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Models of shock wave propagation for diverging metallic structures

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Abstract

In the present study we have considered shock waves propagating freely in some diverging metallic structures. The characteristic equation along with boundary conditions provides a differential equation that is used to evaluate certain flow variables and parameters. The Chisnell-Chester-Whitham (CCW) method forms the basis to obtain the analytical relations for shock velocity, shock strength and pressure behind the shock wave. The variation of pressure with parameter δ (compression behind the shock) and propagation distance r is obtained for cylindrical (barrel) shock waves. It is found that shock velocity decreases and the shock strength increases as the shock wave progresses in both the Al and Cu metals. The pressure behind the shock also decays with propagation distance. The rate of decay of pressure in case of Al differs with that in Cu metal. Adiabatic sound velocity c and the parameter δ both shows a decreasing trend with propagation distance for both materials but the rate of decay is higher for the former.

Key words : Shock waves, adiabatic sound velocity, compression, shock strength.

1. Introduction

The studies of shock propagation in heterogeneous materials are important for the interpretation of impact deformation and impact metamorphosis of rocks, in particular the formation of high-pressure phases and of impact melt. The different behaviour of the components of a composed rock under compression makes its response to shock loading more complicated than that of a single material.

Reflection, refraction, and interference of shock waves caused by inhomogeneities lead to restricted (decentralized) concentrations of pressure, temperature, deformation rate, and to phase transitions. Such effects need to be considered if pressure and temperature during impact loading are gauged by the shock damage of a material. The study of shock wave propagation in gases, liquids and solids are important due to its application in diverse fields such as in aerospace engineering, medicine, mineralogy, rock science etc.

The shock wave propagation through any medium gives rise to a change in the physical characteristics. With an aim to evaluate these changes, the pressure and temperature of the shock waves generated by a picoseconds laser pulse in the layered semiconductor gallium selenide structure were determined by Leung *et al.*[1]. The propagation of waves in solids may lead to the occurrence of several dynamic events causing the materials in question to undergo a series of different possible states governed by plasticity, equation of state, and failure[2]. The propagation of steady plane shock waves in metallic materials has been considered by Molinari and Ravichandran[3]. They have proposed an analytical solution of the steady state propagation of plastic shocks. Laser-driven shock wave propagation in a transparent material such as Plexiglas coated with a thin overlayer of gold was studied by Dhareshwar et al using the technique of high speed optical shadowgraphy[4]. Shock transit time, Shock velocity and Shock pressures determined experimentally agreed well with those obtained through 1D multi-group radiation hydrodynamic simulations. Extensive experimental investigation in the form of large-amplitude, non-linear wave profile measurement manifesting the shock strength and equation of state properties of brittle solids was undertaken by Grady[5]. It was concluded in this study that both crystalline plasticity and brittle fracture may play significant and interconnected roles in the dynamic failure process. Concrete and rocks are supposed to be quasi-brittle material. Fracture of such materials is known to result from the formation and development of microcracks, which lower down the macroscopic value of Young's modulus and hence the corresponding wave velocity. As a result shock wave formation is expected. Masumi et al studied this phenomenon for granite[6]. Through numerical calculations, it was shown that the shape of the incident wave has a significant effect on the forming shock wave. The investigation of the head-on collision of planar shock waves with rigid porous materials was investigated by Ben-Dor *et al.*[7]. The steady shock wave propagation in a ductile metallic material containing voids was studied by Christophe Czarnota *et al.*[8]. Shang Fei and Wang Liangquan studied the influence of pits on the propagation law of explosion shock waves[9]. Numerical simulations were used to calculate the distribution of overpressure peak values at the bottom and rear of the pits. The effect of the spatial mesh size on the shock velocity and peak shock pressure is observed to be significant in certain studies. Simulations can be used as an effective tool for benchmark calculation of laser absorption coefficients.

