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Common Fixed Point Theorems in Fuzzy Metric Space

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Abstract

In the present paper we prove common fixed point theorem for occasionally weakly compatible mapping in fuzzy metric space. The theorem is a version of many fixed point theorem in fuzzy metric spaces, given by many authors announced in the literature.

Key words: Common fixed point, occasionally weakly compatible mappings, fuzzy metric space.

1 Introduction

Zadeh in 1965 defined the fuzzy set. The concept of fuzzy metric space was introduced by Kramosil and Michalek in 1975. Gorge and Veermani in 1994 modified the notion of fuzzy metric spaces with the help of continuous t-norms. Pant in 1998 introduced the concept of reciprocally continuous mappings and proved some common fixed point theorem. Jungck generalized the idea of commuting mapping to compatible mappings and then to weakly compatible mapping. Malhotra et al in 2011 proved some fixed point theorem on occasionally weakly compatible mappings.

The aim of this paper is to obtain a common fixed point theorem for occasionally weakly compatible mappings in fuzzy metric space.

2 Preliminaries:

Definition 2.1 : A binary operation $*$: $[0, 1] \rightarrow [0, 1]$ is a continuous t-norms if it satisfies the following conditions:

- (i) $*$ is associative and commutative;
- (ii) $*$ is continuous;

- (iii) $a * 1 = a$ for all $a \in [0, 1]$;
- (iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ where $a, b, c, d \in [0, 1]$

Definition 2.2 : A fuzzy set A in X is a function with domain X and values in $[0, 1]$.

Definition 2.3 : A triples $(X, M, *)$ is said to be a fuzzy metric space (FM space) if X is an arbitrary set, $*$ is a continuous t -norm and M is a fuzzy set of $X^2 \times (0, \infty)$ satisfying the following conditions, for all $x, y, z \in X$ and $s, t > 0$;

- (i) $M(x, y, t) > 0$;
- (ii) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$.
- (iii) $M(x, y, t) = M(y, x, t)$;
- (iv) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
- (v) $M(x, y, \cdot) : (0, \infty) \rightarrow (0, 1)$ is left continuous.

$(X, M, *)$ denotes a fuzzy metric space, $M(x, y, t)$ can be thought of as degree of nearness between x and y with respect to t . We identify $x = y$ with $M(x, y, t) = 1$ for all $t > 0$.

Definition 2.4 Two self mappings f and g of a fuzzy metric space $(X, M, *)$ are called compatible if $\lim_{n \rightarrow \infty} M(fg x_n, gfx_n, t) = 1$ wherever, $\{x_n\}$ is sequence in X such that $\lim_{n \rightarrow \infty} f x_n = \lim_{n \rightarrow \infty} g x_n = x$ for some x in X .

Definition 2.5 : Two self mappings f and g of a fuzzy metric space $(X, M, *)$ are called reciprocally continuous on X if $\lim_{n \rightarrow \infty} fg x_n = f x$ and $\lim_{n \rightarrow \infty} gfx_n = x$ wherever, $\{x_n\}$ is sequence in X such that $\lim_{n \rightarrow \infty} f x_n = \lim_{n \rightarrow \infty} g x_n = x$ for some x in X .

Definition 2.6 : Let $(X, M, *)$ be a fuzzy metric space. Then

- (i) A sequence $\{x_n\}$ in X is said to converge to x in X if for each $\epsilon > 0$ and each $t > 0$, there exist $n_0 \in \mathbb{N}$ such that $M(x_n, x, t) > 1 - \epsilon$ for all $n \geq n_0$.
- (ii) A sequence $\{x_n\}$ in X is said to be Cauchy in X if for each $\epsilon > 0$ and each $t > 0$, there exist $n_0 \in \mathbb{N}$ such that $M(x_n, x_m, t) > 1 - \epsilon$ for all $n, m \geq n_0$.
- (iii) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 2.7 : Two self maps f and g of a set X are occasionally weakly compatible (owe) iff there is a point in X which is a coincidence point of f and g at which f and g commute.

Lemma 2.8 : Let X be a set and f, g owe self maps of X . If f and g have a unique point of coincidence, $w = f x = g x$, then w is the unique common fixed point of f and g .

