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Bianchi Type IX String Bulk Viscous Cosmological Model Incorporating Barotropic Equation Of State and Dark Energy

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Abstract

Bianchi type IX bulk viscous cosmological model with barotropic equation of state and dark energy is investigated with the consideration of condition $\rho + \lambda = 0$, where ρ and λ are the rest energy density and the tension density of a string cloud respectively. For obtaining deterministic model of the universe we assume that σ (shear) is proportional to θ (expansion) which leads to $B = A^m$, where m is a constant, A and B are metric potentials. The physical and geometrical properties of the model are also discussed.

Key words : Bianchi Type IX, Bulk Viscosity, String, Barotropic Equation, Dark Energy.

1. Introduction

The spatially homogenous models with anisotropy have been extensively researched within the context of general relativity, as they provide a realistic picture of the early universe and its large scale behavior. Therefore the investigation of Bianchi type IX models becomes more significant as they are homogeneous and inherently show anisotropy. Bali and Poonia[3], Chakraborty[6], and Tyagi *et al.*[17] have expressed a strong interest in studying these models. For the same spacetime, Tyagi and Chhajed[16] analyzed the interaction of perfect fluid distribution with the electromagnetic field. Multiple discoveries such as cosmic microwave background (CMB) data and Type Ia Supernovae have explained the rapid growth of the universe. This acceleration is attributed to an unusual force characterized by negative pressure, known as dark energy. Aditya *et al.*[1], Dabgar and Bhabor[8] and Vijaya Santhi *et al.*[18] proposed the dark energy models in different Bianchi spacetimes.

The study of the universe in its initial phase includes cosmic strings as a fundamental component. Strings are believed to contribute to density fluctuations that lead to galaxy formation and possess a significant gravitational pull due to their intrinsic stress-energy. The string cosmological models have been examined in various contexts by many cosmologists viz., Chhajed *et al.*[7], Rao *et al.*[11] and Reddy[13]. Vijayvargiya *et al.*[19] presented a Bianchi type-VI_0 string cosmological model within the framework of Saez-Ballester theory, incorporating a variable cosmological constant (Λ). Although most of the cosmological models are based on the assumption of a perfect fluid, several studies explore the large specific entropy and the noticeable isotropy of the radiations that are believed to generate viscous effects. Banerjee *et al.*[5] investigated Bianchi type II, VIII and IX models incorporating viscosity. Time dependent bulk viscous string model is discussed by Bali and Pradhan[4]. Yadav and Yadav[20] presented cosmological models incorporating bulk viscous and barotropic perfect fluid for Bianchi type-III metric within the framework of Lyra's geometry. Bianchi type-IX cosmological model in the context of bulk viscosity has been studied by Bali and Dave[2] and Pradhan *et al.*[10]. Sharma and Poonia[14] developed a Bianchi type-IX inflationary universe model considering the effects of bulk viscosity and a flat potential. Maheshwari and Poonia[9] examined a Bianchi type-V model showing the inflationary results for the involvement of bulk viscosity. Sharma *et al.*[15] investigated the phenomenon of inflation, incorporating impacts of bulk viscosity and a variable cosmological constant within the context of Creation Field Cosmology.

The above discussion inspired us to investigate Bianchi type IX string cosmological model with bulk viscous fluid distribution and dark energy. The present paper deals with Bianchi type IX model in the context of $\rho + \lambda = 0$ [12]. The symbol ρ represents the proper energy density and λ denotes the string tension density. For obtaining deterministic model we take the σ (shear) varies linearly with θ (expansion), resulting in a relation between metric potentials, *i.e.* $B = A^m$, where m is a constant. We consider barotropic equation of fluid for the determination of coefficient of bulk viscosity ξ. Furthermore, the physical and geometrical characteristics of the model are also examined.

2. Metric and Field Equation

The line element for Bianchi type IX space - time is:

$$ds^2 = -dt^2 + A^2dx^2 + B^2dy^2 + \left(B^2 \sin^2 y + A^2 \cos^2 y\right)dz^2 - 2A^2 \cos^2 y\,dx\,dz \tag{1}$$

Here A and B varies with t only.

