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# Analytical Study of Cylindrical Imploding Strong Shock in a Uniform Real Dusty Gas

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## Abstract

In the present study, we have investigated the motion of cylindrical imploding shock in a real dusty gas having uniform initial density distribution by using (CCW) Chester[2]-Chisnell[3]-Whitham (1958) method. It is considered that the real dusty gas is the mixture of real gas and a large number of small spherical solid particles of uniform size. Initial volume fraction of the solid particles is also assumed constant in this particular study. The particles do not interact with each other therefore their thermal motion is negligible. Initial density of the medium is taken to be constant and medium ahead of the shock front is at rest with small counter pressure. Here the particles behave like a pseudo-fluid. Maintaining the equilibrium flow condition in the flow field, the analytical expressions for the shock velocity, shock strength, pressure, and flow velocity have been derived. The variation of flow variables with propagation distance (r), mass concentration of solid particles in the mixture ($k_p$) and the ratio of the density of solid particles to the initial density of gas (G) are obtained and discussed through figures. The results accomplished are compared with those for dusty ideal gas Yadav *et, al*.[28].

*Key words :* CCW method, shock wave, uninform media, real gas.

## 1. Introduction

The study of the shock wave propagation in a dusty gas has received considerable attention due to its applications to space science, bomb blast, lunar ash flow, coal mine explosions, nozzle flow, missiles and other engineering problems. Many scientists Sedov[16], Pai *et al.*[13], Miura and Glass[10], Gretler and Regenfelder[5], Hirschler and Steiner[6], Igra *et. al.*[7]. Vishwakarma[20] generalized Ray and Bhowmik[14] solution in gas to the mixture of gas and small solid particles with exponentially varying density, using a non-similarity method. He found that the presence of small dust particles in the gaseous medium has significant effects on the variation of flow variables. The problem of propagation of shock wave in dusty ideal gas have been tackled by Vishwakarma[20] using similarity method. Vishwakrma and Pandey[24] have studied one-dimensional unsteady self-similar adiabatic flow of a dusty ideal gas behind a spherical shock wave with time- dependent energy input. Motion of shock wave in a mixture of ideal gas and small dust particles with radiation heat flux and exponentially varying density has been studied by Vishwakarma *et. al.*[25]. Yadav *et. al.*[27] studied the propagation of weak cylindrical shock in the mixture of ideal gas and dust particle in presence of constant axial magnetic field. Singh and Gogoi[17] have used the equation of state for non-ideal gases simplified by Anisimov and Spiner[1], and found similarity solution for the propagation of spherical shock wave in the mixture of non-ideal gas and small solid dust particles.

The aim of the present study is to investigate the motion of cylindrical imploding shock in a real dusty gas having uniform initial density distribution by using (CCW) Chester[2] -Chisnell[3]-Whitham (1958) method. It is assumed that the real (non-ideal) dusty gas is the mixture of real gas and a large number of small spherical solid particles of uniform size. Initial volume fraction of the solid particles and Initial density of the medium is taken to be constant and medium ahead of the shock front is at rest with small counter pressure. Maintaining the equilibrium flow condition in the flow field, the analytical expressions for the shock velocity, shock strength, pressure, and flow velocity have been derived. The variation of flow variables with propagation distance (r), mass concentration of solid particles in the mixture ($k_p$) and the ratio of the density of solid particles to the initial density of gas (G) are obtained and discussed through figures. The results accomplished are compared with those for dusty ideal gas Yadav *et al.*[28]. Change in Entropy and temperature just behind the rotating non-ideal gas on propagation of strong shock wave under the effect of overtaking waves is carried out by Gangwar[4]. Convergence of strong shock waves in an ideal gas with dust particles has been tackled by Singh *et al.*[15].

## 2. *Basic Equations and boundary conditions :*

The fundamental equation for one dimensional, unsteady, adiabatic and cylindrical symmetrical flow of a mixture of real (non-ideal) gas and small spherical solid particles can be written as [Pai *et. al.*[13], Vishwakarma[20], Steiner and Hirschler[19]]

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \tag{1}$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\rho u}{r} = 0 \tag{2}$$

$$\frac{\partial\varepsilon}{\partial t}+u\frac{\partial\varepsilon}{\partial r}+\frac{p}{\rho^2}\left(\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial r}\right)=0 \quad (3)$$

where u, p, ρ, and ε denote respectively, the flow velocity, the pressure and density at a distance '$r$' from the origin at time '$t$'.

It is assumed that the real dusty gas, to be a mixture of a real (non-ideal) gas and small spherical solid particles. The equation of state of the pseudo-fluid of solid particles in the mixture is simply[12].

$$\rho_{sp} = \text{constant} \quad (4)$$

where $\rho_{sp}$ is the species density of solid particles.

Equation of state of the real (non-ideal) gas in the medium is given by [Anisiomov and Spiner[1], Ranga Rao and Purohit[29], Vishwakarma and Nath[23]].

$$p_g = R^*\rho_g\left(1+b\rho_g\right)T_g \quad (5)$$

$$p_g = R^*\left(1-Z\right)\rho_g T_g = \left(1-Z\right)P \quad (6)$$

where $p_g$ and $\rho_g$ are the partial pressure and the partial density of the gas in mixture, $R^*$- is the gas constant of the mixture, $T_g$ is the temperature of the gas and of the solid particles as equilibrium flow condition is mentioned and b is the internal volume of the molecule of gas. In this equation the deviation of a real gas from the ideal state is taken into account. The gas is supposed to be so rarefied that its molecules interact only through binary collisions, while triple, quadruple etc collisions are negligible.

The total pressure of the mixture is p which is obtained from perfect gas law

$$p = R^*\rho_g T_g \qquad \text{since} \quad \rho_g = \rho\left(1-k_p\right) \quad (7)$$

Therefore, the equation of state of the mixture of perfect gas and small solid particles is given by [Vishwakarma and Nath[21], Singh and Gogoi[17]]

$$p=\frac{\left(1-k_p\right)}{1-Z}\left[1+b\bar{\rho}\left(1-k_p\right)\right]\rho R^* T \quad (8)$$

where $R^*$ - is the gas constant, T the temperature, $Z = \frac{V_{sp}}{V}$ is the volume fraction of solid particles in the mixture, $k_p = \frac{m_{sp}}{m}$ the mass concentration of solid particles in the mixture. $V_{sp}$ and $m_{sp}$ are the species volume and mass of the dust particles respectively.