Upon application of dynamic load to a solid object, stress shock wave can be propagated in that solid. When this stress wave surpasses proper yield condition of the solid, plastic deformation and fracture are generated. Shock wave propagation in the solids has been carried out by Koichi[10]. Huang et al studied the shock wave formation and propagation in two dimensional granular materials under vertical vibration by digital high speed photography by exploring the effect of driving parameters and particle number on the shock[11]. L. Q. Wang and D. R. Kong studied the influence of ground impedance on explosive shock wave test accuracy[12]. The numerical simulation analysis of the propagation law of explosion shock waves at the tunnel entrance was performed by X. Liu *et al.*[13]. The propagation of the shock wave generated during pulsed laser ablation in cavities was measured by Zeng et al[14]. The study of shock propagation in solids particularly in Aluminium and Copper has not appeared much in literature.

The study of motion of shock waves in metals was undertaken by Yadav *et al.*[15]. E Cuenca et al studied laser generated shock waves in metals[16]. They performed 3D axisymmetric simulations and the results were compared with those obtained through experiments. An experimental investigation of cylindrical shock waves generated by a moving piston in a rotational axi-symmetric non-ideal gas with conductive and radiative heat fluxes in the presence of azimuthal magnetic field was undertaken by G. Nath[17]. Theoretically, the behaviour of cylindrical shock waves in rotating ideal gas in the presence or absence of magnetic field using Lie group transformation method was analyzed by G. Nath and Sumeeta Singh[18]. This research group assumed adiabatic flow conditions and obtained two kinds of solutions. In one case the solutions were approximate while in the latter case, they were successful in getting exact solutions. Similarity solutions were obtained for magneto gas dynamic cylindrical shock waves in rotating non-ideal gas using Lie Group Theoretic (LGT) method by Sumeeta Singh[19]. As per their study, the shock strength decreases with increase in value of adiabatic exponent. Coupled models for the propagation of shock waves in cylindrical and spherical geometries were also investigated by C. Y. Cao and others[20]. R. Chandra and S. Singh analytically studied about the effect of overtaking disturbances in the propagation of spherical shock waves through solids, particularly, metals such as Aluminium and Copper[21]. In this work, important flow parameters and characteristics were calculated analytically and also estimated numerically. In another work, R. Chandra and S. Singh studied of shock waves propagating in heterogeneous metallic mediums[22]. A shock wave travels through most of the mediums at a speed higher than that of ordinary wave. Yadav et al carried out a study about the very weak imploding shock waves propagating in Al and Cu metals[23].

This paper deals with the analytical study of free propagation of strong diverging shock waves in Al and Cu metals using CCW method. We have obtained analytical relations for shock velocity, shock strength, particle velocity, and pressure behind the shock. The compression behind the shock, adiabatic sound velocity and certain other flow variables have been numerically estimated, tabulated, and compared for Al and Cu metals through several graphs.

2. Equations of the Problem

The conservation principles of mass, momentum and energy govern the shock wave propagation through metallic mediums and respectively expressed through equations

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{\gamma \rho u}{r} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \tag{2}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} + c^2\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0 \tag{3}$$

Where at time t, the symbols used are :

r = Position of shock front; p = Pressure at position r; ρ = Density of metal;

c = Adiabatic Sound velocity; u = Particle velocity

$$\gamma = \begin{cases} 1, \text{cylindrical shock} \\ 2, \text{Spherical shock} \end{cases}$$

The shock velocity U and the particle velocity u are related as

$$U = \alpha + \beta u \tag{4}$$

with α & β representing the constants of the metal.

Involving the weak shock parameter ϵ, the relevant boundary conditions across the shock front are expressible as

$$p_2\beta = \rho_1 c(1+\epsilon)\{c(1+\epsilon) - \alpha\} \tag{5}$$

$$U = (1+\epsilon)c \tag{6}$$

$$u_2\beta = c(1+\epsilon) - \alpha \tag{7}$$

The subscripts '1' and '2' represent various quantities ahead of the shock and behind it.