Lemma 2.9 : Let $(X, M, *)$ be a fuzzy metric space. If there exist $q \in (0, 1)$ such that $M(x, y, qt) \geq M(x, y, t)$ for all $x, y \in X$ and $t > 0$ then $x = y$.

3 Main Theorem :

Theorem 3.1 : Let $(X, M, *)$ be a complete fuzzy metric space and let P, Q, S and T are self-mappings of X . Let the pairs $\{P, S\}$ and $\{Q, T\}$ be owe. If there exist $q \in (0, 1)$ such that $M(Px, Qy, qt) \geq \min \{M(Sx, Ty, t),$

$$\left. \left\{ \frac{M(Sx, Px, t) + M(Qy, Ty, t)}{2} \right\}, M(Px, Ty, t), M(Qy, Sx, t) \right\} \quad (1)$$

For all $x, y \in X$ and for all $t > 0$, then there exists a unique point $w \in X$ such that $Pw = Sw = w$ and a unique point $z \in X$ such that $Qz = Tz = z$. Moreover $z = w$, then there exists a unique common fixed points of P, Q, S and T .

Proof: Let the pair $\{P, S\}$ and $\{Q, T\}$ be owc so there are points $x, y \in X$ such that $Px = Sx$ and $Qy = Ty$. We claim that $Px = Qy$. By inequality (1)

$$\begin{aligned} M(Px, Qy, qt) &\geq \min\{M(Sx, Ty, t), \left\{ \frac{M(Sx, Px, t) + M(Qy, Ty, t)}{2} \right\}, M(Px, Ty, t), M(Qy, Sx, t)\} \\ &\geq \min\{M(Px, Qy, t), \left\{ \frac{M(Px, Px, t) + M(Qy, Qy, t)}{2} \right\}, M(Px, Qy, t), M(Qy, Px, t)\} \\ &\geq M(Px, Qy, t) \end{aligned}$$

So $Px = Qy$ i.e. $Px = Sx = Qy = Ty$.

Suppose that there is another point z such that $Pz = Sz$ then by (1) we have $Pz = Sz = Qy = Ty$ so $Px = Pz$ and $w = Px = Sx$ is unique point of coincidence of P and S .

Similarly there is a unique point $z \in X$ such that $z = Qz = Tz$.

Assume that $w \neq z$. We have

$$\begin{aligned} M(w, z, qt) &= M(Pw, Qz, qt) \\ &\geq \min\{M(Sw, Tz, t), \left\{ \frac{M(Sw, Pz, t) + M(Qz, Tz, t)}{2} \right\}, M(Pw, Tz, t), M(Qz, Sw, t)\} \\ &\geq \min\{M(w, z, t), \left\{ \frac{M(w, z, t) + M(z, z, t)}{2} \right\}, M(w, z, t), M(z, w, t)\} \\ &\geq M(w, z, t) \end{aligned}$$

Therefore by Lemma 2.9 we get $z = w$ and z is a common fixed point of P, Q, S and T . The uniqueness of fixed point can be easily proved by equation (1).

Theorem 3.2 : Let $(X, M, *)$ be a complete fuzzy metric space and let P, Q, S and T are self-mappings of X . Let the pairs $\{P, S\}$ and $\{Q, T\}$ be owe. If there exist $q \vee (0, 1)$ such that $M(Px, Qy, qt) \geq \Phi [\min\{M(Sx, Ty, t), \left\{ \frac{M(Sx, Px, t) + M(Qy, Ty, t)}{2} \right\}, M(Px, Ty, t), M(Qy, Sx, t)\}]$ (2)

for all $x, y \in X$ and $\Phi \in [0, 1] \rightarrow [0, 1]$ such that $\Phi(t) > t$ for all $0 < t < 1$, then there exists a unique common fixed point of P, Q, S and T .

Proof: Let the pair $\{P, S\}$ and $\{Q, T\}$ be owc so there are points $x, y \in X$ such that $Px = Sx$ and $Qy = Ty$. We claim that $Px = Qy$. By inequality (2)

$$\begin{aligned} M(Px, Qy, qt) &\geq \Phi [\min\{M(Sx, Ty, t), \left\{ \frac{M(Sx, Px, t) + M(Qy, Ty, t)}{2} \right\}, M(Px, Ty, t), M(Qy, Sx, t)\}] \\ &\geq \Phi[M(Px, Qy, t)] \end{aligned}$$

Now proof follows from theorem (3.1).

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