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Decision Making Using Hurwicz Criteria

SHRIYA PATANKAR*, PRATIMA OJHA and MAMTA

Department of Mathematics, Madhyanchal Professional University, Bhopal-462001 (India)

*Corresponding Author Email : patankarshriya1996@gmail.com

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Abstract

In this paper we worked on a different type of vehicles performance problem where we have some fuzzy outputs which need to defuzzified to give a crisp result. We have presented defuzzification techniques by area of region (AOR) and also presented Hurwicz criteria method to find the ranking order and suggests that any decision maker with fair or extreme optimistic approach will choose the car as first alternative and the three wheelers like as auto, the second alternative and third alternative two wheelers like as bike. We use Hurwicz criteria method firstly for triangular fuzzy numbers and then same applied for trapezoidal and octagonal fuzzy numbers. The fuzzy numbers are defuzzified using area of region (AOR) and Hurwicz criteria is applied to the defuzzified data.

AMS Subject Classification (2010):- 90B40, 90B50.

Key words : Decision making under uncertainty · States of nature · Hurwicz's criterion · Laplace's criterion · Optimal pure strategy.

1.1 Introduction

Decision making are mostly used in our daily life in many situations. Decision making are forced to rely on their own subjective ideas of the efficiency of possible alternatives and importance of diverse criteria. The Decision making process start with the intelligence phase, where the relative or decision maker has examined the problem and it is identified, statement is defined and analyses decision maker's attitude in the fuzzy part network. Recent research has recognized that multi-criteria decision making (MCDM) should take account of uncertainty, risk and confidence. To illustrate the computation process and demonstrate the feasibility of the results we use a travel problem that has been used previously to assess MCDM. Defuzzification is method that's allowing the transfer of a fuzzy value into a crisp output.[13] Norman Fenton and Wei Wang are presented a method which is mostly important to solve this type of decision making problems. They also studies in multi criteria decision making for triangular fuzzy numbers and also extend to trapezoidal and octagonal fuzzy numbers.[10,11] There are many different types of methods of defuzzification available for trapezoidal fuzzy numbers. The center of gravity method and the mean of maxima method are mostly used in fuzzy mathematics.

The area method for triangular, trapezoidal and octagonal fuzzy number exhibits the property of continuity that makes them suitable for fuzzy controller.[12] Malini and Kennedy defined octagonal fuzzy numbers and their arithmetic operations.

The Hurwicz Criterion firstly presented in a chapter 1951, is probably the earliest novel contribution to the field of economics for which Leo has been recognized. It provides a formula for balancing pessimism and optimism in decision making under uncertainly that is when future conditions are to some extent unknown. This method is also known as decision making criterion. The Hurwicz Criterion rule is commonly used in decision making that provided the best and the worst possible outcomes to decision makers.[8, 18] For each given action the decision makers chooses a "coefficient of pessimism", and "coefficient of optimism" called delta (∂) and ($1 - \partial$), where ∂, ($1-\partial$) are maximum pay off and minimum payoff and calculate the balance weighted outcome in uncertain situations. And find an even distribution of weights between the two absolute optimism and pessimism and $\partial \in [0, 1]$. If the coefficient of pessimism is 0.8, then the emphasis on the best outcomes will be 0.2 otherwise it's consequently conditions is true and $\partial = 0.5$ show that DM is neither optimistic nor pessimistic. In this chapter, using Hurwicz criteria method firstly for triangular fuzzy numbers and then same applied for trapezoidal and octagonal fuzzy numbers.[9] The fuzzy numbers are defuzzified using area of region (AOR) and Hurwicz criteria is applied to the defuzzified data.

1.2. *Definition By Area Of Region And Decesion Making:*

In this section we have discuss some basic definition which used chapter and some important components are defined

Definition 1.2.1. Fuzzy Set :

If x is an element of X, where X is a universal set, then a fuzzy set A defined on X is given by

$$A = \{(x, m(x)), \forall x \in X\}$$

Where, m(x) is a membership function.

Definition 1.2.2. Hurwicz Criteria :

The Hurwicz criteria joined a weighted value from the maximum payoff and the Minimum payoff of the strategies. Basically Hurwicz criteria are a combination of optimistic and pessimistic decision, where optimistic (1- ∂) is also known as Maximin and pessimistic (∂) is also known as Maximax. First select coefficient of realism ", with a value between 0 and 1. When ∂ is close to 1, decision maker is optimistic about future and when ∂ is close to 0, decision maker is pessimistic about future.