The relation between $k_p$ and Z is as follows[12]

$$k_p = \frac{Z\rho_{sp}}{\rho} \tag{9}$$

where $\rho_{sp}$ is the species density of solid particles. In equilibrium flow, $k_p$ is a constant in the whole flow-field. Therefore, from the equations (4) and (9)

$$k_p = Z/\rho = \text{constant}$$

we have the relation for Z is [Pai[12], Naidu *et. al.*[11]].

$$Z = \frac{k_p}{\sigma(1-k_p)+k_p} \tag{10}$$

where $\sigma = \rho_{sp}/\rho$ the ratio of density to solid particle to the specific density of a non-ideal gas (specific density or density ratio), Hence the fundamental parameters of the Pai model are $K_p$ and $\sigma$ which describe the effects of the dust-loading. For the dust loading parameter $\sigma$, we have range of $\sigma$=1 to $\sigma\to\infty$ . The internal energy of the mixture can be written as follows[12]

$$\varepsilon = \left[k_p C_{sp} + (1-k_p)C_v\right]T = C_{vm}T \tag{11}$$

where $C_{sp}$ is the specific heat of solid particles, $C_v$ the specific heat of the gas at constant volume process and $C_{vm}$ the specific heat of the mixture at constant volume process. The specific heat of the mixture at constant pressure process is

$$C_{pm} = k_p C_{sp} + (1-k_p)C_p \tag{12}$$

where $C_p$ is the specific heat of the gas at constant pressure process. The ratio of specific heat of the mixture is given by [Marble[9]; Pai *et. al.*[13]]

$$\Gamma = \frac{C_{pm}}{C_{vm}} = \frac{\gamma+\delta\Phi}{1+\delta\Phi} \tag{13}$$

where $\gamma = \frac{C_p}{C_v}$, $\delta = \frac{k_p}{1-k_p}$ and $\Phi = \frac{C_{sp}}{C_v}$, (14)

The internal energy is therefore, given by [Anisiomov and Spiner[1], Vishwakarma and Nath[23])]

$$\varepsilon = \frac{p(1-Z)}{\rho(\Gamma-1)\left[1+b\rho(1-k_p)\right]} \tag{15}$$

Let $u_o$, $p_o$, $\rho_o$, $F_o$ and $Z_o$ denote the undisturbed values of particle velocity, pressure, density, heat flux and volume fraction of solid particles in the mixture just ahead of the shock and u, p, $\rho$, F and Z be the values of respective quantities at any point immediately after the passage of the shock (just behind the shock). It is assumed that a cylindrical strong shock is propagating into the real dusty gas having constant initial density $\rho_o$ at rest ($u_o$= 0) with negligible small counter pressure $p\approx 0$. The boundary conditions at the shock are given by

$$\rho\left(U-u\right)=\rho_o U \tag{16}$$

$$p=\rho u\left(U-u\right) \tag{17}$$

$$\varepsilon+\frac{p}{\rho}+\frac{1}{2}\left(U-u\right)^2-\frac{F}{\rho_o U}=\frac{1}{2}U^2-\frac{F_o}{\rho_o U} \tag{18}$$

$$\frac{Z}{\rho}=\frac{Z_o}{\rho_o} \tag{19}$$

where U=dR/dt denotes the shock velocity, R is the shock radius, $U/a_o=M$ is the Mach number, the suffix "$_o$" refers to the values in front of the shock. The initial volume fraction of the solid particles $Z_o$ is, given by[11]

$$Z_o=k_p\Big/\left[\sigma\left(1-k_p\right)+k_p\right] \tag{20}$$

where $\sigma=\rho_{sp}/\rho_g$ the ratio of density to solid particle to the initial specific density of a non-ideal gas( specific density or density ratio), Hence the fundamental parameters of the Pai model are $K_p$ and $\sigma$ which describe the effects of the dust-loading. For the dust loading parameter σ, we have range of σ=1 to $\sigma\rightarrow\infty$ . The speed of sound 'a' in the equilibrium two phase flow for an isentropic change of state of the mixture of the real gas and small spherical solid particles may be calculated as[17]

$$a^2=\left(\frac{dp}{d\rho}\right)_s=\frac{\left[\Gamma+\left(2\Gamma-Z\right)b\rho\left(1-k_P\right)\right]p}{\left(1-Z\right)\left\{1+b\rho\left(1-k_P\right)\right\}\rho}$$

Neglecting $b^2\rho^2$ and higher order terms, where the subscript 's' is the process of constant entropy. On simplifying above equation, we get

$$a^2=\frac{p}{\rho\left(1-Z\right)}\left[\Gamma+\left(\Gamma-Z\right)b\rho\left(1-k_P\right)\right] \tag{21}$$

Now, at the equilibrium state $u=0=\partial/\partial t$ the equation (1) becomes

$$\rho_o=\text{constant},\quad p_o=\text{constant}$$

Therefore, the speed of sound in unperturbed medium $a_o$' is given by

$$a_o=\sqrt{\left[\frac{p_o\left\{\Gamma+\left(\Gamma-Z_o\right)\bar{b}\left(1-k_P\right)\right\}}{\rho_o\left(1-Z_o\right)}\right]} \tag{22}$$

To represent the quantities u, p, ρ and Z in terms of their undisturbed values, the jump conditions across the strong shock are given by [Pai[12], Vishwakarma and Nath[22]]

$$u = (1-\beta)U \tag{23}$$

$$\rho = \frac{\rho_o}{\beta} \tag{24}$$

$$p = (1-\beta)\rho_o U^2 \tag{25}$$

$$Z = \frac{Z_0}{\beta} \tag{26}$$

The quantity $\beta(0<\beta<1)$ is obtained by the relation

$$\frac{2(\beta - Z)\beta}{\rho(\Gamma-1)\left[\beta+\bar{b}(1-k_P)\right]} + \beta - \left[1+\frac{2(F_2-F_1)}{pU}\right] = 0 \tag{27}$$

where $\bar{b} = b\rho_o$ (28)

As the shock is strong, we assume $F_1$-$F_2$ to be negligible in the comparison with the product of p and U [Laumbach and Probstein[8], Singh and Gogoi[18]], therefore above equation may be written as

$$\beta^2(\Gamma+1) + \beta\left[\left\{\bar{b}(1-k_P)-1\right\}(\Gamma-1) - 2Z_o\right] - (\Gamma-1)\bar{b}(1-k_P) = 0 \tag{29}$$

where the quantity β is shock density ratio which is an unknown parameter to be determined.

