ISSN 2231-3478 (Print)

JUSPS-B Vol. 35(2), 16-23 (2023). Periodicity-Monthly

Section B

(Online) ISSN 2319-8052

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES

An International Open Free Access Peer Reviewed Research Journal of Physical Sciences

website:- www.ultrascientist.org

Estd. 1989

Inflationary Scenario in Bianchi Type – V Cosmological Model with Bulk Viscosity in General Relativity

JYOTI SINGH[1], ATUL TYAGI[1], JAIPAL SINGH[2*] and DHIRENDRA CHHAJED[1]

[1]Department of Mathematics and Statistics, University College of Science,
MLS University, Udaipur- 313001 (India)
[2]Department of Basic Science Mathematics, CTAE, College MPUAT University,
Udaipur- 313001 (India)
Corresponding Author Email: jaipalsingh075@gmail.com
http://dx.doi.org/10.22147/jusps-B/350201

Acceptance Date 18th March, 2023, Online Publication Date 30th March, 2023

Abstract

In present paper we have investigated the inflationary scenario in Bianchi Type V cosmological model under the effect of bulk viscosity in general relativity. To get deterministic model of the universe, we assume the shear (σ) is proportional to the expansion (θ) by which a condition $A_1 = (A_2 A_3)^n$ between metric potentials is obtained. We consider the region is flat which leads to the scalar potential $V(\phi)$ is constant. We found the spatial volume increases with time, which represent the inflationary scenario in the model. Some physical and geometrical properties of the model are also discussed in detail. The model is found to be compatible with the results of recent observations.

Key word : Bianchi Type-V, bulk viscous fluid, inflationary, general relativity.

1. Introduction

Bianchi Type-V space-times are interesting to the study because of their richer structure both physically and geometrically than standard Friedmann–Robertson–Walker (FRW) models and significance of Bianchi type-V model is that the space of constant negative curvature is contained in it as a special case. A general orthogonal Bianchi type-V space time model containing stiff matter and source-free electromagnetic field considered by Roy and Singh[17]. In order to understand in a better way the observed small amount of anisotropy in the universe, several authors have studied Bianchi Type Models. Bali *et al.*,[6] examined a cosmological scenario proposing a variation law for Hubble parameter H in the background of homogeneous, anisotropic Bianchi type-V space-time. The Bianchi type-V universe is generalization of the open universe in FRW cosmology. Adhav *et al.*,[1] studied the space-time geometry corresponding to Bianchi type-V filled with perfect fluid and dark energy in five dimensions.The Bianchi Type-V universe has been considered for a mixture of a perfect fluidand dark energy given by cosmological constant by Singh and Chaubey[23].

Pradhan and Srivastava[15] described a new class of conformally flat tilted Bianchi type-V magnetized cosmological models with a bulk viscous fluid as the source of matter. In most cosmological models the universe can be described by dust or at the early stages of universe viscous effects do play a role. Bali and Singh[7] investigated Bianchi Type-V bulk viscous fluid string & dust cosmological model in General Relativity. Bulk viscosity driven inflation is primarily due to the negative bulk viscous pressure giving rise to a total negative effective pressure which may overcome the pressure due to the usual gravity of matter distribution in the universe.Singh *et al.*,[22] presented four models of Bianchi type-V cosmological solutions to field equation with viscous fluid in the presence of a cosmological term Λ in general relativity. The behaviour of a viscous fluid with a cosmological constant in the framework of a Bianchi type-V space time discussed by Kandalkar *et al.*,[11]. Tiwari and Tiwari[26] investigate Bianchi type-V cosmological models with bulk viscous fluid distribution in general relativity.

Bulk viscous models have prime roles in getting inflationary phases of the universe.Inflation is the extremely rapid exponential expansion of the early universe by a factor of at least 10^{78} in volume driven by a negative pressure vacuum energy density. Following the inflationary period, the universe continued to expand but at a slower rate. The inflationary hypothesis was originally proposed by Guth[9] who named it inflation. In particular, self-interacting scalar fields play a central role in the study of inflationary cosmology. The idea of inflation can be realised in a much better way in the chaotic inflation scenario which was not based on the theory of high temperature phase transitions in the early universe discovered by Linde[13]. Katore *et al.*,[12] obtained a five-dimensional inflationary universe in the presence of massless scalar field.