3. Diverging shock wave propagation

The freely diverging shock wave is expressible through the characteristic equation

$$dp + \rho c du + \frac{\gamma \rho c^2 u}{u-c}\frac{dr}{r} = 0 \tag{8}$$

The adiabatic sound velocity (c) is related to Huguenot a_H by the expression

$$c = a_H(1-\Delta)^{1/2} \tag{9}$$

where, $a_H = \frac{\alpha}{\{\beta - \delta(\beta-1)\}}\left[\frac{\delta + \beta(\delta-1)}{\{\beta-\delta(\beta-1)\}}\right]^{1/2}$

and $$\Delta = \frac{\Gamma(\delta-1)}{2}\left[1 - \left\{\frac{U-u_2}{a_H}\right\}\right]^2 \tag{10}$$

Now

$$\rho\Gamma = \rho_0\Gamma_0 \tag{11}$$

Where,

Γ_0 = Constant of Mc Gruneisen's equation of state at the initial density (ρ_0) of the metal, and

Γ = parameter of Mc Gruneisen's equation of state of the medium.

Differentiation of equations (5), (6) & (7) followed by putting obtained expressions in equation (8), results into the following equation after simplification

$$\frac{dr}{r} = -\frac{1}{\gamma\beta c}\frac{[c(1+\epsilon) - \alpha + \beta c][2c(1+\epsilon) - \alpha + c]d\epsilon}{[c(1+\epsilon) - \alpha]} \tag{12}$$

Integration of equation (12) provides us the equation

$$\gamma\beta c^2 \log r = -\left[\{c(1+\epsilon)-\alpha\}^2 + (c+\alpha)\{c(1+\epsilon)-\alpha\} + 2\beta c\{c(1+\epsilon)-\alpha\} + \beta c(c+\alpha)log\{c(1+\epsilon)-\alpha\}\right] + \gamma\beta c^2 \log r_0 \quad (13)$$

Where, log r_0 is the constant of integration.

4. Results and Discussion

Expression (13) which represents the relation between propagation distance (r/r_0), density (ρ), and metal constants α & β is used to compute the shock velocity and shock strength of the diverging shock propagating freely in the metals Al and Cu. The values of delta (δ) are calculated with propagation distance for these metals which are used to calculate the numerical values of shock velocity (U), pressure (p), and shock strength (U/c) with help of equations (5), (6) and (7) and are shown in tables (1) & (2). Particle velocity is calculated numerically by the use of equation (4) and adiabatic sound velocity from equation (9) with the help of (10). These variations are also shown in figures (1) to (4).

Table 1. The variation of shock velocity, shock strength, pressure, particle velocity, adiabatic sound velocity and compression behind the shock (δ) with propagation distance (r/r_0) for freely propagating cylindrical shock wave in the metal Al (α =5.328, β =1.338, ρ =2.785)

Propagation Distance r/r_0	Shock Velocity U	Shock Strength U/c	Pressure p	Particle Velocity u	Adiabatic Sound velocity c	Delta δ
1.0	159.9687	0.5033	51487.3700	115.5687	317.8101	3.6029
1.2	100.3539	0.6621	15962.6350	70.5885	151.5697	3.3715
1.4	69.4564	0.6823	9231.4073	47.8231	101.7988	3.2260
1.6	34.9496	0.8157	2155.1787	22.1419	42.8435	2.7288
1.8	24.6445	0.8677	990.3710	14.4376	28.4015	2.4145
2.0	19.4618	0.8927	572.7496	10.5670	21.8009	2.1880
2.2	16.1071	0.9084	480.8461	8.0528	17.7315	2.0013
2.4	13.6521	0.9193	236.5328	6.2212	14.8502	1.8372
2.6	11.7042	0.9267	155.3788	4.7667	12.6298	1.6871
2.8	10.0731	0.9489	100.1745	2.5637	10.6155	1.5458
3.0	8.5642	0.9954	85.2541	2.3654	9.5423	1.3325

Table 2: The variation of shock velocity, shock strength, pressure, particle velocity, adiabatic sound velocity and delta with propagation distance (r/r_0) for freely propagating cylindrical shock wave in the metal Cu (α =3.940, β =1.489, ρ =8.950)

