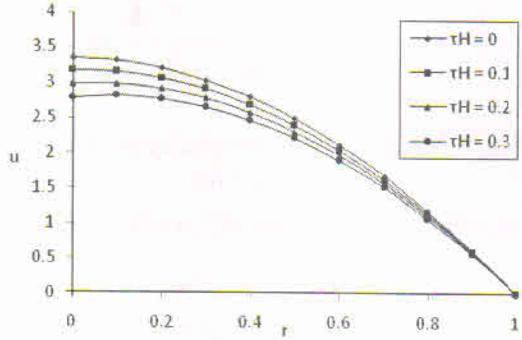

Fig. 5. Variation of axial velocity with radial distance r for

$e = 2,\ B = 1,\ \omega = 1,\ \phi = 0.2,\ \alpha = 0.1, u_s = 0.09$

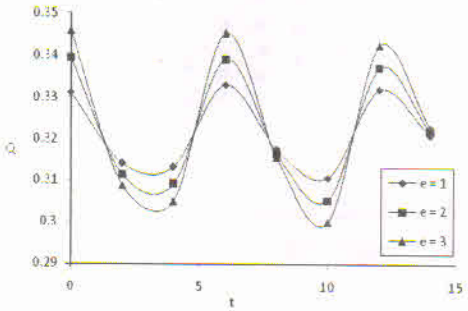

Fig. 6. Variation of Volumetric flow rate Q with time for t for

$\tau_H = 0.1, \mathrm{B} = 1, \alpha = 0.2, \omega = 1, \phi = 0.2, u_s = 0.09$

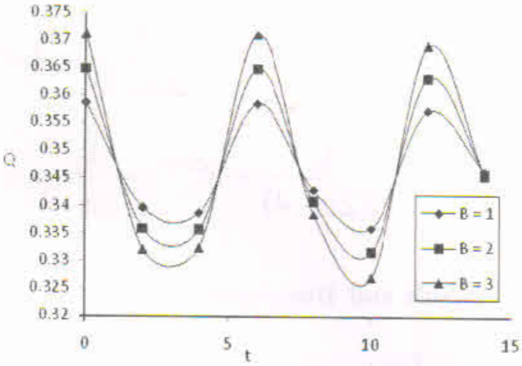

Fig. 7. Variation of Volumetric flow rate Q with time for t for

$\tau_H = 0.1, e = 1,\ \omega = 1, \phi = 0.2,\ \alpha = 0.2, u_s = 0.09$

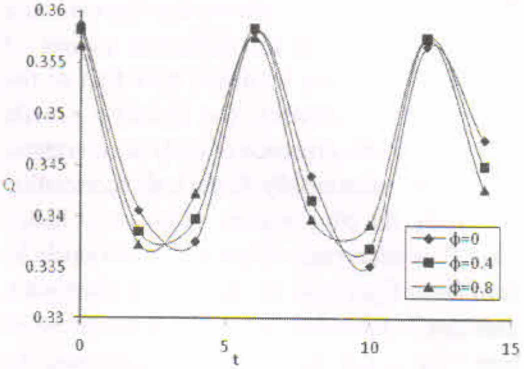

Fig. 7. Variation of Volumetric flow rate Q with time for t for

$\tau_H = 0.1, e = 2,\ \omega = 1, B = 1,\ \alpha = 0.1, u_s = 0.09$

Vi. Conclusion

The present study deals with a theoretical investigation of the characteristics of the pulsatile flow of blood through with stenoses artery with periodic body acceleration. Blood is represented by Herschel – Bulkley fluid model. Using appropriate boundary conditions, analytical expressions for the velocity profile and volumetric flow rate have been obtained.

It is clear from the above result and discussions that the body acceleration effects largely on the axial velocity of blood flow.

A proper understanding of interactions of body acceleration with blood flow could be useful in the diagnosis and therapeutic treatment of some health problems (joint pain, vision loss and vascular disorder) to better design of protective pads and machines.

Hence from all the above discussions we can conclude that a careful choice of the values of the parameters of body acceleration, yield stress will affect the flow characteristics and hence can be utilised for medical and engineering applications.