Using boundary conditions (23) - (26), the speed of sound is given by

$$a = \left[\frac{(1-\beta)\left\{\Gamma\beta^2 + (\Gamma\beta - Z_o)\bar{b}(1-k_P)\right\}}{(\beta - Z_o)}\right]^{1/2} U \tag{30}$$

*3. Theory*

*3.1 Analytical expressions for shock velocity and shock strength*

*3.1.1 Freely Propagation of shock (FP)*

For cylindrical imploding shocks, the characteristic form of the system of equations (1) - (3), *i.e.* the form in which each equation contain derivatives in only one direction in (r, t) plane, is

$$dp - \rho a du + \frac{2\rho a^2 u}{(u-a)}\frac{dr}{r} = 0 \tag{31}$$

Substituting the values from equations (23)-(26) and (30) in above equation, at constant initial density distribution ($\rho_o$= constant), we have

$$\frac{dU}{U} + X\left(\beta, \Gamma, Z_o, k_P, \bar{b}\right)\frac{dr}{r} = 0 \tag{32}$$

where $$X\left(\beta,\Gamma,Z_o,k_P,\bar{b}\right)=\frac{1}{\left[\dfrac{2\beta\left(\beta-Z_o\right)}{\left\{\Gamma\beta^2+\left(\Gamma\beta-Z_o\right)\bar{b}\left(1-k_P\right)\right\}}-\sqrt{\dfrac{\left(1+\beta\right)^2\left(\beta-Z_o\right)}{\left(1-\beta\right)\left\{\Gamma\beta^2+\left(\Gamma\beta-Z_o\right)\bar{b}\left(1-k_P\right)\right\}}}+1\right]} \tag{33}$$

*(a) Expression for Shock Velocity :*

On integrating the equation (32), we have

$$\log_e U + X\left(\beta,\Gamma,Z_o,k_P,\bar{b}\right)\log_e r = \log_e K$$

where K is the constant of integration. On simplifying, we get

$$\mathbf{U=Kr^{-X\left(\beta,\Gamma,Z_o,k_P,\bar{b}\right)}} \tag{34}$$

This is the expression for the freely propagation of shock velocity just behind the cylindrical strong shock

*(b) Expression for Shock Strength :*

Using equation (22), the expression for the propagating shock strength in the region behind the shock is given by

$$\mathbf{\frac{U}{a_o}=K^*\sqrt{\frac{\left(1-Z_o\right)}{\left\{\Gamma+\left(\Gamma-Z_o\right)\bar{b}\left(1-k_P\right)\right\}}}\,r^{-X\left(\beta,\Gamma,Z_o,k_P,\bar{b}\right)}} \tag{35}$$

where $K^* = K\sqrt{p_o/\rho_o}$

*(c) Expression for Non-dimensional Pressure :*

The expression for non-dimensional pressure ($p/p_0$) in the region behind the shock is given by on using equation (22) and substituting the value of $U/a_o$ from equation (35) in the above expression for the non-dimensional pressure ($p/p_o$) is given by

$$\mathbf{\frac{p}{p_o}=\left(1-\beta\right)K^{*2}\,r^{-2X\left(\beta,\Gamma,Z_o,k_P,\bar{b}\right)}} \tag{36}$$

*(d) Expression for Flow Velocity :*

The expression for non-dimentional flow velocity in the region behind the shock is given by

$$\frac{u}{a_o}=(1-\beta)K^*\sqrt{\frac{(1-Z_o)}{\left\{\Gamma+(\Gamma-Z_o)\bar{b}(1-k_P)\right\}}}\,r^{-X(\beta,\Gamma,Z_o,k_P,\bar{b})} \tag{37}$$

*3.1.2 Effect of Overtaking Disturbances (EOD) :*

It is assumed that shock propagates along $C_-$ characteristic and produces velocity increment $du_-$ whereas overtaking wave propagating along $C_+$ characteristic, creat velocity increment $du_+$. For $C_-$ disturbance generated by the shock, the fluid velocity increment from equation (23)

$$du_- = (1-\beta)\,dU \qquad \left\{\text{since} \quad u = (1-\beta)\,dU\right\} \tag{38}$$

Using equation (32) and simplifying, the equation (38), becomes

$$du_- = -(1-\beta)X\left(\beta,\Gamma,Z_o,k_P,\bar{b}\right)U\frac{dr}{r} \tag{39}$$

To estimate the strength of overtaking disturbance (*i.e.* effect of flow behind the shock) an independent $C_+$ disturbances is considered. The characteristics equation for $C_+$ disturbances is given by

$$dp+\rho a du+\frac{\rho a^2 u}{(u+a)}\frac{dr}{r}=0 \tag{40}$$

Substituting the values from equations (23)-(26) and (30) in above equation, and taking initial density distribution of medium $\rho_o$ is constant *i.e.* $d\rho_o = o$, and on solving the above equation becoms

or $$\frac{dU}{U}+Y\left(\beta,\Gamma,Z_o,k_P,\bar{b}\right)\frac{dr}{r}=0 \tag{41}$$

where $$Y\left(\beta,\Gamma,Z_o,k_P,\bar{b}\right)=\frac{1}{\left[\sqrt{\frac{(\beta-Z_o)(1+\beta)^2}{(1-\beta)\left\{\Gamma\beta^2+(\Gamma\beta-Z_o)\bar{b}(1-k_P)\right\}}}+\frac{2\beta(\beta-Z_o)}{\left\{\Gamma\beta^2+(\Gamma\beta-Z_o)\bar{b}(1-k_P)\right\}}+1\right]}$$

The fluid velocity increment due to overtaking disturbances[26]

$$du_+ = (1-\beta)dU \qquad \left\{\text{since} \quad u=(1-\beta)dU\right\}$$

Using equation (41) we have

$$du_+ = -(1-\beta)Y\left(\beta,\Gamma,Z_o,k_P,\bar{b}\right)U\frac{dr}{r} \tag{42}$$

The resultant fluid velocity increment will be due to both $C_-$ and $C_+$ disturbances. Therefore the net increase in fluid velocity [Yadav(1992)] is given by

$$du_- - du_+ = (1-\beta)\, dU^* \qquad \{\text{since } u = (1-\beta)\, dU\} \tag{43}$$

Where '*' represents the modified values of respective variables under the influence of overtaking waves.