Propagation Distance	Shock Velocity	Shock Strength	Pressure	Particle Velocity	Adiabatic Sound velocity	Delta
r/r_0	U	U/c	p	u	c	δ
1.0	491.9090	0.1975	1467369.7000	327.7423	2490.7454	2.9964
1.2	227.2762	0.2842	317490.1300	150.0212	799.5259	2.9419
1.4	160.9739	0.3356	154133.0600	105.4810	479.6852	2.9008
1.6	69.3313	0.4883	27248.0160	43.9119	141.9779	2.7275
1.8	46.5027	0.5790	11976.7830	28.5436	80.4465	2.5966
2.0	36.4056	0.6154	7139.0334	21.8130	59.1510	2.4948
2.2	30.4024	0.6547	4845.7042	17.7743	46.4383	2.4075
2.4	26.2224	0.6848	3512.1991	14.9652	38.2909	2.3294
2.6	23.0408	0.7083	2657.4874	12.8336	32.5304	2.2573
2.8	20.5414	0.7251	2071.7689	11.1881	28.2158	2.1898
3.0	19.2543	0.7563	1845.2451	9.2145	25.1463	1.9546

4.1. Shock velocity versus propagation distance

The variations in shock velocity with propagation distance r/r_0 are displayed in fig. 1 & fig. 2. Shock velocity decreases with increase in propagation distance as diverging cylindrical shock wave moves in Al and Cu metals. The rate of decrease of shock velocity is higher in the case of Cu compared to Al, possibly due to higher density of latter medium.

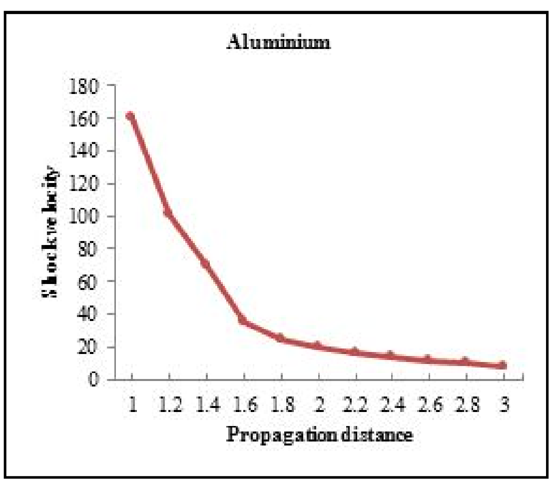

Fig. 1: Shock velocity versus propagation distance for Al

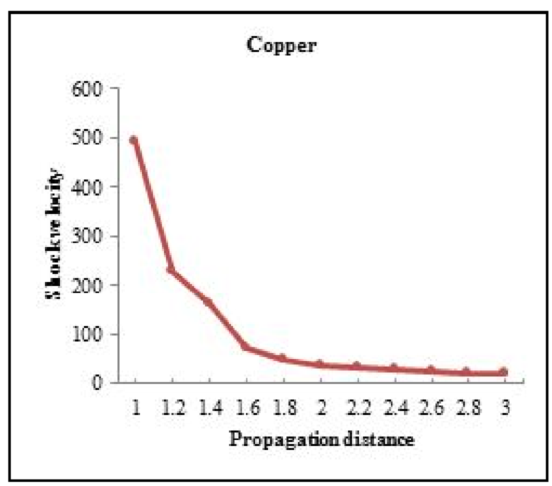

Fig 2: Shock velocity versus propagation distance for Cu

4.2. Shock strength versus propagation distance

The variations of shock strength with propagation distance r/r_0 are shown in fig. 3 & 4. Shock strength increases with increase in propagation distance for both Al and Cu as cylindrical shock moves in these metals. The variation in the Shock strength is higher for the Cu metal as compared to Al metal.

