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A new Type of Fixed Point Theorem in a Complete Dislocated Metric Space

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Abstract

In this paper a fixed point theorem have been proved in dislocated metric spaces using a class of continuous function G_4.

Key words: fixed point, dislocated metric space.

AMS Subject Classification: 47H10, 54H25.

Introduction

The Polish mathematician Stefan Banach in 1922 proved a theorem which ensures, under appropriate conditions, the existence and uniqueness of a fixed point. It is well known as a Banach fixed point theorem. The existence of a fixed point plays an important role in several areas of mathematics, physics and engineering branches. This principle has been generalized by many authors in various ways. Hitzler and Seda[8] introduced the concept of dislocated metric space, which is generalized by Zeyada *et al.*[3] by introducing the concept of dislocated quasi-metric space. The aim of this paper is to obtain a fixed point theorem in dislocated metric space. Our result generalizes the main result of Kastriot and Elida[7].

Preliminaries:

Definition[3] *:* Let X be a non empty set and let $d : X \times X \to [0, \infty)$ be a function satisfying following definitions

(i) $d(x, y) = d(y, x) = 0$ implies $x = y$.

(ii) $d(x, y) \leq d(x, z) + d(z, y)$ for all $x, y, z \in X$.

Then d is called a dislocated quasi-metric on X. If d satisfies $d(x, y) = 0$, then it is called a quasi-metric on X. If d satisfies $d(x, y) = d(y, x)$, then it is called dislocated metric.

Definition[3] *:* A sequence $\{x_n\}$ in dq-metric space (dislocated quasi-metric space) (X, d) is called Cauchy sequence if for, given $\varepsilon > 0$, there exist $n_0 \in N$, such that $\forall$ m, $n \geq n_0$, implies $d(x_m, x_n) < \varepsilon$ or $d(x_n, x_m) < \varepsilon$ *i.e.* $\min\{d(x_m, x_n), d(x_n, x_m)\} < \varepsilon$.

Definition[3] *:* A sequence $\{x_n\}$ dislocated quasi-convergent to x if

$\lim_{n\to\infty} d(x_n, x) = \lim_{n\to\infty} d(x, x_n) = 0$

In this case x is called a dq-limit of $\{x_n\}$ and we write $x_n \rightarrow x$.

Definition[3]*:* A dq-metric space (X, d) is called complete if every Cauchy sequence in it is a dq-convergent.

Definition[4]*:* Let (X, d) be a dq-metric space. A map $T : X \rightarrow X$ is called contraction if there exists $0 \leq \lambda \leq 1$ such that

$$d(Tx, Ty) \leq \lambda\, d(x, y), \text{ for all } x,y \in X.$$

Main Result

We consider the set G_4 of all continuous functions $g : [0, \infty)^4 \rightarrow [0, \infty)$ with the following properties

(1) g is non decreasing in respect to each variable.

(2) $g(t, t, t, t) \leq t$ for all $t \in [0, \infty)$.

For example $g(t_1, t_2, t_3, t_4) = \max\{t_1, t_2, t_3, t_4\}$ or $\max\{t_1, t_2, t_2, t_3, t_3 + t_4, t_4 + t_1\}$.

Theorem : Let (X, d) be a complete dislocated metric space and f, $g : X \rightarrow X$ be two continuous mappings such that

$$d(fx, gy) \leq cg[d(x,y), d(x, fx), d(y, gy), d(fy, gx)] \tag{1}$$

for all $x, y \in X$ where $g \in G_4$ and $0 \leq \lambda < 1$. Then f and g have a unique common fixed point in X.

Proof : Let $x_o \in X$ and define a sequence $\{x_n\}$ in X such that

$fx_0 = x_1, gx_1 = x_2 \ldots\ldots\ldots\ldots, gx_{2n-1} = x_{2n}, fx_{2n} = x_{2n+1}$

Consider, $d(x_{2n+1}, x_{2n+2}) = d(fx_{2n}, gx_{2n+1})$

$\leq \lambda\, g[d(x_{2n}, x_{2n+1}), d(x_{2n}, fx_{2n}), d(x_{2n+1}, gx_{2n+1}), d(fx_{2n+1}, gx_{2n})]$

$\leq \lambda\, g[d(x_{2n}, x_{2n+1}), d(x_{2n}, fx_{2n+1}), d(x_{2n+1}, x_{2n+2}), d(x_{2n+2}, x_{2n+2})]$

$\leq \lambda\, d(x_{2n}, x_{2n+1})$

So $d(x_{2n+1}, x_{2n+2}) \leq \lambda\, d(x_{2n}, x_{2n+1})$

In a similar way we have, $d(x_{2n}, x_{2n+1}) = d(gx_{2n-1}, fx_{2n})$

$= d(fx_{2n}, gx_{2n-1})$

$\leq \lambda\, g[d(x_{2n}, x_{2n-1}), d(x_{2n}, fx_{2n}), d(x_{2n-1}, gx_{2n-1}), d(fx_{2n}, gx_{2n-1})]$

$\leq \lambda\, g[d(x_{2n}, x_{2n-1}), d(x_{2n}, fx_{2n+1}), d(x_{2n-1}, x_{2n}), d(x_{2n}, x_{2n+1})]$

$\leq \lambda\, d(x_{2n-1}, x_{2n})$

Continuing this process we get

$d(x_{2n+1}, x_{2n+2}) \leq \lambda\, d(x_{2n}, x_{2n+1}) \leq .... \leq \lambda^{2n}\, d(x_0, x_1)$

Since $0 \leq \lambda < 1$, as $n \to \infty$, $\lambda^n \to 0$ which implies that $d(x_{2n+1}, x_{2n+2}) \to 0$. Hence (x_n) is a cauchy sequence in complete dislocated metric space X. So by completeness of X this sequence (x_n) must be convergent to u in X. Thus both the subsequence (fx_{2n}) and (gx_{2n}) also converges to u. Since f, g : X → X are continuous mapping we get fu = u and gu = u.

Uniqueness : Let $v \neq u$ be another fixed point of f and g where fu = u and gv = v. Then by given condition, we have $d(u, v) \leq d(fu, gv)$

$$\leq \lambda\, g[d(u, v), d(u, fu), d(v, gv), d(fv, gu)] \qquad (2)$$

$$\leq \lambda\, g[d(u, v), d(u, u), d(v, v), d(v, u)]$$

Replacing v = u in (2) we get, $d(u, u) \leq d(fu, gu)$

$$\leq \lambda\, g[d(u, u), d(u, fu), d(u, gu), d(fu, gu)]$$

$$\leq \lambda\, g[d(u, u), d(u, u), d(u, u), d(u, u)]$$

$$\leq \lambda\, d(u, u)$$

Thus we have d(u, u) = 0, since 0 ≤ λ B 1.

Now replacing u = v in (2) we get d(v, v) = 0.

Again from (2), we have

d(u, v) d” λ d(u, v) since 0 ≤ λ B 1 we get u = v.

Thus fixed point is unique.

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