On putting the values from (39) and (42) into equation equation (43), we have

$$\frac{dU^*}{U^*} + X^*\left(\beta, \Gamma, Z_o, k_P, \bar{b}\right)\frac{dr}{r} = 0 \tag{44}$$

where $X^*\left(\beta, \Gamma, Z_o, k_P, \bar{b}\right) = \left[X\left(\beta, \Gamma, Z_o, k_P, \bar{b}\right) - Y\left(\beta, \Gamma, Z_o, k_P, \bar{b}\right)\right]$

*(a) Expression for Modified Shock Velocity*

After simplifying, we have

$$\mathbf{U^* = Kr^{-X^*\left(\beta,\Gamma,Z_o,k_P,\bar{b}\right)}} \tag{45}$$

The above expression represents the shock velocity ($U^*$) modified by the effect of overtaking disturbances.

*(b) Expression for Modified Shock Strength :*

The expression for the shock strength modified by the overtaking disturbances($U^*/a_o$) may be obtained as

$$\frac{\mathbf{U^*}}{\mathbf{a_o}} = \mathbf{K^*}\sqrt{\frac{\left(\mathbf{1-Z_o}\right)}{\left\{\mathbf{\Gamma + \left(\Gamma - Z_o\right)\bar{b}\left(1-k_P\right)}\right\}}}\,\mathbf{r}^{-\mathbf{X^*\left(\beta,\Gamma,Z_o,k_P,\bar{b}\right)}} \tag{46}$$

where $K^* = K\sqrt{p_o/\rho_o}$

*(c) Expression for Modified Non-dimensional Pressure :*

The expression for non-dimensional pressure ($p^*/p_o$) in the region behind the shock in the influence of overtaking disturbances can be given by

Using equation (46), we have

$$\frac{\mathbf{p^*}}{\mathbf{p_o}} = \mathbf{\left(1-\beta\right)K^{*2}r^{-2X^*\left(\beta,\Gamma,Z_o,k_P,\bar{b}\right)}} \tag{47}$$

*(d) Expression for Modified Flow Velocity ;*

The expression for non-dimensional particle velocity (flow velocity) ($u^*/a_o$) in the region behind the shock in the influence of overtaking disturbances can be given by

$$\frac{u^*}{a_o}=(1-\beta)K^*\sqrt{\frac{(1-Z_o)}{\{\Gamma+(\Gamma-Z_o)\bar{b}(1-k_P)\}}}\,r^{-X^*(\beta,\Gamma,Z_o,k_P,\bar{b})} \tag{48}$$

## 4. Results and discussion

The study of the variation of shock velocity(U), shock strength($U/a_o$), non-dimensional pressure($p/p_o$) and flow velocity($u/a_o$) behind cylindrical imploding strong shock propagating adiabatically in a dusty real gas (mixture of non-ideal gas and small spherical solid inert particles) with constant initial density distribution is carried out for both the cases viz 1. Freely propagating and 2. Modified under the effect of overtaking disturbances (EOD). The solutions are presented for the case of cylindrical symmetry and the gas of adiabatic index $\gamma$=1.4. Parameters used in our study [Pai *et.al.*[13], Vishwakarma and Nath[21], Vishwkarma[20]]. Adiabatic index of gas $\gamma$ =1.4, Adiabatic index of the mixture $\Gamma$=1.36, density ratio $\beta$ = 0.1615, realness of the gas $\bar{b}$ = 0.01, 0.02, 0.03, 0.04, mass concentration of solid particles $k_p$ = 0.1, 0.2, 0.3, 0.4, 0.5 ….. , Specific density ratio $\sigma$ =50, 100, 200, 300, 500……The value $k_p$ = 0 corresponds to the dust free gas and $k_p$=0, $\bar{b}$ =0 to the ideal dust free gas.

### 4.1 *Shock Velocity Profile :*

The expressions for shock velocity just behind the imploding cylindrical strong shock, for freely propagating and under the effect of overtaking (EOD), are given by equation (34) and (45), respectively. These equations are used for computation. The solutions depend on the propagation distance (r), the parameter of realness ($\bar{b}$), the mass concentration of solid particles in the mixture ($k_p$), the density ratio between the solid and the real gas ($\sigma$) and the ratio of the specific heats of the mixture $\Gamma$.

#### *4.1.1 Variation of shock velocity (U) with propagation distance(r):*

Initially taking r =1.1, $\bar{b}$=0.01, $k_p$=0.1, $\sigma$ =50, $u/a_0$=12, $\gamma$ =1.36, $\beta$=0.1615 and K=9.9531, variation of Shock velocity (U) with propagation distance (r) has been computed from equation (34) & (45) and presented through figure (1). It is found that shock velocity increases as cylindrical imploding strong shock wave propagates in a uniform real dusty gas. Figure (1) depicts the variation of shock velocity with propagation distance (r) for different values of $\bar{b}$, $k_p$ and $\sigma$. It is observed that increase in velocity is enhanced when realness is taken in to account in comparison with ideal gas as in figure (1). The comparison of modified shock velocity under the effect of overtaking disturbances (EOD) and freely propagation shows that shock velocity increases more rapidly under the effect of overtaking disturbances as depicted through figures (1).

*4.1.2 Effect of realness ($\bar{b}$) on shock velocity (U) :*

The effect of parameter of realness ($\bar{b}$) of the mixture on the velocity of cylindrical imploding strong shock is calculated numerically and is shown through graph which is depicted in figure (1), which shows that shock velocity increases gradually with parameter of realness with some enhancement with effect of overtaking disturbances.

*4.1.3 Effect of mass concentration of dust particles ($k_p$) on shock velocity (U) :*

Effect of mass concentration of dust particles in the mixture on shock velocity behind the shock front may be calculated from equations (34) & ( 45) respectively for freely as well as under the effect of overtaking disturbances. Variation of shock velocity (U) and modified shock velocity ($U^*$) with concentration of solid particles ($k_p$) is shown in figure (2) which indicates that it increases with concentration of solid particles.