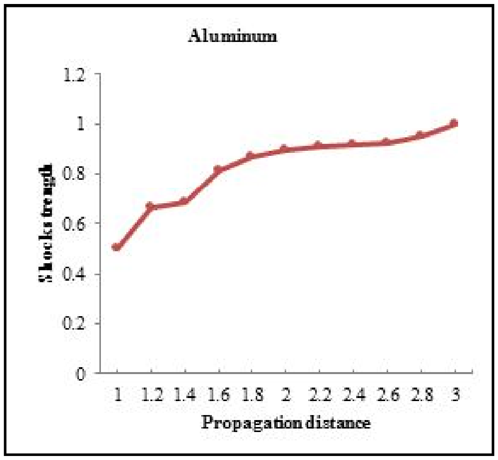

Fig 3: Shock strength versus propagation distance for Al

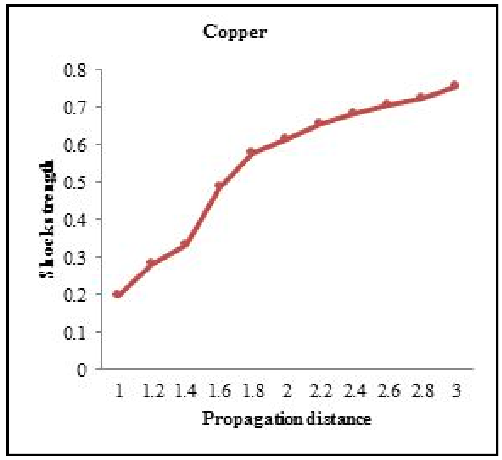

Fig 4: Shock strength versus propagation distance for Cu

4.3. Pressure versus propagation distance

Pressure variations with propagation distance r/r_0 are shown in fig. 5 & fig. 6 for Al and Cu metals. Pressure decreases at a faster rate in Cu compared to Al with increase in propagation distance as cylindrical shock moves in Al and Cu metals.

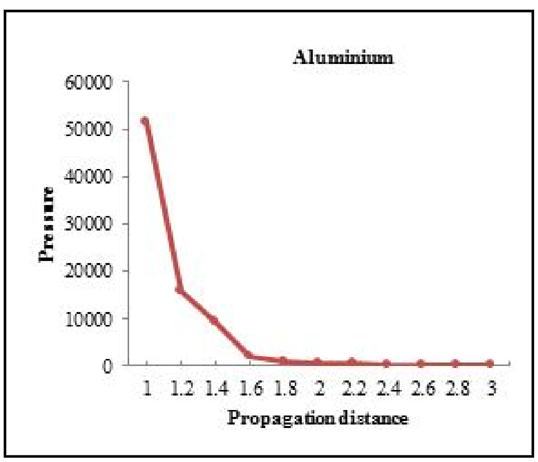

Fig 5: Pressure versus propagation distance for Al

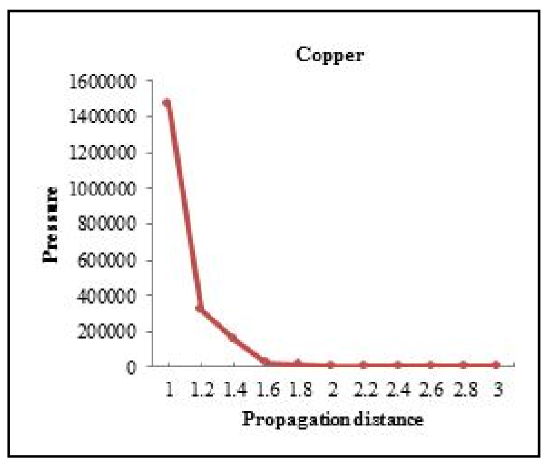

Fig 6: Pressure versus propagation distance for Cu

4.4 Particle velocity versus propagation distance

The variations in particle velocity with propagation distance r/r_0 are displayed in fig. 7 & fig. 8. The particle velocity in the medium decreases as the propagation distance increases with the propagation of cylindrical shock waves in the Al and Cu metals. For initial small values of r/r_0, the range of variation is quite large in the case of Cu metal in comparison to Al metal. However, for larger values of r/r_0 the variation pattern of particle velocity is almost similar in both the mediums.

