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Overtaking Effects on Spherical Shock Wave Propagation in Metallic Mediums

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Abstract

The propagation of spherical shock waves through metallic mediums have been considered in this paper. The effect of overtaking disturbances in the propagation of spherical shock waves through solids, especially metals such as Aluminium and Copper, has been analysed by employing Chisnell-Chester-Whitham method. Under the influence of spherical shock waves flow variables have been obtained analytically as well as estimated numerically. Expressions for the pressure and the particle velocity behind the shock wave have also been obtained. It is found that shock strength in both the metals decreases as shock propagates through them. Shock velocity and modified shock velocity depict a decreasing pattern with increasing propagation distance. These observations suggest many industrial and medical applications of shock waves.

Key words : Shock waves, shock strength, overtaking disturbances, picosecond laser, peak shock pressure.

1. Introduction

The study of shock waves propagation in gases, liquids and solids have gained importance due to its application in many real-life problems, particularly in the medical field for the treatment of various diseases. To be specific, shock wave assisted lithotripsy, probably one of the most handy and indubitable treatments for kidney stones and gall bladder diseases is extensively used in modern times. The shock waves used for this purpose are of microsecond duration with peak pressures lying in the range 35 to 120 MPa in a single pulse. Other notable applications of shock waves in the field of medical include the treatment of pancreatic and salivary stones, and in orthopedics too these have shown their utility. Further, these shock waves are also employed for the treatment of nephrolithiasis and orthopedic diseases such as nonunion of bone fractures and heel spur pain.

The pressure and temperature of the shock waves generated by a picosecond laser pulse in the layered semiconductor gallium selenide (GaSe) were determined by Leung *et al*.[1]. The peak pressure was measured to be about 13 kilobar, while the temperature remained the same during the measured time range. Laser-driven shock wave propagation in a transparent material such as plexiglass (a transparent thermoplastic homopolymer) coated with a thin over layer of gold was studied by Lalitha Dhareshwar et al. using the technique of high speed optical shadowgraphy[2]. Shock pressures and scaling of pressure with laser intensity were found to be in line with the values obtained through simulations. Shock pressures in gold-coated plexiglass target was observed to be considerably higher compared to those in uncoated targets. Fracture of quasi-brittle material such as concrete and rocks is known to result from the formation and development of microcracks. Microcracks reduce the macroscopic value of Young's modulus and therefore the corresponding wave velocity, possibly giving way to shock wave formation. Masumi et al. studied the phenomenon for granite as an example of quasi-brittle materials[3]. It was shown that the shape of the incident wave has a significant effect on the shock wave so formed. The investigation of the head-on collision of planar shock waves with rigid porous materials was investigated by Ben-Dor *et al*.[4].

Rutkevich *et al*. obtained and applied a thermodynamic description of metals in the range of high pressure and temperature to the problem of the stability of strong shock waves under spontaneous emission of sound[5]. A three-term form of the equation of state was employed to evaluate the contributions of the cold elastic pressure, the thermal atomic pressure, and the thermal pressure of the free electrons to the total pressure. The full determination of the equation of the state was performed from the experimental Hugoniot data and from a model connecting the atomic Gruneisen parameter and the atomic cold pressure. From the equation of state, the behavior of the Mach number, the temperature, and the entropy along the Hugoniot adiabatic was obtained and analyzed.

The studies of propagation of stress waves induced in solid material by shock wave has been investigated by Yosuke *et al*.[6]. An analytical model for laser-generated shock waves in solids has been investigated by Romain *et al*.[7]. The interpretation was given of the irradiated target surface. A comprehensive numerical simulation study of laser-driven shock wave propagation in planar aluminium foils has been investigated by Senecha *et al*.[8]. The effect of the spatial mesh size on the shock velocity and peak shock pressure was found to be significant in the quoted study. They concluded that simulations can be used as an effective tool for benchmark calculation of laser absorption coefficients. When a dynamic load is applied to solid object, stress shock wave propagates in the solid. When the stress wave exceeds proper yield condition of the solid, plastic deformation and fractures are generated. Shock wave propagation in the solids has been carried out by Koichi[9]. Huang *et al*. studied the shock wave formation and propagation in two dimensional granular materials under vertical vibration by digital high-speed photography by exploring the effect of driving parameters and particle number on the shock[10].