*4.1.4 Effect of specific density of dust particles (σ) on shock velocity (U):*

The variation of shock velocity with dust loading ratio (σ) is depicted through figure (3) and shows the decreasing pattern. The effect of overtaking disturbances has little impact on variation of shock velocity with realness while has some more effect on variation with concentration of solid particles but has considerable effect on variation with dust loading ratio.

*4.2 Shock Strength Profile :*

The expressions for shock strength behind the cylindrical imploding strong shock, for freely propagating and under the effect of overtaking (EOD), are given by equations (35) and (46), respectively.

*4.2.1 Variation of shock strength ($U/a_0$) with propagation distance(r) :*

Variation of shock velocity ($U/a_0$) for cylindrical strong shock imploding in the uniform medium is calculated from equation (35& 46). Initially taking r=1.1, $\bar{b}$ =0.01, $k_p$=0.1, σ =50, $U/a_0$=12, γ =1.36, β=0.1615 and K=9.9531, variation of shock strength with propagation distance (r) has been computed and presented through figures (4). The resultant modification in the strength of imploding shock ($U^*/a_0$) under EOD is estimated using equation (46). It is found that shock strength increases as cylindrical imploding strong shock wave propagates in a uniform real dusty gas. Figures (4) depicts the variation of shock strength with propagation distance (r) for different values of $\bar{b}$. It is observed that increase in strength is enhanced when realness is taken in to account in comparison with ideal gas [figure (4)]. The comparison of modified shock strength under the effect of overtaking disturbances and freely propagation shows that shock strength increases more rapidly under the effect of overtaking disturbances as depicted through figure (4).

*4.2.2 Effect of realness ($\bar{b}$) on shock strength ($U/a_0$) :*

The effect of parameter of realness ($\bar{b}$) of the mixture on the strength of cylindrical imploding strong shock for freely as well as modified case (EOD) is calculated numerically and shown through graph which is depicted in figure (4), which shows that shock is strengthened slowly with increase in the realness of gas with some inflation with effect of overtaking disturbances.

*4.2.3 Effect of mass concentration of dust particles ($k_p$) on shock strength ($U/a_o$) :*

From the figure (5), the variation of shock strength with concentration of solid particles ($k_p$) is represented. Which reports that strength of imploding cylindrical shock increases as concentration of solid particles ($k_p$) increases in the mixture. Under the effect of overtaking waves, it increases much rapidly.

*4.2.4 Effect of specific density of dust particles (σ) on shock strength ($U/a_o$) :*

The variation of shock strength ($U/a_0$) and its modified values ($U^*/a_0$) with dust loading ratio (σ) is depicted through figure (6) and shows the decreasing pattern. The effect of overtaking disturbances has little impact on variation of shock strength with realness while has some more effect on variation with concentration of solid particles but has considerable effect on variation with dust loading ratio.

*4.3 Pressure Profile :*

The expressions for pressure behind the cylindrical imploding strong shock, for freely propagating and under the effect of overtaking (EOD), are given by equation (36) and (47), respectively

*4.3.1 Variation of pressure ($p/p_0$) with propagation distance(r) :*

From the expressions (36) and (47), the variation of non-dimensional pressure both for freely ($p/p_0$) and under EOD ($p^*/p_0$) is numerically calculated. Initially taking $r = 1.1$, $\bar{b} = 0.01$, $k_p = 0.1$, $\sigma = 50$, $U/a_0 = 12$, $\gamma = 1.36$, $\beta = 0.1615$ and $K = 9.9531$, The variation of pressure with propagation distance (r) has been computed and represented through figure (7). It is found that pressure increases as cylindrical imploding strong shock wave propagates in a uniform real dusty gas. Figure (7) depicts the variation of pressure with propagation distance (r) for different values of $\bar{b}$, $k_p$ and σ. It is analyzed that increase in pressure is inflated when realness is taken in to account in comparison with ideal gas [figure (7)]. The comparison of modified pressure under the effect of overtaking disturbances and freely propagation shows that pressure increases more speedily under the effect of overtaking disturbances as displayed through figures (7).

*4.3.2 Effect of realness ($\bar{b}$) on pressure ($p/p_o$) :*

The variation of pressure with realness ($\bar{b}$) is shown through graph which is described in figure (7), which shows that non-dimensional pressure increases gradually with realness with some enhancement with effect of overtaking disturbances.

*4.3.3 Effect of mass concentration of dust particles ($k_p$) on pressure ($p/p_0$) :*

The variation of non-dimensional pressure ($p/p_0$) and modified pressure ($p^*/p_0$) for the strong cylindrical imploding shock with concentration of solid particles ($k_p$) is exhibited in figure (8) which indicates that it increases with concentration of solid particles.

*4.3.4 Effect of specific density of dust particles (σ) on pressure ($p/p_0$) :*

The variation of pressure with dust loading ratio (σ) is depicted through figure (9) and portray the decreasing pattern. The effect of overtaking disturbances has little impact on variation of pressure with realness while it has some more effect on variation with concentration of solid particles but has noticeable effect on variation with dust loading ratio.

*4.4 Flow Velocity Profile :*

The expressions for Flow velocity behind the cylindrical imploding strong shock, for freely propagating and under the effect of overtaking (EOD), are given by equation (37) and (48) respectively.

*4.4.1 Variation of flow velocity ($u/a_0$) with propagation distance(r) :*

For the purpose of numerical calculations, taking initially r=1.1, $\bar{b}$=0.01, $k_p$= 0.1, σ =50, $U/a_0$=12, γ=1.36, β= 0.1615 and K=9.9531. Variation of flow velocity with propagation distance (r) has been computed and displayed through figure (10). It is found that flow velocity increases as cylindrical imploding strong shock wave moves in a uniform real dusty gas. Figure (10) render the variation of flow velocity with propagation distance (r) for different values of $\bar{b}$, $k_p$ and σ. It is observed that increase in flow velocity is strengthened when non idealness is taken in to account in comparison with ideal gas figure (10). The comparison of modified flow velocity under the effect of overtaking disturbances and freely propagation shows that flow velocity increases more speedily under the effect of overtaking disturbances as demonstrated through figure (10).