An inflationary stage is a very general property of the solutions concluded by Belinski and the concept of quantum "creation" of the universe with a subsequent inflationary stage probably does not require fulfilment of any too special conditions.The inflationary scenario is a satisfactory solution to some of the conceptional issues in cosmology, but it is not understood in the standard Big-bang theory. Bali and Poonia[5] solved inflationary cosmological model for bianchi type-VI_0. Bali and Kumari[2] found inflationary scenario in Bianchi type-VI_0 space time with flat potential and bulk viscosity in general relativity. Recently Sharma *et al.*,[19] derived synchronized bianchi space-VI cosmological model in existence of flat potential under scalar fields which is purely mass less and in which $V(\phi)$ is constant.Bianchi type-VI_0 cosmological model with bulk viscosity and dust distribution in C-field theory investigated by Singh *et al.*[21].

The system of fields equations are basically set of non-linear differential equations we required solutions in various applications in astrophysics and cosmology. Singh and Kumar[20] stated that during the inflationary era the dominant contributions to the cosmic energy density came from a vacuum energy and a scalar field. This has been designed as a two-component fluid, consisting of a vacuum fluid and a Zel'dovich fluid. Tinker[25]has considered a scalar potential as an exponential function of Higg's field (ϕ), for the study of an inflationary framework in spatially homogeneous and anisotropic Bianchi Type-II space-time and tried to describe that inflationary framework can be proposed for an anisotropic and homogeneous metric with exponential potential and the model attains isotropy in a special case.

The explanation for the flatness of the universe is that it has undergone a period of exponential expansion at its early stage of evolution. Reddy[16] studied Bianchi type-V anisotropic, homogeneous and inflationary cosmological model in the presence of mass less scalar field with a flat potential. The inflation will take place if the potential $V(\phi\,)$ has a 'flat' region and in this region the ϕ field evolves slowly but the universe expands in an exponential way due to vacuum field energy by Stein-Schabes[24]. It is known that self-interacting scalar fields contribute significantly in the study of inflationary cosmology. Locally symmetric Bianchi Type-I space time undertaken in framework of massless scalar field with flat potential is constructed by Sharma and Poonia[18]. Bali and Jain[8] investigated Bianchi type-I inflationary cosmological model in the presence of massless scalar field with a flat potential.

In this paper, we have investigated the inflationary scenario in Bianchi Type V cosmological model under the effect of bulk viscosity in general relativity. To get deterministic model of the universe, we assume the shear (σ) is proportional to the expansion $(\theta\,)$ by which a condition $A_1 = (A_2A_3)^n$ between metric potentials is obtained. We consider the region is flat which leads to the scalar potential $V(\phi\,)$ is constant. We found the spatial volume increases with time, which represent the inflationary scenario in the model. Some physical and geometrical properties of the model are also discussed in detail, which supposed by Bali and Swati[3], Bali and Poonia[4], Poonia and Sharma[14] and Jat *et al.*[10].

2. Metric and Field Equations :

We Consider Bianchi Type-V line-element in orthogonal form as

$$ds^2 = -dt^2 + {A_1}^2dx^2 + e^{2x}{A_2}^2dy^2 + e^{2x}{A_3}^2dz^2 \tag{1}$$

Where A_1, A_2 and A_3 are metric potentials and are function of t-alone.
We assume the co-ordinates to be commoving so that

$$v^1 = v^2 = v^3 = 0,\ \ v^4 = 1.$$

The action of gravitational field minimally coupled to a scalar field with potential $V(\phi\,)$ is givenby Stien-Schabes[24] as

$$L = \int \sqrt{-g}\,[R - \frac{1}{2}g^{ij}\,\partial_{ij}\,\phi\partial_{ij}\,\phi - V(\phi)]d^4x \tag{2}$$

The Einstein's field equations (in the gravitational unit $8\pi G=1$) in case of massless scalar field ϕ with potential $V(\phi)$ are given by

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \tag{3}$$

With energy momentum tensor (T_{ij}) for scalar field in presence of viscosity is given by

$$T_{ij} = (\rho + p)v_i v_j + pg_{ij} + \partial_i\phi\partial_j\phi - \left[\frac{1}{2}\partial_\rho\phi\partial^\rho\phi + V(\phi)\right]g_{ij} - \xi\theta(g_{ij} + v_i v_j) \tag{4}$$

Where V is the effective potential, ϕ is Higgs field, ξ is the coefficient of bulk viscosity and θ is the expansion in the model.