The study of motion of shock waves through certain metals was undertaken by Yadav *et. al.*[11]. The propagation of shock wavs in uniform real gas atmosphere for plane, cylindrical and spherical symmetries of the wave was also studied by the group of Yadav *et. al.*[12]. Rana et al. studied the plane hydromagnetic shock waves through the uniform and non-uniform media and observed some interesting and peculiar results[13]. G. Nath performed an experimental investigation of cylindrical shock waves generated by a moving piston in a rotational axisymmetric non-ideal gas with conductive and radiative heat fluxes in the presence of azimuthal magnetic field[14]. Theoretical studies concerning the presence or absence of magnetic field on the behaviour of cylindrical shock waves in rotating ideal gas employing Lie group transformation method were also undertaken by Nath G. and Sumeeta Singh[15]. They presumed the adiabatic flow conditions. Two kinds of solutions i.e., power law and exponential law shock path were examined, however exact solutions were obtainable only in the latter case. Sumeeta Singh studied for the similarity solutions for magnetogasdynamic cylindrical shock waves in rotating non-ideal gas using Lie Group theoretic method[16]. Coupled models for the propagation of shock waves in cylindrical and spherical geometries were also investigated by the research group of C.Y. Cao[17].

This paper investigates the propagation of strong diverging and converging shock waves in metals Aluminium and Copper for evaluating the effects of overtaking disturbances. Chisnell-Chester-Whitham method has been employed to obtain the analytical relations for shock velocity and shock strength. Expressions for the pressure and the particle velocity behind the shock wave have also been derived. The flow variables are numerically estimated and discussed through various figures and tables.

2. Theoretical Modeling :

The conservation of mass, momentum and energy can be expressed through the equations-

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\alpha \rho u}{r} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \tag{2}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} + c^2\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0 \tag{3}$$

Here,

r = position of shock front at time t,

p, ρ, c and u = pressure, density, sound velocity and particle velocity respectively $\alpha = 1$ (cylindrical shock waves) and $\alpha = 2$ (spherical shock waves).

The shock velocity (U) and particle velocity (u) in the metal are supposed to be related as

$$U = a + bu \tag{4}$$

where, a and b are constants of the metals.

3. Boundary Conditions :

Let subscript '2' and '1' denote the quantities behind and ahead of the shock front, then mechanical jump conditions across the shock front take the form

$$p_2 = \frac{\rho_1 a^2 \delta(\delta - 1)}{\{b-\delta\}(b-1)^2} \tag{5}$$

$$U = \frac{a\delta}{\{b-\delta(b-1)\}} \tag{6}$$

$$u_2 = \frac{a(\delta - 1)}{\{b-\delta(b-1)\}} \tag{7}$$

Where $\delta = \frac{\rho_2}{\rho_1}$, represent the compression behind the shock front and p = pressure.

4. Mathematical Description

4.1. Free propagation of diverging shock waves

The characteristic form of basic equations (1)-(3) for freely diverging shock is

$$\mathrm{dp} + \rho \mathrm{cdu} + \frac{\alpha\rho c^2 u}{u+c}\frac{\mathrm{dr}}{\mathrm{r}} = 0 \tag{8}$$

Use of boundary conditions (5), (6) and (7) in equation (8) and simplification results into

$$\begin{aligned}\alpha \log\frac{r}{r_0} &= -\delta - \Gamma\left(\frac{\delta^2}{2} - \delta\right) + \frac{\Gamma}{2b}\delta^2 + b\log\delta \\ &+ \frac{b\Gamma}{2}(\delta - \log\delta) + \frac{b^3\Gamma}{4}\left(\delta - 3\log\delta - \frac{3}{\delta} + \frac{1}{2\delta^2}\right) + \frac{b^2}{2}\left(\log\delta + \frac{1}{\delta}\right) \\ &+ \frac{b^3}{2}\left\{\log\delta - \frac{1}{2\delta^2} + \frac{2}{\delta}\right\} + \frac{b^2\Gamma}{\delta} + b^2\Gamma\log\delta + \frac{1}{(b-1)}\log\{b - \delta(b-1)\} \\ &+ \frac{b^4\Gamma}{4}\left\{5\delta - 9\log\delta - \frac{6}{\delta} - \frac{1}{2\delta^2} + \frac{2}{\delta^3} - \frac{1}{4\delta^4}\right\}\end{aligned} \tag{9}$$

where, log r_0 is constant of integration. Now differentiating equation (7), we get