*4.4.2 Effect of realness ($\bar{b}$) on flow velocity ($u/a_0$) :*

Variation of flow velocity ($u/a_0$) and under EOD modified flow velocity ($u^*/a_0$) with increase in parameter of realness ($\bar{b}$) at r=1.1, is represented through graph which is depicted through figure (10), which shows that flow velocity increases gently with realness with some magnification with effect of overtaking disturbances.

*4.4.3 Effect of mass concentration of dust particles ($k_p$) on flow velocity ($u/a_0$) :*

The variation of non-dimensional flow velocity ($u/a_0$) and modified non-dimensional flow velocity with EOD ($u^*/a_0$) with concentration of solid particles ($k_p$) is exhibited in figure (11) which indicates that it increases with concentration of solid particles.

#### *4.4.4 Effect of specific density of dust particles (σ) on flow velocity ($u/a_0$):*

The variation of flow velocity ($u/a_0$) and modified non-dimensional flow velocity with EOD ($u^*/a_0$) with dust loading ratio (σ) is shown through figure (12) and portray the decreasing pattern. The effect of overtaking disturbances has little impact on variation of flow velocity with realness while has some more effect on variation with concentration of solid particles but has considerable effect on variation with dust loading ratio.

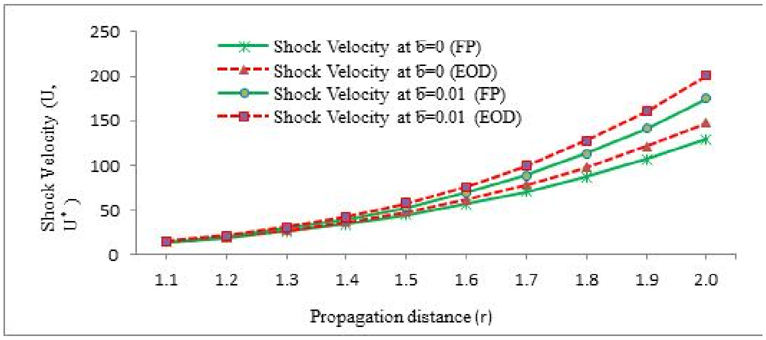


Figure 1: Variation of shock velocity (U, $U^*$) with propagation distance (r) for $k_p$=0.1, σ=50

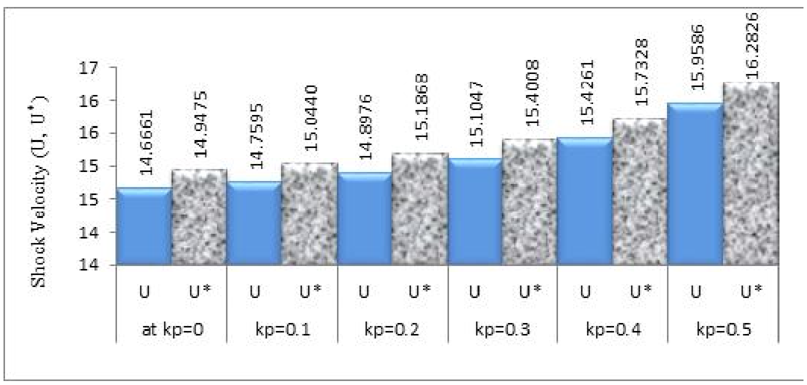


Figure 2: Variation of shock velocity with concentration of solid particles ($k_p$) for $\bar{b}$=0.01, σ =5

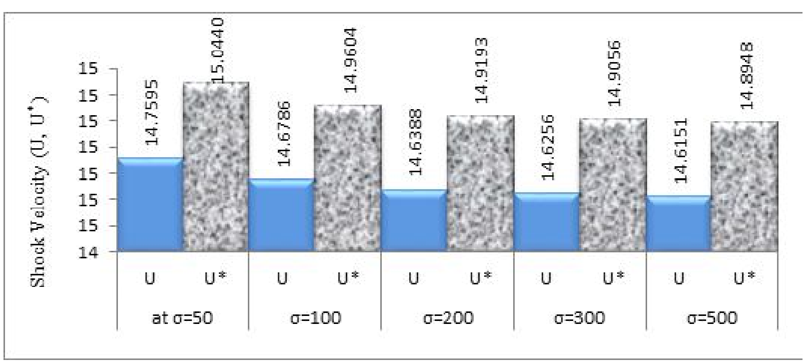


Figure 3: Variation of shock velocity with dust loading ratio (σ) for $\bar{b}$ =0.01, $k_p$= 0.1

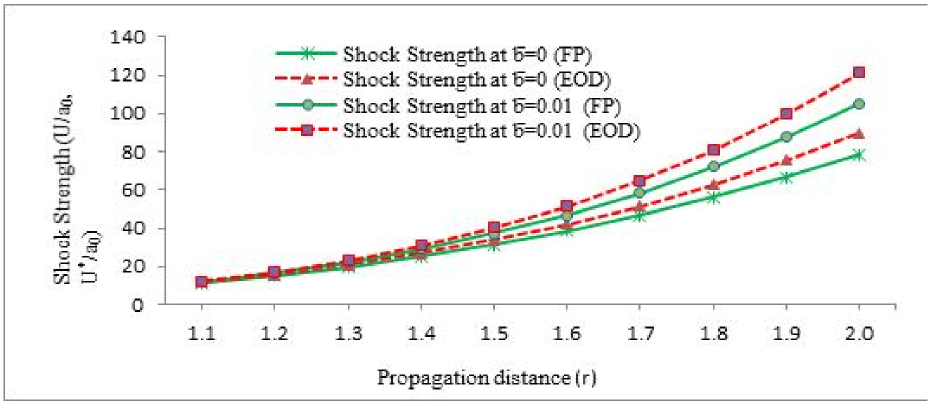


Figure 4: Variation of shock strength ($U/a_0$, $U^*/a_0$) with propagation distance (r) for $k_p$=0.1, σ =50