References

1. Agarwal, R., and Varshney, N.K., "MHD pulsatile flow of couple stress fluid through an inclined circular tube with periodic body acceleration", *Jour. PAS*, Vol. 17 *(Mathematical Science)*, 277-293 (2011).
2. Biswas, D. and Chakraborty, U.S., "Pulsatile blood flow through through a catheterized artery with an axially non symmetrical stenosis", *Applied mathematical sciences*, *4(58)*, 2865-2880 (2010).
3. Biswas, D. and Laskar, R.B., "Study flow of blood through a stenosed artery: A non-Newtonian fluid model, *physical science and technology"*, *7(11)*, 144-153 (2011).
4. Chaturani, P. and Palanisami, V. "Pulsatile flow of power law fluid model for blood flow under periodic body acceleration, *Biorheol"*. *27*, 747-758 (1990).
5. Chaturani, P. and Palanisamy, V. "Pulsatile flow of blood with periodic body acceleration", *Int. J. Eng. Sci. 29*, pp. 113-121 (1991).
6. Chaturani, P. and Samy, R.P. "Pulsatile flow of Casson's fluid through stenosed arteries with applications to blood flow, *Biorheol.*", *23*, pp. 499-511 (1986).
7. Chien, S. "Hemorheology in clinical medicine", *Recent Advances in Cardio-vascular Diseases*, Vol. *2*, pp. 21–26 (1981).
8. El-Shahed, M. "Pulsatile flow of blood through a stenosed porous medium under periodic body acceleration", *Applied Mathematics and Computation 138*, pp. 479-488 (2003).
9. Elshehawey, E. F., Elsayed, M. E., Afifi, N. A. S. and El-Shahed, M. "Pulsatile flow of blood through a porous medium under periodic body acceleration", *Int. Journal of theoretical Physics,39(1)*, pp. 183-188 (2000).
10. Mandal, P. K., Chakravarthy, S., Mandal, A. and Amin, N., "Effect of body acceleration on unsteady pulsatile flow of non-Newtonian fluid through a stenosed artery", *Applied Mathematics and Computation*, *189*, pp. 766-779 (2007).
11. Maruthi Prasad, K., Radhakrishnamacharya, G. "Effect of multiple stenoses on Herschel-Bulkley fluid through a tube with non-uniform cross-section, International e-journal of engineering mathematics", *Theory and Application*, *1*, pp. 69-76 (2007).
12. Maruthi Prasad, K. and Radhakrishnamacharya, G. "flow of Herschel-Bulkley fluid through an inclined tube of non-uniform cross-section with multiple stenoses", *Arch. Mech.*, 60, 2, pp. 161-172 (2008).
13. Merrill, E. W., Benis, A. M., Gilliland, E. R., Sherwood, T. K. and Salzman, E. W., "Pressure flow relations of human blood

in hollow fibre at low shear rates", *Appl. Physiol. 20,* pp. 954-967 (1965).

14. Nagarani, P. and Sarojamma, G. "Effect of body acceleration on pulsatile flow of Casson fluid through a mild stenosed artery", *Korea- Australia Rheology Journal* Vol. *20,* no. 4, pp. 189-196 (2008).
15. Sankar, D.S. and Hemalatha, K. "Pulsatile flow of Herschel-Bulkey fluid through stenosed arteries a mathematical model", *International Journal of Non-Linear Mechanics*, Vol. *41,* no. 8, pp. 979–990 (2006).
16. Sarojamma, G. and Nagarani, P. "Pulsatile flow of Casson fluid in a homogeneous porous medium subject to external acceleration", *Int. J. Non-Linear Differ. Eqns. Theor. Models Appl.*, *7,* pp. 50-64 (2002).
17. Shukla, J.B., Parihar, R.S. and Rao, B.R.P. "Effects of stenosis on non-Newtonian flow through an artery with mild stenosis", *Bull. Math. Biol.*, *42,* pp. 283-294 (1980).
18. Siddiqui, S.U. and Mishra, S. "A study of modified Casson's fluid in modeled normal and stenotic capillary – tissue diffusion phenomenal, *Appl. Math. Comput.*, *189,* pp. 1048-1057 (2007).
19. Siddiqui, S.U., Gupta, R.S., Verma, N.K. and Mishra, S. "Mathematical modelling of pulsatile flow of Casson's fluid in arterial stenosis", *Appl. Math. Comput.*, *210(1),* pp. 1-10 (2009).
20. Tu, C. and Deville, M. "Pulsatile flow of non-Newtonian fluids through arterial stenoses", *J. Biomech.*, *29,* pp. 899–908 (1996).
21. Vajravelu, K., Sreenadh, S. and Ramesh Babu, V. "Peristaltic transport of a Herschel-Bulkley fluid in an inclined tube", *Int. J. of Non-Linear Mech.*, *40,* pp. 83-90 (2005).
22. Young, D.F., "Effect of time dependent steonosis on flow through a tube" J. Engng. For Ind. Trans of ASME, *90,* 248-254 (1968).