The energy conservation law coincides with the equation of motion for ϕ and we have

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\partial^\mu\phi\right) = -\frac{dV}{d\phi} \tag{5}$$

Where scalar field ϕ is the function of t-alone.

The Einstein field equation (3) for the metric (1) and energy momentum tensor (4) leads to the following system of equations

$$\frac{\ddot{A_2}}{A_2} + \frac{\ddot{A_3}}{A_3} + \frac{\dot{A_2}\dot{A_3}}{A_2A_3} - \frac{1}{{A_1}^2} = -\frac{\dot{\phi}^2}{2} + V(\phi) - (p - \xi\theta) \tag{6}$$

$$\frac{\ddot{A_1}}{A_1} + \frac{\ddot{A_3}}{A_3} + \frac{\dot{A_1}\dot{A_3}}{A_1A_3} - \frac{1}{{A_1}^2} = -\frac{\dot{\phi}^2}{2} + V(\phi) - (p - \xi\theta) \tag{7}$$

$$\frac{\ddot{A_1}}{A_1} + \frac{\ddot{A_2}}{A_2} + \frac{\dot{A_1}\dot{A_2}}{A_1A_2} - \frac{1}{{A_1}^2} = -\frac{\dot{\phi}^2}{2} + V(\phi) - (p - \xi\theta) \tag{8}$$

$$\frac{\dot{A_1}\dot{A_2}}{A_1A_2} + \frac{\dot{A_1}\dot{A_3}}{A_1A_3} + \frac{\dot{A_2}\dot{A_3}}{A_2A_3} - \frac{3}{{A_1}^2} = \rho + \frac{\dot{\phi}^2}{2} + V(\phi) \tag{9}$$

$$2\frac{\dot{A_1}}{A_1} - \frac{\dot{A_2}}{A_2} - \frac{\dot{A_3}}{A_3} = 0 \tag{10}$$

The dot $(\cdot)$ denotes ordinary differentiation with respect to t.

3. Solutions of Field Equations :

We are interested in inflationary solution so flat region is considered. Thus $V(\phi)$ is constant (K). Now by equation (5) for the scalar field (ϕ) leads to

$$\ddot{\phi} + \left[\frac{\dot{A_1}}{A_1} + \frac{\dot{A_2}}{A_2} + \frac{\dot{A_3}}{A_3}\right]\dot{\phi} = 0 \tag{11}$$

Equation (11) leads to

$$\dot{\phi} = \frac{l}{A_1A_2A_3} \tag{12}$$

Where l is constant of integration.
Equation (10) leads to

$$A_1{}^2 = A_2 A_3 \tag{13}$$

From equations (7) and (8), we obtain

$$\frac{\ddot{A_2}}{A_2} - \frac{\ddot{A_3}}{A_3} + \frac{\dot{A_1}}{A_1}\left[\frac{\dot{A_2}}{A_2} - \frac{\dot{A_3}}{A_3}\right] = 0 \tag{14}$$

Einstein field equations (6)–(10) with unknowns $A_1, A_2, A_3, \phi, \rho, p, \xi, \theta$. To get the deterministic solution, we assume that shear (σ) is proportional to expansion (θ) which leads to the condition between metric potential

$$A_2 = (A_3)^n \tag{15}$$

Using equation (15) in (14) we obtain

$$(n-1)\frac{\ddot{A_3}}{A_3} + \frac{(n-1)(3n+1)}{2}\frac{\dot{A_3}{}^2}{A_3{}^2} = 0 \tag{16}$$

Which leads to

$$\frac{\ddot{A_3}}{\dot{A_3}} = -\frac{(3n+1)}{2}\frac{\dot{A_3}}{A_3} \tag{17}$$

From equation (17), we get

$$\dot{A_3} = \frac{m_1}{A_3{}^{(3n+1)/2}} \tag{18}$$

Which leads to

$$A_3{}^{3(n+1)/2} = \gamma t + \delta \tag{19}$$

Where

$$\frac{3(n+1)}{2}m_1 = \gamma \quad \text{and} \quad \frac{3(n+1)}{2}m_2 = \delta \tag{20}$$

