ISSN 2231-3478 (Print)
JUSPS-B Vol. 33(1), 1-4 (2021). Periodicity-Monthly
Section B
(Online) ISSN 2319-8052

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES
An International Open Free Access Peer Reviewed Research Journal of Physical Sciences
website:- www.ultrascientist.org
Estd. 1989

Inequalities Among Related Quadruple of Fibonacci Numbers

MANJEET SINGH TEETH[1] and SANJAY HARNE[2],

[1]Christian Eminent College, Indore (India)
[2]Government Mata Jija Bai Girls PG College, Indore (India)
Corresponding Author Email:- manjeetsinghteeth@gmail.com
http://dx.doi.org/10.22147/jusps-B/330101

Acceptance Date 21st March, 2021, Online Publication Date 27th March, 2021

Abstract

In this paper we consider Fibonacci Inequalities and relate them through the sequence $\left\{s_r\right\}_{r=1}^{n}$ defined by $s_r = F_r F_{n+1-r} F_{n+2-r} F_{n+3-r}$ where n is a fixed natural number and $F_1, F_2, F_3, \ldots$ are the ordinary Fibonacci numbers.

Key word : Fibonacci Number, Fibonacc Sequence
Mathematical Subject Classification: 11B39

1. Introduction

Fibonacci numbers[5,8] were introduced in 1202 in Fibonacci book Liber Abaci.The Fibonacci sequence is a source of many identities[2,6]. The sequence of Fibonacci Numbers $\left\{F_n\right\}$ is defined by

$$F_n = F_{n-1} + F_{n-2}, \quad n \ge 2, \quad F_0 = 1, \quad F_1 = 1 \tag{1.1}$$

The Binet'sformula for Fibonacci sequence is given by

$$F_n = \frac{\alpha^n - \beta^n}{\alpha - \beta} = \frac{1}{\sqrt{5}} \left\{ \left(\frac{1+\sqrt{5}}{2} \right)^n - \left(\frac{1-\sqrt{5}}{2} \right)^n \right\} \tag{1.2}$$

Where $\alpha = \frac{1+\sqrt{5}}{2} = \textit{Golden ratio} \approx 1.618$

and $\beta = \frac{1-\sqrt{5}}{2} \approx -1.618$

Atanassov[4] consider the Fibonacci inequalities and relate them through the sequence $\{m_r\}_{r=1}^{n}$ defined by $s_r = F_r F_{n+1-r}$ where n is a fixed natural number and $F_1, F_2, F_3, \ldots$ are the ordinary Fibonacci numbers.

S. Harne and B. Singh[7] consider the Fibonacci inequalities and relate them through the sequenc $\{m_r\}_{r=1}^{n}$ *defined by* $m_r = F_r F_{n+1-r} F_{n+2-r}$ *where n is a fixed natural number and* $F_1, F_2, F_3, \ldots$ *are the ordinary Fibonacci numbers.*

we consider Fibonacci Inequalities and relate them through the sequence $\{s_r\}_{r=1}^{n}$ defined by $s_r = F_r F_{n+1-r} F_{n+2-r} F_{n+3-r}$ where n is a fixed natural number and $F_1, F_2, F_3, \ldots$ are the ordinary Fibonacci numbers.

2. Inequalities among related quadruple of Fibonacci numbers :

Theorem 2.1. For every natural number *k*, the following inequalities for the elements ofthe sequence $\{s_r\}_{r=1}^{n}$ are valid:

For $n = 4k$

$$(i)\ F_1 F_{4k} F_{4k+1} F_{4k+2} > F_3 F_{4k-2} F_{4k-1} F_{4k} > \ldots > F_{2k+1} F_{2k} F_{2k+1} F_{2k+2}$$
$$> F_{2k+2} F_{2k-1} F_{2k} F_{2k+1} > F_{2k+4} F_{2k-3} F_{2k-2} F_{2k-1} > \ldots > F_{4k} F_1 F_2 F_3$$

$$(ii)\ F_2 F_{4k-1} F_{4k} F_{4k+1} > F_4 F_{4k-3} F_{4k-2} F_{4k-1} > \ldots > F_{2k} F_{2k+1} F_{2k+2} F_{2k+3}$$
$$> F_{2k+1} F_{2k} F_{2k+1} F_{2k+2} > F_{2k+3} F_{2k-2} F_{2k-1} F_{2k} > \ldots > F_{4k-1} F_2 F_3 F_4$$

Theorem 2.2. For every natural number *k*, the following inequalities for the elements of the sequence $\{s_r\}_{r=1}^{n}$ are valid :