Weighted Outcome = ∂ (maximum payoff) + (1- ∂) (minimum payoff)

Definition 1.2.3. Triangular Fuzzy Number :

Triangular Fuzzy Number is a number such as A = (a, b, c) if its membership function $\mu(x)$ is defined as

$$\mu_A(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a < x \le b \\ 1, & x = b \\ \dfrac{c-x}{c-b}, & b < x \le c \\ 0, & x \ge c. \end{cases}$$

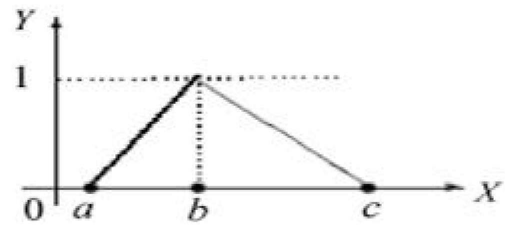

Figure 1.1: Triangular fuzzy number

This number is based on three-value judgment a, b, and c, where a, b and c is the lower, most and upper possible value.

Definition 1.2.4. Trapezoidal Fuzzy Number :

We can define Trapezoidal Fuzzy Number Ã as (a, b, c, d), the membership function $\mu(x)$ of this fuzzy number will be interpreted as follows

$$\mu_A(x) = \begin{cases} 0, & x \le a \\ \dfrac{x-a}{b-a}, & a < x \le b \\ 1, & b \le x \le c \\ \dfrac{c-x}{d-c}, & c < x \le d \\ 0, & x \ge d \end{cases}$$

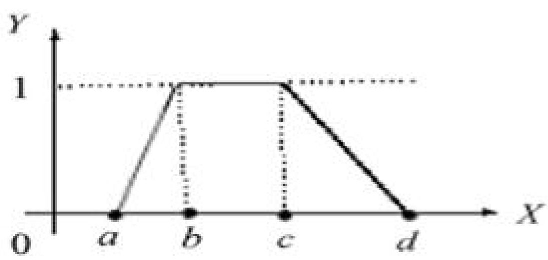

Figure 1.2: Trapezoidal fuzzy number

Definition 1.2.1. Octagonal Fuzzy Number:

A fuzzy number A denoted by (a, b, c, d, e, f, g, h) is a normal octagonal fuzzy number whose membership function $\mu(x)$, where a, b, c, d, e, f, g, h are real numbers is given as

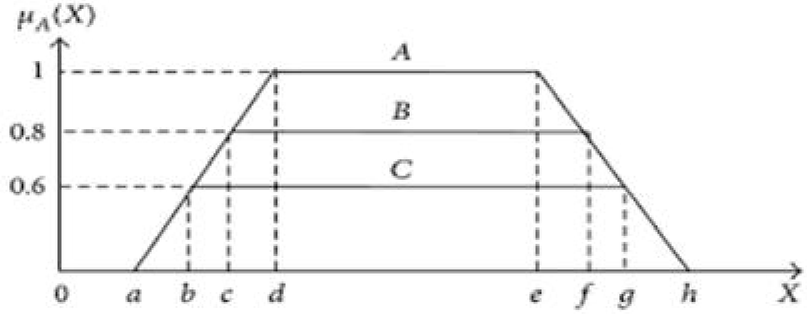

Figure 1.3: Octagonal fuzzy number

Definition 1.2.6. Fuzzy Mcdm Approach :

A fuzzy MCDM for a general multi criteria decision problem with m alternatives $\tilde{A}_i$ (i = 1, 2, 3, …, m) and n criteria C_j (j = 1, 2,…, n) can be expressed as D = (x_{ij} where i = 1, 2,..., m and j = 1, 2,…, n where D is decision matrix (where the entry x_{ij} represents the rating of alternative A_i (with respect to criteria C_j). Basically criteria may be classified in two ways as Benefit criteria or Cost criteria, Where Benefit criteria shows higher value of x_i for the DM and Cost criteria shows lower the value of x_{ij} for DM.

Since, it was proposed to consider fuzzy, as opposed to crisp values in D and W, the following notation D[x_{ij}] and weight vector of DM is represent as W= (w_j)= (w_1, w_2, $w_{3, \ldots,}$ w_m) satisfied $\sum_{j=1}^{m} W_j = 1$, where w_j represents the fuzzy weight of criterion C_j.