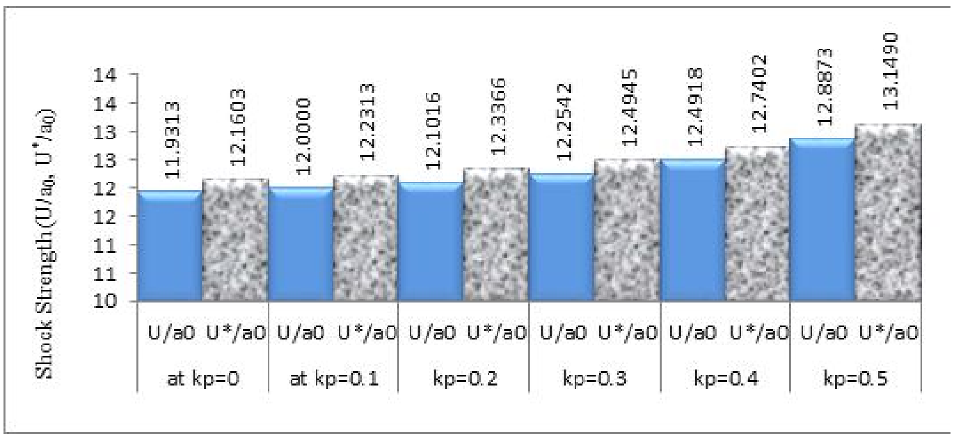


Figure 5: Variation of shock strength with concentration of solid particles ($k_p$) for $\bar{b}$=0.01, σ =50

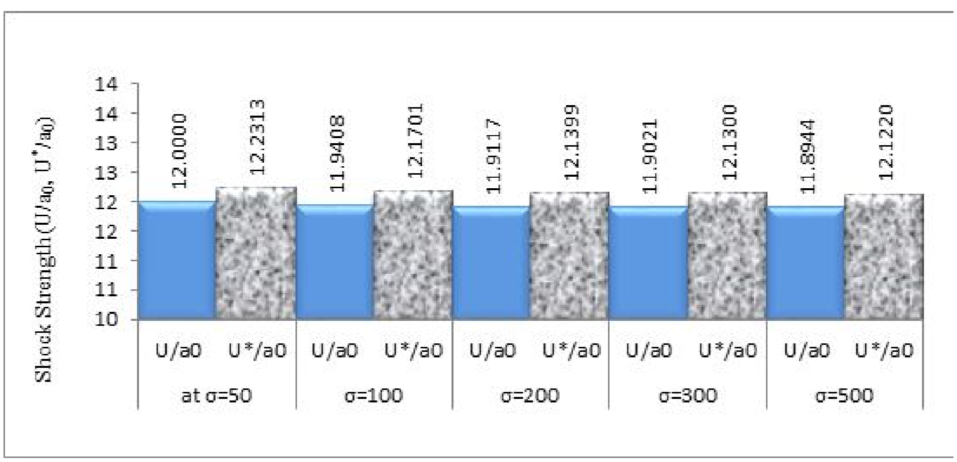


Figure 6: Variation of shock strength with dust loading ratio (σ) for $\bar{b}$=0.01, $k_p$=0.1

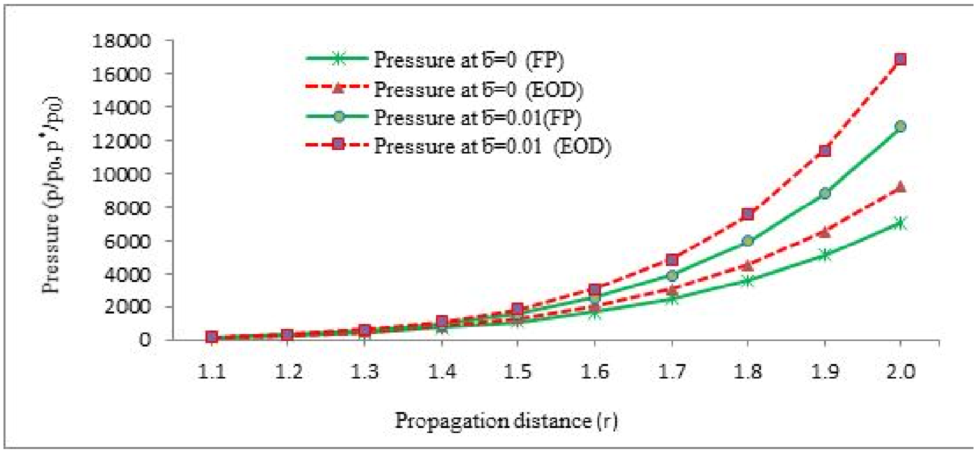


Figure 7: Variation of pressure ($p/p_0$, $p^*/p_0$) with propagation distance (r) for $k_p$=0.1, σ =50

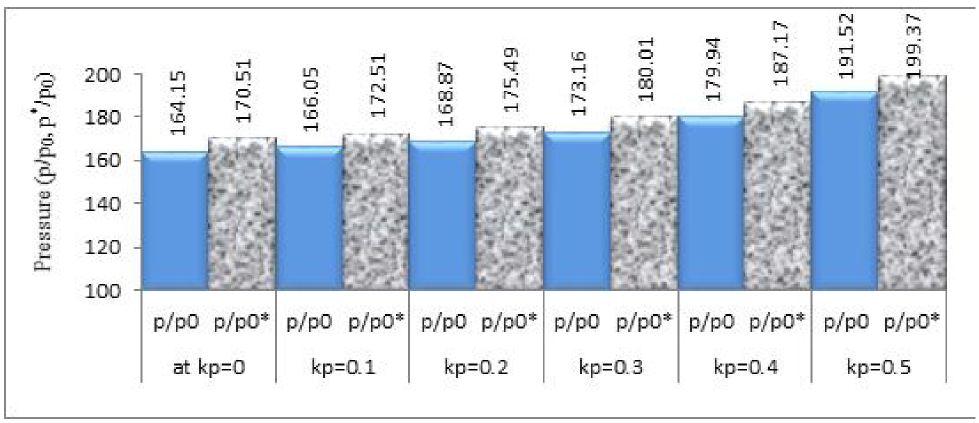


Figure 8: Variation of pressure with concentration of solid particles ($k_p$) for $\bar{b}$=0.01, σ=50