$$du_{+} = \frac{ad\delta_1}{\{b - \delta_1(b-1)\}^2} \tag{10}$$

4.2. The effect of overtaking disturbances on diverging shock waves :

Let us take the differential equation

$$dp - \rho c du + \frac{\alpha \rho c^2 u}{u - c}\frac{dr}{r} = 0 \tag{11}$$

Using boundary conditions and equation (10) provides us

$$\alpha \frac{dr}{r} = \frac{\{\delta + b(\delta-1)\}d\delta}{\{\delta + b(\delta-1) - b\Gamma(\delta-1)^2\}} + \frac{\{\delta + b(\delta-1)d\delta}{(\delta-1)\{b-\delta(b-1)\}^{1/2}\{\delta + b(\delta-1) - b\Gamma(\delta-1)^2\}^{1/2}}$$
$$+ \frac{d\delta}{\{b-\delta(b-1)\}\{\delta + b(\delta-1) - b\Gamma(\delta-1)^2\}^{1/2}} - \frac{d\delta}{(\delta-1)\{b-\delta(b-1)\}} \tag{12}$$

Now differentiating equation (8), we get

$$du_{-} = \frac{ad\delta_2}{\{b-\delta_2(b-1)\}^2} \tag{13}$$

Therefore, resultant particle velocity increment will be

$du_{+} + du_{-} = (1/b)\, dU^{*}$ (14)

Substituting the values of du_{+} and du_{-} in equation (14) and then simplifying, we get modified shock velocity as

$$U^{*} = \frac{ab}{(b-1)}\left[\frac{1}{\{b-\delta_1(b-1)\}} + \frac{1}{\{b-\delta_2(b-1)\}}\right] \quad + \quad \text{const.} \tag{15}$$

and modified pressure

$$P^{*} = \frac{\rho_0 a^2}{(b-1)}\left[\frac{b}{\{b-\delta_1(b-1)\}^2} - \frac{(b+1)}{\{b-\delta_1(b-1)\}} + \frac{b}{\{b-\delta_2(b-1)\}^2} - \frac{(b+1)}{\{b-\delta_2(b-1)}\right] + \text{const.} \tag{16}$$

These expressions represent the shock velocity, particle velocity and the pressure modified by overtaking disturbances.

5. Results and Discussion

Expression (9) that relates propagation distance (r/r_0), density ratio (δ), metal constants a and b, is put to be used to determine numerically the shock wave parameters such as the shock velocity and shock strength for the shock wave propagating freely in the metallic medium (Aluminium and Copper metals). The values of delta (δ) so obtained for the metals under consideration are used to get the numerical values of shock velocity (U), pressure (P) with the help of relations (5), (6) and (7) for freely propagating shock and are shown in tables 1 to 3. Figures 1 to 4 show these variations for freely propagating shock. The value of modified shock velocity (U*) and modified pressure (P*) are calculated using equations (14) and (15) and are also listed in tables 1 to 3.

5.1. Variation of shock velocity with propagation distance (r/r_0) :

The variation of shock velocity with propagation distance is displayed in figure 1. Shock velocity decreases from 59.9687 to 8.5642 (for Al) as spherical shock moves from propagation distance 1.2 to 2.6 [table 1 and figures 1and 2].

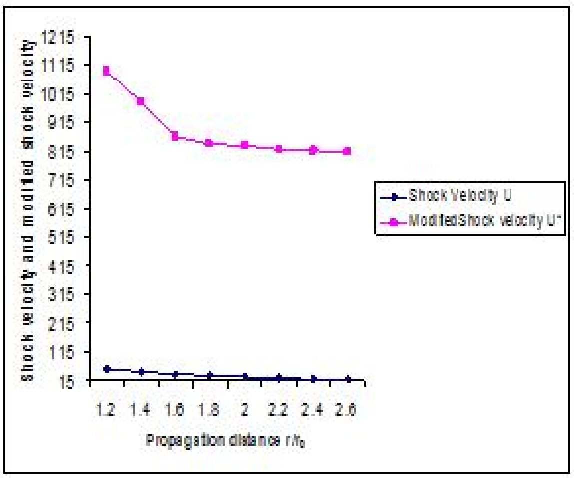

Figure-1

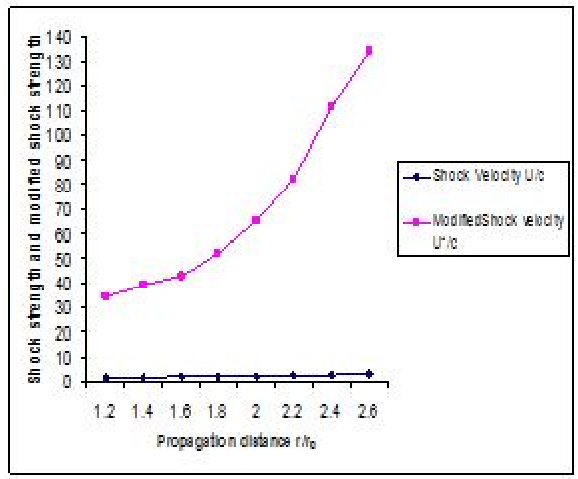

Figure-2

Table 1. The variation of shock velocity and shock strength with distance parameter (r/r_0) for spherical diverging shock waves for the case of Aluminium metal (a = 5.328, b = 1.338, ρ = 2.785)