For $n = 4k+1$,

$$(i)\ F_1 F_{4k+1} F_{4k+2} F_{4k+3} > F_3 F_{4k-1} F_{4k} F_{4k+1} > \ldots > F_{2k+1} F_{2k+1} F_{2k+2} F_{2k+3}$$
$$> F_{2k+2} F_{2k} F_{2k+1} F_{2k+2} > F_{2k+4} F_{2k-2} F_{2k-1} F_{2k} > \ldots > F_{4k} F_2 F_3 F_4$$

$$(ii)\ F_2 F_{4k} F_{4k+1} F_{4k+2} > F_4 F_{4k-2} F_{4k-1} F_{4k} > \ldots > F_{2k+1} F_{2k+1} F_{2k+2} F_{2k+3}$$

$$> F_{2k+3}F_{2k-1}F_{2k}F_{2k+1} > F_{2k+5}F_{2k-3}F_{2k-2}F_{2k-1} > \ldots > F_{4k+1}F_1F_2F_3$$

Theorem 2.3. For every natural number *k*, the following inequalities for the elements of the sequence $\{s_r\}_{r=1}^{n}$ are valid :

For $n = 4k+2$,

$$(i)\ F_1F_{4k+2}F_{4k+3}F_{4k+4} > F_3F_{4k}F_{4k+1}F_{4k+2} > \ldots > F_{2k+1}F_{2k+2}F_{2k+3}F_{2k+4}$$
$$> F_{2k+2}F_{2k+1}F_{2k+2}F_{2k+3} > F_{2k+4}F_{2k-1}F_{2k}F_{2k+1} > \ldots > F_{4k+2}F_1F_2F_3$$

$$(ii)\ F_2F_{4k+1}F_{4k+2}F_{4k+3} > F_4F_{4k-1}F_{4k}F_{4k+1} > \ldots > F_{2k+1}F_{2k+2}F_{2k+3}F_{2k+4}$$
$$> F_{2k+3}F_{2k}F_{2k+1}F_{2k+2} > F_{2k+5}F_{2k-2}F_{2k-1}F_{2k} > \ldots > F_{4k+1}F_2F_3F_4$$

Theorem 2.4. For every natural number *k*, the following inequalities for the elements of the sequence $\{s_r\}_{r=1}^{n}$ are valid :

For $n = 4k+3$,

$$(i)\ F_1F_{4k+3}F_{4k+4}F_{4k+5} > F_3F_{4k+1}F_{4k+2}F_{4k+3} > \ldots > F_{2k+1}F_{2k+3}F_{2k+4}F_{2k+5}$$
$$> F_{2k+2}F_{2k+2}F_{2k+3}F_{2k+4} > F_{2k+4}F_{2k}F_{2k+1}F_{2k+2} > \ldots > F_{4k+2}F_2F_3F_4$$

$$(ii)\ F_2F_{4k+2}F_{4k+3}F_{4k+4} > F_4F_{4k}F_{4k+1}F_{4k+2} > \ldots > F_{2k+1}F_{2k+3}F_{2k+4}F_{2k+5}$$
$$> F_{2k+3}F_{2k+1}F_{2k+2}F_{2k+3} > F_{2k+5}F_{2k-1}F_{2k}F_{2k+1} > \ldots > F_{4k+3}F_1F_2F_3$$

Theorem 2.5. For every natural number **n** the maximal element[1, 3] of the sequence $\{l_r\}_{r=1}^{n}$ is $F_1F_nF_{n+1}F_{n+2}$.

Theorem 2.6. For every natural number**n** the minimal element is $F_1F_2F_3F_n$.

3. Conclusion

The main results of this paper are six theorems. There are many known identities for Fibonacci numbers. This paper extended the results of inequalities among related triplets of Fibonacci numbers. The research can derive inequalities among related quadruple of Lucas numbers.

Acknowledgements

The authors would like to thank the referee for their very helpful and detailed comments.

References :

1. A.F. Alameddine. “Bounds on the Fibonacci Number of a maximal Outerplannar Graph.” The Fibonacci Quarterly *36.3,* 206-10 (1998).
2. D. Han. “On the Chebyshev Polynomials and some of their newidentities.” Advances in ifference Equations (2020).
3. K.T. Atanassov. “One Extremal Problem.” Bulletian of Number Theory & Related Topics *8.3*, 6-12 (1984).
4. K.T. Atanassov & Ron Knott. “Inequalities among related Pairs of Fibonacci Numbers.” The Fibonacci Quarterly 20-22 (2003).
5. N.N. Vorobyov. “The Fibonacci Numbers.” D.C. Health Company, Boston, (1963).
6. O. Engin “3-Fibonacci Polynomials in the family of Fibonacci Numbers.” Journal of Science and Technology Vol. *12*, No. 2, 926-933 (2019).
7. S. Harne & B. Singh Inequalities among related Triplets of Fibonacci Numbers.” Applied Mathematical Sciences, Vol. *8,* No.102, 5079-5082 (2014).
8. V.E. Hoggatt, Jr. Fibonacci and Lucas Numbers, p. 59, Boston Houghton-Miffin, (1969).