Definition 1.2.7. Normalization: [14, 20]

The fuzzy numbers in the decision matrix are normalized as the Performance matrix: P□ = [p_{ij}]. This process is applied to deal with criteria of different scales. It's denoted as (P_{ij}), where

$$P_{ij} = \left(\frac{xij_1}{M}, \frac{xij_2}{M}, \frac{xij_3}{M}\right), \quad M = \max(x_{ij3}) \text{ for benefit criteria}$$

or

$$P_{ij} = \left(\frac{N\text{-}xij1}{N}, \frac{N\text{-}xij2}{N}, \frac{N\text{-}xij3}{N}\right) \quad N = \max(x_{ij3}) \text{ for cost criteria.}$$

This method preserves ranges of normalized triangular fuzzy numbers to [0, 1].

Definition 1.2.8. Weighting The Criteria :

The multiplication of weight vector with decision matrix represent the weighted performance matrix and it's denoted by P□ written as P□ = [p_{ij}], where

$Pij_1 = w_{j1} \times P_{ij1}$, $P_{ij2} = w_{j2}\ P_{ij2}$ and $P_{ij3} = w_{j3}\ P_{ij3}$, where i= 1, 2, 3, …, m and j = 1, 2, …, n.

1.3. *Methodology :*

In this section we have discuss applying deffuzification method Hurwicz criteria method for triangular, trapezoidal and octagonal fuzzy numbers and find the result according to time, distance and satisfaction.

1.3.1 *Procedure:*

The algorithm was carried out by the following steps:
1) Express the problem in the form of fuzzy decision matrix.
2) For Each element normalizing of the matrix then apply the performance matrix.
3) Calculated the weighted performance matrix by multiplying its corresponding weight and the performance matrix.
4) Suppose that each triangular fuzzy number as the vertex of a triangle in the form (a, 0), (b, 1), (c, 0) and find the area of each triangle.
5) Then using Hurwicz criteria to find the weighted outcome for each row.
6) If the entire weighted outcome will be provided higher value then shows that best action.

1.3.2. *Numerical Example :*

Table 1 shows the decision matrix and its corresponding weight for the different types of vehicle feature problem. In this example, the different types of vehicle judge time and distance according to maximum speed of vehicles are measured in unit minute and meter respectively. Let as suppose that a ranking order scale from 0-10 for different value of ∂. Let us consider the ratings in the decision matrix expressed as triangular fuzzy numbers (for example, two wheeler vehicles like a scooty journey to office most typically time 31 minutes but it can be as low as 29 minutes and as high as 33 minutes).[11] The problem was illustrated with 3 different assumptions. The ratings in the decisions matrix were expressed as A) triangular, B) trapezoidal and C) octagonal fuzzy numbers:

A. [21] Triangular Fuzzy Numbers: - Table 1.3.1: Decision Matrix

Table 1.1: Decision Matrix

Types of vehicles	Maximum speed of vehicles	Time of vehicles $\omega = 0.3$	Distance of vehicles $\omega = 0.5$	Satisfy $\omega = 0.2$
Scooty	(25, 35, 45)	(20, 31, 33)	(141.67 ,300.72, 326.17)	(5, 6, 7)
Auto rickshaw	(20, 30, 40)	(30, 40, 55)	(102.50, 190.68, 330.80)	(6, 8, 9)
Car	(60, 70, 70)	(32, 43, 55)	(498.72, 723.01, 910.50)	(2, 5 ,7)

For Each element normalizing of the matrix then apply the performance matrix was constructed and shown in Table 1.2

Table 1.2: Performance matrix

Types of vehicles	Time of vehicles ω = 0.3	Distance of vehicles ω = 0.5	Satisfy ω = 0.2
Scooty	(0.400, 0.436, 0.636)	(0.641, 0.669, 0.840)	(0.222, 0.333, 0.444)
Auto rickshaw	(0, 0.272, 0.454)	(0.636, 0.790, 0.887)	(0, 0.111, 0.333)
Car	(0.036, .0.218, 0.418)	(0, 0.205, 0.457)	(0.222,0.444, 0.777)

Table 1.2: Performance matrix

Calculated the weighted performance matrix by multiplying its corresponding weight and the performance matrix show in Table 1.3

$$A=\begin{bmatrix} (0.120, 0.130, 0.190) & (0.320,\ 0.334, 0.420\ ) & (\ 0.444,\ 0.666,\ 0.888\ ) \\ (0\ , 0.0816\ , 0.136) & (0318, 0.395, 0.443) & (0, 0.022, 0.066) \\ (0.0108, 0.0654, 0.125) & (\ 0, 0.102, 0.228) & (\ 0.044, 0.088, 0.1554) \end{bmatrix}.$$