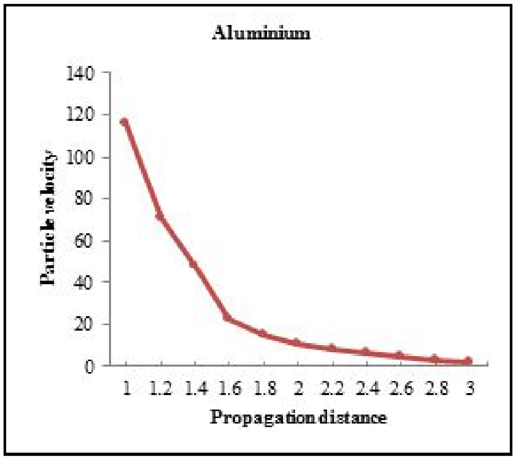

Fig 7: Particle velocity versus propagation distance for Al

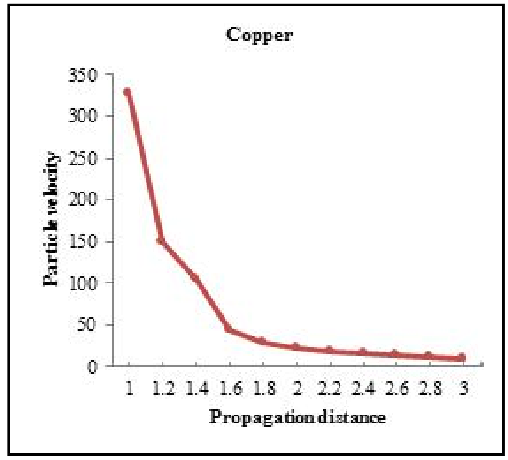

Fig 8: Particle velocity versus propagation distance for Cu

4.5 Adiabatic sound velocity versus propagation distance

As a cylindrical shock wave moves forward, the adiabatic sound velocity decreases at a fast rate in both Al and Cu metals for initial small increase in propagation distance r/r_0 and then for higher values of r/r_0 it acquires nearly stable values as displayed respectively in fig. 9 & fig. 10.

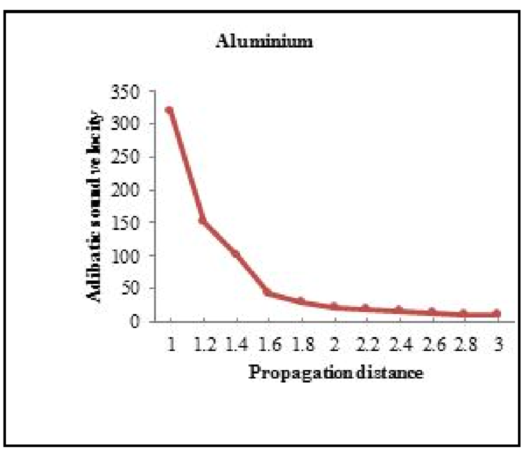

Fig 9: Adiabatic sound velocity versus propagation distance for Al

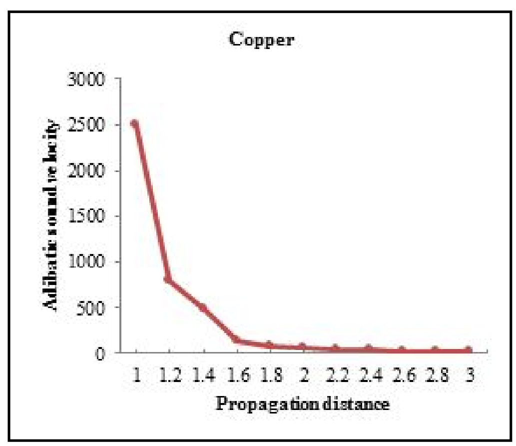

Fig 10: Adiabatic sound velocity versus propagation distance for Cu

4.6 Compression behind shock (δ) versus propagation distance

The variations in shock parameter δ with propagation distance r/r_0 of the shock wave in the metals Al and Cu are displayed respectively in the fig. 11 & fig. 12. The compression behind the shock δ decreases as the propagation distance increases as a cylindrical shock wave moves in Al and Cu metals. The variation pattern in Cu metal is relatively flatter in Cu metal compared to that Al metal in which the curve is relatively steeper.