Thus, the metric potential are as follows

$$A_1 = A_3{}^{(n+1)/2} = [\gamma t + \delta]^{1/3} \tag{21}$$

$$A_2 = A_3{}^n = [\gamma t + \delta]^{2n/3(n+1)} \tag{22}$$

$$A_3 = [\gamma t + \delta]^{2/3(n+1)} \tag{23}$$

Therefore the metric (1) leads to

$$ds^2 = -dt^2 + [\gamma t + \delta]^{2/3}dx^2 + e^{2x}[\gamma t + \delta]^{4n/3(n+1)}dy^2 + e^{2x}[\gamma t + \delta]^{4/3(n+1)}dz^2 \tag{24}$$

4. Some Physical and Geometrical Aspects :

For the model (24), the rate of Higg's field

$$\phi = \frac{l}{\gamma}\log[\gamma t + \delta] + M \tag{25}$$

Where M is constant of integration.

The Spatial volume (R^3) for the model (24) is

$$R^3 = A_1A_2A_3 = \gamma t + \delta \tag{26}$$

The expansion (θ) is

$$\theta = \frac{\dot{A}_1}{A_1} + \frac{\dot{A}_2}{A_2} + \frac{\dot{A}_3}{A_3} = \frac{\gamma}{[\gamma t + \delta]} \tag{27}$$

The shear (σ) for the model (24) is

$$\sigma = \frac{1}{\sqrt{3}}\left[2\frac{\dot{A}_2}{A_2} - \frac{\dot{A}_3}{A_3}\right] = \frac{1}{\sqrt{3}}\frac{2\gamma(2n-1)}{3(n+1)[\gamma t + \delta]} \tag{28}$$

Thus

$$\frac{\sigma}{\theta} = constant \neq 0 \tag{29}$$

The decelerating parameter (q) is

$$q = -\frac{\ddot{R}/R}{\dot{R}^2/R^2} = 2 \tag{30}$$

The Hubble parameter (H) is

$$H = \frac{\theta}{3} = \frac{\gamma}{3[\gamma t + \delta]} \tag{31}$$

The energy density (ρ) is

$$\rho = \frac{\dot{A}_1\dot{A}_2}{A_1A_2} + \frac{\dot{A}_1\dot{A}_3}{A_1A_3} + \frac{\dot{A}_2\dot{A}_3}{A_2A_3} - \frac{3}{{A_1}^2} - \frac{\dot{\phi}^2}{2} - V(\phi)$$

Which leads to

$$\rho = \frac{4\gamma^2 - 9l^2}{18[\gamma t + \delta]^2} + \frac{4n\gamma^2}{9(n+1)^2[\gamma t + \delta]^2} - \frac{3}{[\gamma t + \delta]^{2/3}} - k \tag{32}$$

5. Conclusion

We found that the spatial volume for the model (24), increases with time. When $t \to \infty$ then the spatial volume $R^3 \to \infty$. Thus inflation is possible in Bianchi type V model with a massless scalar field in the potential which has flat space region. The deceleration parameter $q > 0$, hence the universe is decelerating. Thus the model represents that the universe is not only expanding but decelerating also. Since $\frac{\sigma}{\theta} \neq 0$ in general, therefore anisotropy is maintained. The model has Point type singularity at $t = -\delta/\gamma$. The presence of bulk viscosity tends to increase the inflationary phase. The Hubble parameter decreases with time.

Scope and application:- The inflationary scenario explains several problems of modern cosmology like homogeneity, isotropy, flatness and horizon problem of the observed universe. The present investigation will be very helpful to the researchers who are engaged for research work in inflationary cosmological models. The present study did not receive any financial aid from any government / non-government agency.