Let assume that each triangular fuzzy number as the vertex of a triangle in the form (a, 0), (b, 1), (c, 0). The triangular fuzzy decision matrix can be written as

$$A=\begin{bmatrix} (0.120, 0.130, 0.190) & (0.320,\ 0.334, 0.420\ ) & (\ 0.444,\ 0.666,\ 0.888\ ) \\ 0, 0.0816\ , 0.136) & (0318, 0.395, 0.443) & (0,\ 0.022, 0.066) \\ (0.0108, 0.0654, 0.125) & (0, 0.102, 0.228) & (\ 0.044, 0.088, 0.1554) \end{bmatrix}.$$

The area of a triangle is defined as the total space that is enclosed by any particular triangle. The basic formula to find the area of a given triangle is $A = \frac{1}{2} \times b \times h$, where b is the base and h is the height of the given triangle.

(Area of triangle) $A = \frac{1}{2} \times \text{base} \times \text{height}$,

$$A = \frac{1}{2} \times (c\text{- } a), \text{ since } (h=1)$$

Each triplet represents a triangle. Table 1.3 was obtained by area of triangle (Fig-1.1)

Table 1.3*:* Defuzzified matrix

Table 1.3*:* Defuzzified matrix

Types of	Time of vehicles	Distance of	Satisfy
vehicles	ω = 0.3	vehicles ω = 0.5	ω = 0.2
Scooty	0.035	0.050	0.022
Auto rickshaw	0.068	0.062	0.033
Car	0.0575	0.114	0.055

Table 1.4: Maximum and Minimum Payoffs

Types of vehicles	Time of vehiclesω = 0.3	Distance of vehiclesω = 0.5	Satisfy ω = 0.2	Row Maximum	Row Minimum
Scooty	0.035	0.050	0.022	0.050	0.022
Auto rickshaw	0.068	0.062	0.033	0.068	0.033
Car	0.0575	0.114	0.055	0.114	0.055

Table 1.4: Maximum and Minimum Payoffs

Hurwicz criteria were used to find the weighted outcome in Table 4.

Weighted outcome = ∂ (maximum payoff) + (1- ∂) (minimum payoff).

Table 1.5: Weighted Outcome, for different values of ∂

Types of vehicles	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
Scooty	0.022	0.0248	0.0276	0.0.304	0.0332	0.036	0.0377	0.046	0.044	0.047	0.05
Auto rickshaw	0.033	0.0365	0.0400	0.0435	0.047	0.054	0.0575	0.060	0.064	0.067	0.068
Car	0.055	0.609	0.0668	0.0727	0.0786	0.084	0.0904	0.096	0.102	0.108	0.114

Table 1.5: Weighted Outcome, for different values of ∂

Defuzzification by area of region and decision making

Table 1.6: Ranking Order, for different values of ∂

Types of vehicles	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
Scooty	3	3	3	3	3	3	3	3	3	3	3
Auto rickshaw	2	2	2	2	2	2	2	2	2	2	2
Car	1	1	1	1	1	1	1	1	1	1	1

Table 1.6: Ranking Order, for different values of ∂

Table 6 gives the ranking order under different risk attitude of the decision maker. The ranking order suggests that any decision maker with fair or extreme optimistic approach will choose the car as first alternative and auto rickshaw the second alternative and third alternative like as scooty.

B) [23] *Trapezoidal Fuzzy Number:*

The ratings in the decision matrix are assumed as trapezoidal fuzzy numbers and same method apply in this case.

Table 1.7*:* Decision Matrix

Types of vehicles	Maximum speed of vehicles	Time of vehicles ω = 0.3	Distance of vehicles ω = 0.5	Satisfy ω = 0.2
Scooty	(25, 35, 40, 45)	(20,31, 32, 33)	(141.67, 300.72, 311.35, 326.17)	(5, 6, 6, 7)
Auto rickshaw	(20, 30, 35, 40)	(30, 40,47, 55)	(102.50, 190.67, 238.59, 330.70)	(6, 7, 8, 9)
Car	(60, 70, 75, 70)	(32,43, 50, 55)	(498.72, 723.01, 860.20, 910.50)	(2, 5 ,6, 7)

Table 1.7*:* Decision Matrix

Normalized weighted performance and defuzzified matrix were found using area of trapezium