Distance Parameter (r/r_0)	Velocity of Shock waves (U)	Modified Shock velocity (U*)	Shock Strength (U/c)	Modified shock Strength (U*/c)
1.2	55.4708	1093.6503	1.7475	34.4531
1.4	45.6409	986.2688	1.8136	39.1897
1.6	38.2101	865.9995	1.8943	42.9333
1.8	32.3757	839.6563	1.9970	51.7916
2.0	27.6492	834.1386	2.1481	64.8066
2.2	23.7355	824.4341	2.3722	82.3981
2.4	20.4248	817.5988	2.7778	111.1925
2.6	17.5649	814.0166	2.9014	134.4615

From table 1 it is obvious that shock velocity (U) and modified shock velocity (U*) decreases whereas shock strength (U/c) and modified shock strength (U*/c) increases with increase in distance parameter representing propagation of shocks (r/r_0). [Also see figures 1 and 2]. However, there is a three-fold decrease in the velocity of shocks as against around 21% reduction in the modified shock velocity for the considered range of distance parameter. But the observed pattern for shock strength is reverse of the pattern for the shock velocity as the modified shock strength shows a greater range for its variation.

5.2. Variation of shock strength with propagation distance :

Shock strength increases from 0.1975 to 0.7563 and 0.5033 to 0.9954 for Cu metal, as spherical shock moves from distances 1.2 to 2.6 [Refer to table 2 and figures 3 and 4].

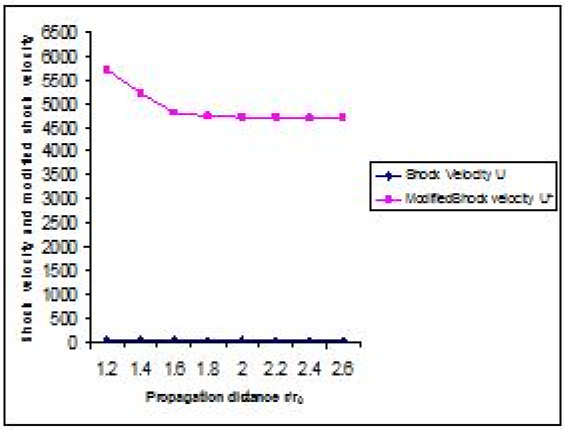

Figure 3

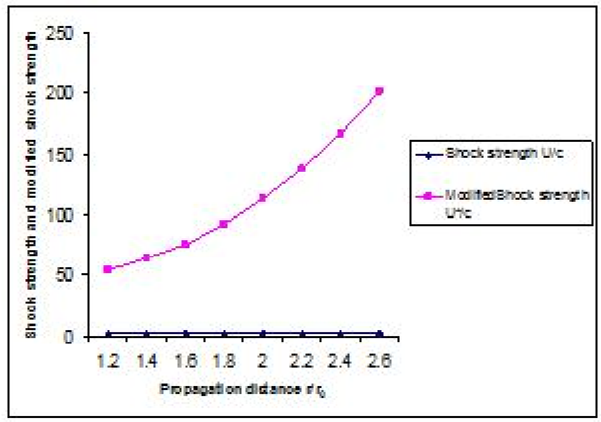

Figure 4

Table 2: The variation of shock velocity and shock strength with distance parameter (r/r_0) for spherical diverging shock waves in the metal Copper (a = 3.94, b = 1.489, ρ = 8.95)

Distance Parameter (r/r_0)	Velocity of Shock waves (U)	Modified Shock velocity (U*)	Shock Strength (U/c)	Modified shock Strength (U*/c)
1.2	55.4708	5734.3412	1.5199	53.7497
1.4	45.6409	5225.0464	1.5611	64.2332
1.6	38.2101	4803.1369	1.5987	75.2530
1.8	32.3757	4743.2266	1.6308	92.4197
2.0	27.6492	4720.0200	1.6650	113.5223
2.2	23.7355	4706.5846	1.6959	137.9963
2.4	20.4248	4697.3154	1.7247	166.6589
2.6	17.5649	4690.2146	1.7516	200.7024

A critical examination of table 2 prompts us to conclude that shock velocity (U) and modified shock velocity (U*) decreases whereas shock strength (U/c) and modified shock strength (U*/c) increases with propagation distance (r/r_0). [Also refer to figures 3 and 4].