References

1. Adhav, K.S. *et. al*. Higher Dimensional Bianchi Type-V Cosmological Models with Perfect Fluid and Dark Energy, Int. J Theor Phys., *50,* 2573–2581 (2011).
2. Bali R. and Kumari P., Inflationary Universe Scenario in Bianchi Type VI_0 Space Time with Flat Potential and Bulk Viscosity in General Relativity, IOSR-JAP, *9(1),* 34-40 (2017).
3. Bali, R. and Swati, Bianchi Type VIII Inflationary Universe with Massless Scalar Field in General Relativity, Prespacetime Journal, *6(8),* 679-683 (2015).
4. Bali, R. and Poonia, L., Bianchi Type-III Inflationary Cosmological Model with Bulk Viscosity in General Relativity, Prespacetime Journal, *5(10),* 981-986 (2014).
5. Bali, R. and Poonia, L., Bianchi type VI_0 inflationary cosmological model inGeneral relativity, Int. J. Mod. Phys., *22,* 593–602 (2013).
6. Bali, R., Singh, P. and Singh, J.P., Bianchi Type V Viscous Fluid Cosmological Models in Presence of Decaying Vacuum Energy, Astrophys. Space Sci., *341,* 701-706 (2012).
7. Bali, R. and Singh, D.K., Bianchi Type-V Bulk Viscous Fluid String Dust Cosmological Model in General Relativity, Astrophysics and Space Science, *300,* 387–394 (2005).
8. Bali, R. and Jain, V.C., Bianchi type I inflationary universe in general relativity, Pramana journal of physics, 59(1), 1-7 (2002).
9. Guth, A., Inflationary universe: A possible solution to the horizon and flatness problems, Physical Review D, *23(2),* 347-356 (1981).
10. Jat, L.L., Tyagi A. and Tripathi, B.R., Bianchi Type-I Inflationary Cosmological Models with Constant Deceleration Parameter For Perfect Fluid Distribution in General Relativity, JUSPS-A, *34(1),* 20-27 (2022).
11. Kandalkar, S., Khade, P. and Gaikwad, M., Bianchi-V Cosmological Models with Viscous Fluid andConstant Deceleration Parameter in General Relativity, Turk J Phys., *36,* 141-154 (2012).
12. Katore, S.D., *et. al.*, A higher-dimensional Bianchi type-I inflationary Universein general relativity, Pramana journal of physics, *78(1),* 101–107 (2012).
13. Linde, A., The Inflationary Universe, Rep. Prog. Phys., *47,* 925-986 (1984).
14. Poonia, L. and Sharma, S., Inflationary Scenario in Bianchi Type II Space with Bulk Viscosity in General Relativity, Annals of R.S.C.B., *25(2),* 1223-1229 (2021).
15. Pradhan, A. and Srivastava, S.K., Tilted Bianchi Type V Bulk Viscous Cosmological Models in General Relativity, Romanian Reports in Physics, *59(3),* 749–761 (2007).
16. Reddy, D.R.K., Bianchi Type-V Inflationary Universe in GeneralRelativity, Int. J. Theor Phys., *48,* 2036–2040 (2009).
17. Roy, S.R. and Singh, J.P., A Bianchi Type VUniverse with Stiff Fluid and Electromagnetic Radiation, Aust. J. Phys., *38,* 763-768 (1985).

18. Sharma, S and Poonia, L., Bianchi Type I Inflationary Cosmological Model with BulkViscosity in General Relativity, Journal of Xi'an University of Architecture & Technology, *11(12),* 1348-1354 (2019).
19. Sharma, S., Poonia, L. and Kumawat, S., Bianchi -VI inflationary model with flat potential in generalrelativity, Journal of Physics, 1921 (2021).
20. Singh, C.P. and Kumar, S., Bianchi Type-II inflationary models with constantdeceleration parameter in general relativity, Pramana journal ofphysics, *68(5),* 707-720 (2007).
21. Singh, J., Tyagi, A., & Singh, G.P., Bianchi Type- VI_0 Cosmological Model with Bulk Viscosity & Dust Distribution in C-field Theory, Prespacetime Journal, *11(7),* 617-625 (2020).
22. Singh, J.P., Baghe, P. S. and Singh A., Viscous Fluid Bianchi Type V Cosmological Models with Late Time Acceleration, EJTP, *13(36),* 117–130 (2016).
23. Singh, T. and Chaubey, R., Bianchi Type-V Model with A Perfect Fluid and -Term, Pramana journalof physics, *67(3),* 415-428 (2006).
24. Stein-Schabes, J.A., Inflation in Spherically Symmetric Inhomogeneous Models, FERMILAB, 86 (1986).
25. Tinker, S., Inflationary Bianchi type-II space-time with exponentialpotential, GANITA, *71(1),* 235-242 (2021).
26. Tiwari L.K. and Tiwari R.K., Bianchi Type-V Cosmological Models with Viscous Fluid & Varying E, Prespacetime Journal, *8(14),* 1509-1520 (2017).