The energy momentum tensor T_i^j incorporating bulk viscous fluid that includes one dimensional cosmic string is described by :

$$T_i^j = (\bar{p} + \rho)v_i v^j + \bar{p} g_i^j - \lambda x_i x^j \tag{2}$$

The symbol λ represents the density of string, ρ denotes the matter density, $\bar{p}$ expresses the effective pressure, is a unit vector in space that shows the string's direction and v^i is the four - velocity vector that obeys the specific relation given by

$$v_i v^i = -1 = -x_i x^i ; v_i x^i = 0 \tag{3}$$

The effective pressure $\bar{p}$ is related to equilibrium pressure p by relation

$$\bar{p} = p - \xi\theta \tag{4}$$

where ξ is the co-efficient of bulk viscosity and θ is the expansion scalar.
The co-moving frame gives

$$v^j = (0,0,0,1) \text{ and } x^i = \left(\frac{1}{A},0,0,0\right) \tag{5}$$

In the geometrized units $(8\pi G = c = 1)$, the Einstein's field equations are expressed as

$$R_i^j - \frac{1}{2} R g_i^j + \Lambda g_i^j = -T_i^j \tag{6}$$

The metric (1) together with equations (2) and (6) leads to following system of equations

$$\frac{2B_{44}}{B} + \frac{B_4^{\,2}}{B^2} + \frac{1}{B^2} - \frac{3A^2}{4B^4} + \Lambda = -p + \xi\theta + \lambda \tag{7}$$

$$\frac{A_{44}}{A} + \frac{B_{44}}{B} + \frac{A_4 B_4}{AB} + \frac{A^2}{4B^4} + \Lambda = -p + \xi\theta \tag{8}$$

$$\frac{2A_4 B_4}{AB} + \frac{B_4^{\,2}}{B^2} + \frac{1}{B^2} - \frac{A^2}{4B^4} + \Lambda = \rho \tag{9}$$

When the symbols A and B are followed by the number 4, they denote the ordinary differentiation with respect to 't'.

The scalar expansion is expressed by

$$\theta = v^i_{;i} = \frac{A_4}{A} + \frac{2B_4}{B} \tag{10}$$

The shear σ can be obtained as

$$\sigma^2 = \frac{2}{3}\left(\frac{B_4}{B} - \frac{A_4}{A}\right)^2 \tag{11}$$

The Hubble's parameter H using (10) can be derived as

$$H = \frac{\theta}{3} = \frac{1}{3}\left(\frac{A_4}{A} + \frac{2B_4}{B}\right) \tag{12}$$

Also corresponding value of the deceleration parameter can be written as

$$q = -1 + \frac{d}{dt}\left(\frac{1}{H}\right) \tag{13}$$

3. *Solution of Field Equation :*

The system of three equations (7) to (9) is obtained in eight unknown quantities p, ρ, ξ, θ, λ, Λ, A and B. For complete solution we take expansion (θ) is directly related to shear (σ) which results in

$$B = A^m \tag{14}$$

Here 'm' is a constant.
Also Λ is inversely proportional to R^3 which gives

$$\Lambda = \frac{\beta}{R^3} = \frac{\beta}{AB^2} \tag{15}$$

where β is the constant of proportion.

Now as we consider $\rho + \lambda = 0$, the equations (7) to (9) with conditions (14) and (15) lead to following expression

$$A_{44} + \frac{3m^2 A_4^{\,2}}{(m-1)A} = \frac{5}{4(m-1)A^{4m-3}} - \frac{2}{(m-1)A^{2m-1}} - \frac{\beta}{(m-1)A^{2m}} \tag{16}$$

Now substituting $A_4 = l(A)$ and $A_{44} = ll'$ into equation (16), allow us to obtain

$$\frac{dl^2}{dA} + \frac{6m^2 l^2}{(m-1)A} = \frac{5}{2(m-1)A^{4m-3}} - \frac{4}{(m-1)A^{2m-1}} - \frac{2\beta}{(m-1)A^{2m}} \tag{17}$$

Equation (17) leads to

$$l^2 = \frac{5}{4(m^2+4m-2)A^{4m-4}} - \frac{2}{(2m^2+2m-1)A^{2m-2}} - \frac{2\beta}{(4m^2+3m-1)A^{2m-1}} + \frac{M}{\frac{6m^2}{A^{m-1}}} \tag{18}$$

In this context, M denotes the constant of integration. Referring to equation (18), we obtain

$$\left(\frac{dA}{dt}\right)^2 = \frac{5}{4(m^2+4m-2)A^{4m-4}} - \frac{2}{(2m^2+2m-1)A^{2m-2}} - \frac{2\beta}{(4m^2+3m-1)A^{2m-1}} + \frac{M}{\frac{6m^2}{A^{m-1}}} \tag{19}$$

Equation (19) leads to

$$\frac{dA}{dt}=\sqrt{\frac{5}{4\left(m^2+4m-2\right)A^{4m-4}}-\frac{2}{\left(2m^2+2m-1\right)A^{2m-2}}-\frac{2\beta}{\left(4m^2+3m-1\right)A^{2m-1}}+\frac{M}{A^{\frac{6m^2}{m1}}}} \tag{20}$$