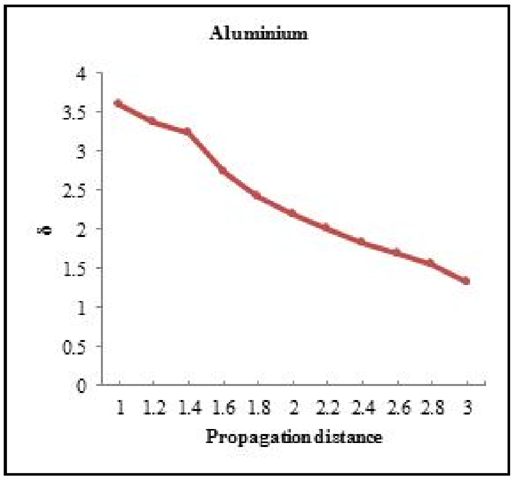

Fig 11: Compression behind shock versus propagation distance for Al

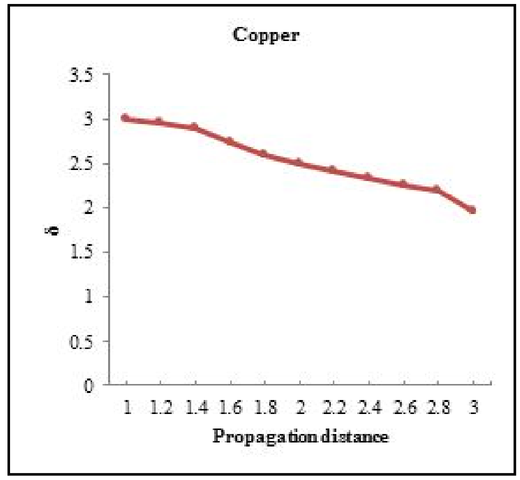

Fig 12: Compression behind shock versus propagation distance for Cu

5. Conclusions

Numerical computations about shock velocity, shock strength, pressure, particle velocity, adiabatic sound velocity, and compression behind the shock have been performed for the cylindrical diverging shock wave propagation in the Al and Cu metals with mathematical formulation using CCW method. Fig. 1 & Fig. 2 clearly show that shock velocity decreases with increasing propagation distance for both Al and Cu whereas reverse is the trend for shock strength as displayed in Fig. 3 and Fig. 4. Pressure variation follows the trend shown by that of shock velocity for both Al and Cu as is clear from Fig. 5 and Fig. 6. Particle velocity (Fig. 7 and Fig. 8) and adiabatic sound velocity (Fig. 9 and Fig. 10) also manifests a decrease with the advancement of the cylindrical shock in these metals. Finally, the compression behind the shock (δ) also shows a gradual decrease with increasing propagation distance for the cylindrical shock wave propagation along the metals under consideration. However, the curve representing the compression behind the shock is little flatter for Cu in comparison to Al.

The difference in the variation pattern of the shock wave characteristics in the metallic mediums is largely attributed to the difference in their density and elasticity properties. These studies seems to be suitable for applications in various fields of engineering and technology such as medicine, Geophysics, hypersonic flight exportations, plasma physics, aviation mechanical engineering, and astronomy etc. This study may be particularly useful for rock related issues.

Scope of Future Work

We have done this study for diverging cylindrical shock wave propagation in Al and Cu metals. The same study may be carried out to spherical and other types of shock wave propagation through these metals in converging and diverging cases of shock waves. Further, it can also be extended to other metals in the cylindrical diverging case itself with a view of their industrial and medical applications.

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Statement of Competing Interests

The authors declare that there is no competing financial interest of whatsoever nature between them.

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