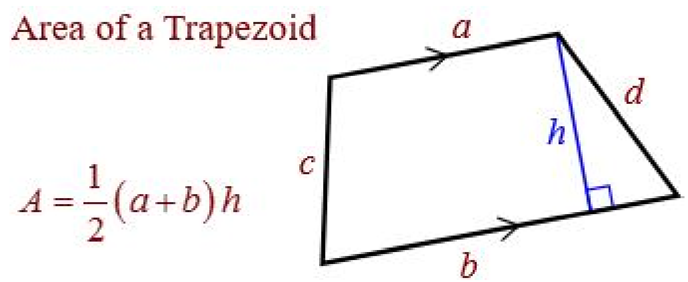

Figure 1.3: Area of Trapezoidal

Where, h = height

(Area of trapezium) A = $\frac{1}{2}$ × [(d-a) + (c-d)] × height,

Area of Trapezium = $\frac{1}{2}$ × [(d-a) + (c-d)] ×h

Area of Trapezium = $\frac{1}{2}$ × [(d-a) + (c-d)], (since h=1)

The same procedure was adopted to find the ranking order and shown in Table 7.

Table 1.8: Ranking Order (for different values of ∂)

Types of vehicles	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1**
Scooty	3	3	3	3	3	3	3	3	3	3	3
Auto rickshaw	2	2	2	2	2	2	2	2	2	2	2
Car	1	1	1	1	1	1	1	1	1	1	1

Table 1.8: Ranking Order (for different values of ∂)

Table 1.8 gives the same ranking order for trapezoidal fuzzy number.

C) *Octagonal Fuzzy Number:*

The same ratings are assumed for octagonal fuzzy numbers in the decision matrix

Table 1.9*:* Decision Matrix

Types of vehicles	Maximum speed of vehicles	Time of vehicles $\omega = 0.3$	Distance of vehicles $\omega = 0.5$	Satisfy $\omega = 0.2$
Scooty	(25, 27, 30, 33, 35, 38, 40, 43, 45)	(20, 23, 25, 27, 29, 31, 32, 33)	(141.67,180.70, 230.81, 259.80, 271.40, 300.72, 311.35, 326.17)	(5,1.5, 1.5 6, 6.5, 6.7, 6.8, 7)
Auto rickshaw	(20, 23, 27, 30, 33, 35,38, 40)	(30, 34, 37, 40, 43, 47, 50, 55)	(102.50, 131.46,184.036, 190.68, 210.74, 238.59, 289.25, 330.80)	(6, 6.3, 6.7, 7, 7, 7.5, 7.8, 8, 9)
Car	(60, 63, 66, 70, 73, 75, 77, 80)	(32, 36, 40, 43, 47, 50, 52, 55)	(498.72, 550.43, 648.74, 723.01, 784.70, 860.20, 892.70, 910.50)	(2,3,3.5, 4, 5, 6, 6.5, 7)

Table 1.9*:* Decision Matrix

Normalized weighted performance and defuzzified matrix were found using area of trapezium (Fig-1.3)

Area of Region = Area of Trapezium ABCD + Area of Trapezium EFGH

Area of Region = $\frac{1}{2} \times h□ \times [(h - a) + (g - b)] + \frac{1}{2} \times h2 \times [(f - c) + (e - d)]$, (since h□, h□, = 0.5)

The same procedure was adopted to find the ranking order and shown in Table 10

Table 1.10: Ranking Order (for different values of ∂)

Types of vehicles	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Scooty	3	3	3	3	3	3	3	3	3	3	3
Auto rickshaw	2	2	2	2	2	2	2	2	2	2	2
Car	1	1	1	1	1	1	1	1	1	1	1

Table 1.10: Ranking Order (different values of ∂)

Table 10 gives the ranking order for octagonal fuzzy numbers with same input. It was noted that the ranking order for triangular, trapezoidal and octagonal fuzzy numbers were same for this problem.

1.4. Conclusion

In this paper we worked on a different type of vehicles performance problem where we have some fuzzy outputs which need to defuzzified to give a crisp result. We have presented defuzzification techniques by area of region (AOR) and also presented Hurwicz criteria method to find the ranking order and suggests that any decision maker with fair or extreme optimistic approach will choose the car as first alternative and the three wheelers like as auto, the second alternative and third alternative two wheelers like as bike. As per experimental observations, both methods have given approximately same output for the triangular fuzzy numbers. Similarly same output for the trapezoidal and the octagonal fuzzy numbers using input value were same for this problem. In this chapter region (AOR) and Hurwicz criteria defuzzification method are very useful for multi criteria decision making problem to find the best action under different risk Attitudes of the decision Maker.

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