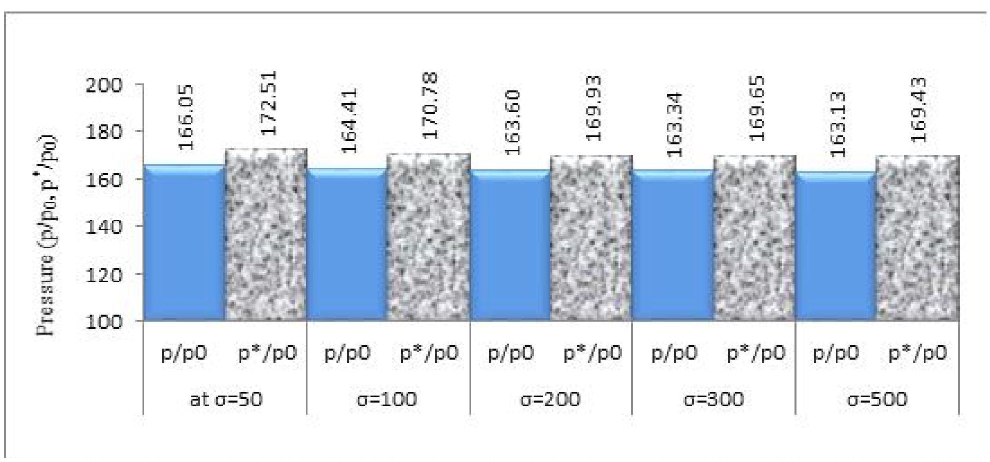


Figure 9: Variation of pressure with dust loading ratio (σ) for $\bar{b}$=0.01, $k_p$=0.1

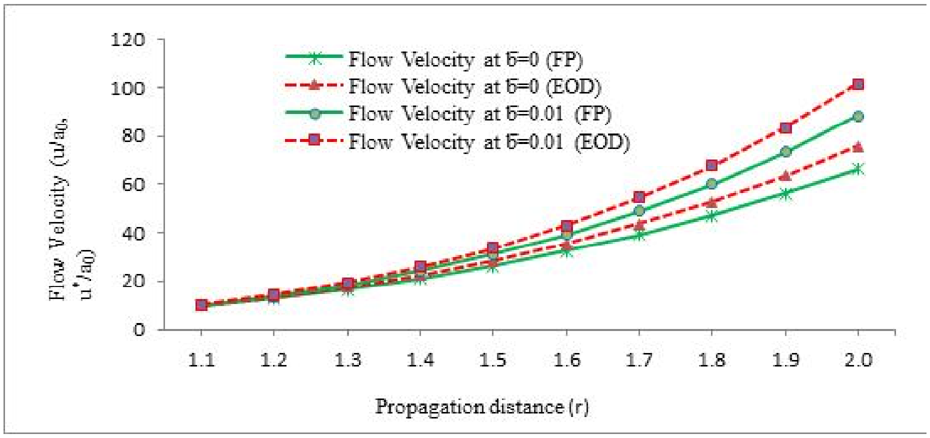


Figure 10: Variation of flow velocity ($u/a_0$, $u^*/a_0$) with propagation distance (r) for $k_p$=0.1, σ=50

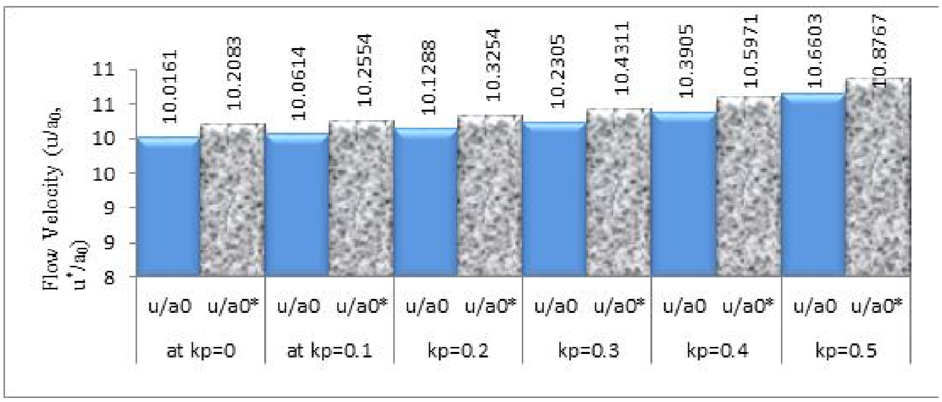


Figure 11: Variation of flow velocity with concentration of solid particles ($k_p$) for $\bar{b}$=0.01, σ=50

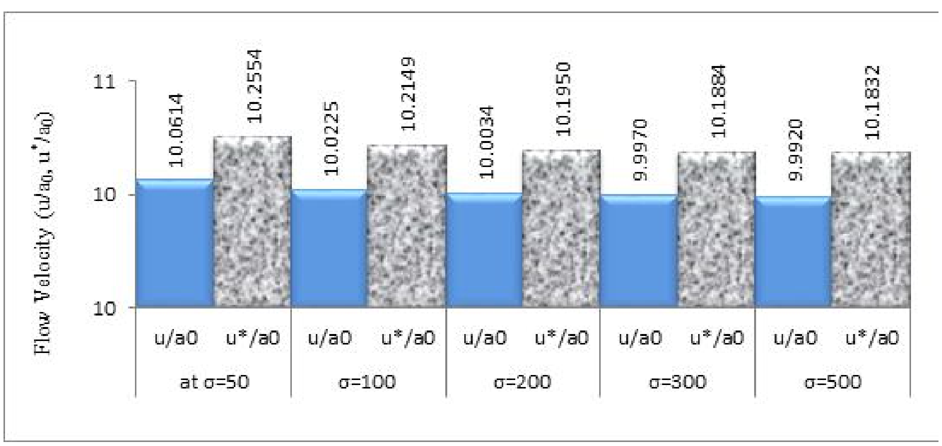


Figure 12: Variation of flow velocity with dust loading ratio (σ) for $\bar{b}$ =0.01, $k_p$ =0.1

## Conclusion

The problem of shock wave propagation a mixture of real gas and small spherical solid particles having uniform initial density distribution has been carried out by using well known CCW method. It is found that the presence of dust particles plays a significant effect on all flow parameters behind the strong shock front. The strength of shock wave increases on adding dust particles in real gas. The role of overtaking waves is also calculated numerically and its presence also increases the flow parameters.

**Scope of Future work:** In the present study, we have considered only the uniform initial density distribution of medium. Authors plan to implement CCW method to investigate the different type of density distribution as well as the effect of rotation and self-gravitation on shock propagation in dusty gas, will form the content of future work.

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