The variation of pressure and particle velocity along with corresponding values with overtaking disturbances P* and U* immediately behind the shock have been calculated and their variation with propagation distance r/r_0 are shown in table 1 (for Aluminium metal) and table 2 (for Copper metal) for spherical shock.

5.3. Variation of pressure with propagation distance :

The pressure behind the shock is calculated using equations (15) and (16). The variation of pressure for spherical shock waves with parameter d are shown in table 3 and figure 3 with propagation distance. The pressure decreases from 51487.3700 to 85.2541 and 1467569.7000 to 1845.2451 for Al metal, as spherical shock moves from distance 1.2 to 2.6 [table 3 and figures 5 and 6].

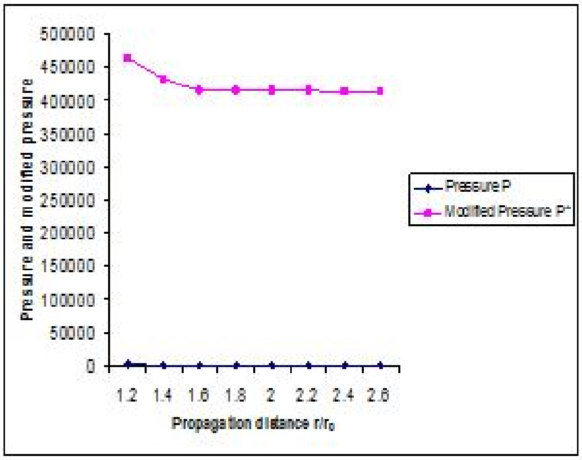

Figure-5

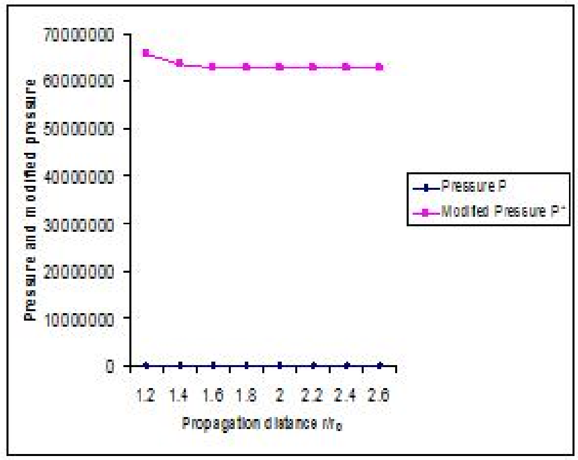

Figure-6

Table 3: The variation of pressure and modified pressure with distance parameter (r/r_0) for spherical diverging shock waves in the metal Aluminium and Copper (a = 5.328, b = 1.338, ρ = 2.785 and a = 3.94, b = 1.489, ρ = 8.95).

Distance Parameter (r/r_0)	Pressure (p)	Modified pressure (p*)	Pressure (P)	Modified pressure (P*)
1.2	1191.2820	462890.3932	17183.5874	65962476.7478
1.4	622.0580	432056.4709	11439.3093	63719604.1861
1.6	474.1091	415059.5645	7870.4684	62972367.6602
1.8	287.7818	413908.6094	5535.8076	62948436.2826
2.0	162.3264	413788.2733	3940.1677	62943809.0827
2.2	78.4483	413601.4512	2824.1839	62942064.9051
2.4	24.8688	413520.8066	2023.7948	62941194.6901
2.6	7.4367	413492.6451	1438.7689	62940694.8686

6. Conclusions :

It is concluded from above computations that the pressure (P), the modified pressure (P*), the particle velocity (u) and the modified particle velocity (u*) decrease with propagation distance r/r_0. The shock velocity and modified shock velocity decreases as the spherical shock waves propagate along the metallic mediums. However, the shock strength and the modified shock strength increases with the advancement of shock in the metallic mediums such as Al and Cu. So, the present study might prove to be useful in the identification and location of defects in solids (*i.e.*, in fractures and cracks in metals). The study will also open new avenues for the treatment of various human diseases as the modifications in the shock strength may extend the range of extracorporeal shock wave applications.

Scope of Future Work:

The analysis presented in this paper done for two specific metals can be extended to certain other metals useful for industrial and medical applications. Herein we have relied on numerical and analytical calculations using freely available tools. So future studies can be experimental as well as theoretical using other advanced numerical calculation software and a critical comparison of theory with experiments can be targeted.

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