Integrating (20) we get

$$\int\frac{dA}{\sqrt{\frac{5}{4\left(m^2+4m-2\right)A^{4m-4}}-\frac{2}{\left(2m^2+2m-1\right)A^{2m-2}}-\frac{2\beta}{\left(4m^2+3m-1\right)A^{2m-1}}+\frac{M}{A^{\frac{6m^2}{m-1}}}}}$$

$$\int dt+M'=t+M' \tag{21}$$

In this expression M' is representing the integrating constant. The value of A can be derived from equation (21). Thus by implementing an appropriate transformation of co-ordinates, where A is replaced by τ, co-ordinates x, y, z are converted to X, Y and Z respectively, the metric (1) modified according as

$$ds^2=-\frac{d\tau^2}{\frac{5}{4\left(m^2+4m-2\right)\tau^{4m-4}}-\frac{2}{\left(2m^2+2m-1\right)\tau^{2m-2}}-\frac{2\beta}{\left(4m^2+3m-1\right)\tau^{2m-1}}+\frac{M}{\tau^{\frac{6m^2}{m-1}}}}$$

$$+\tau^2dX^2+\tau^{2m}dY^2+\left(\tau^{2m}\sin^2Y+\tau^2\cos^2Y\right)dZ^2-2\tau^2\cos YdXdZ \tag{22}$$

4. *Physical And Geometrical Characteristics :*

The energy density ρ and pressure p for the model (22) are derived as

$$\rho=\frac{(m+1)(2m+1)}{2\left(m^2+4m-2\right)\tau^{4m-2}}-\frac{(2m+1)}{\left(2m^2+2m-1\right)\tau^{2m}}$$

$$+\frac{\left(2m^2-m-1\right)\beta}{\left(4m^2+3m-1\right)\tau^{2m+1}}+\frac{m(m+2)M}{\tau^{\frac{6m^2+2m-2}{m-1}}} \tag{23}$$

$$p=\xi\theta+\frac{(m-2)(m+1)}{\left(m^2+4m-2\right)\tau^{4m-2}}+\frac{2}{\left(2m^2+2m-1\right)\tau^{2m}}$$

$$+\frac{2\left(2m^3-3m+1\right)\beta}{(1-m)\left(4m^2+3m-1\right)\tau^{2m+1}}+\frac{2m^2(m+2)M}{(m-1)\tau^{\frac{6m^2+2m-2}{m-1}}} \tag{24}$$

The string tension density λ for this model is obtained as

$$\lambda=-\frac{(m+1)(2m+1)}{2\left(m^2+4m-2\right)\tau^{4m-2}}+\frac{(2m+1)}{\left(2m^2+2m-1\right)\tau^{2m}}$$

$$-\frac{(2m^2-m-1)\beta}{(4m^2+3m-1)\tau^{2m+1}}-\frac{m(m+2)M}{\tau^{\frac{6m^2+2m-2}{m-1}}} \tag{25}$$

To obtain value of bulk viscosity coefficient ξ we apply barotropic equation of state for fluid, i.e.,

$$p=\gamma\rho\ ;\ 0<\gamma\le 1 \tag{26}$$

$$\xi\theta=\frac{[(4+\gamma)+2m(\gamma-1)](m-1)}{2(m^2+4m-2)\tau^{4m-2}}-\frac{(2+2m\gamma+\gamma)}{(2m^2+2m-1)\tau^{2m}}$$
$$+\frac{[(2m^2+1)(2+\gamma)-3m(2+m\gamma)]\beta}{(m-1)(4m^2+3m-1)\tau^{2m+1}}+\frac{m(m+2)(m\gamma-2m-\gamma)M}{(m-1)\tau^{\frac{6m^2+2m-2}{m-1}}} \tag{27}$$

Also here expansion θ is given by

$$\theta=(2m+1)\left(\frac{M}{\tau^{\frac{6m^2+2m-2}{m-1}}}+\frac{a_1\beta}{\tau^{2m+1}}+\frac{a_2}{\tau^{4m-2}}+\frac{a_3}{\tau^{2m}}\right)^{1/2} \tag{28}$$

where $a_1=-\frac{2}{(4m^2+3m-1)}$, $a_2=\frac{5}{4(m^2+4m-2)}$ and $a_3=-\frac{2}{(2m^2+2m-1)}$

The shear scalar σ, Hubble parameter H and deceleration parameter q are presented as follows (the values of a_1, a_2 and a_3 are defined above)

$$\sigma=\frac{\sqrt{2}(m-1)}{\sqrt{3}}\left(\frac{M}{\tau^{\frac{6m^2+2m-2}{m-1}}}+\frac{a_1\beta}{\tau^{2m+1}}+\frac{a_2}{\tau^{4m-2}}+\frac{a_3}{\tau^{2m}}\right)^{1/2} \tag{29}$$

$$H=\frac{(2m+1)}{3}\left(\frac{M}{\tau^{\frac{6m^2+2m-2}{m-1}}}+\frac{a_1\beta}{\tau^{2m+1}}+\frac{a_2}{\tau^{4m-2}}+\frac{a_3}{\tau^{2m}}\right)^{1/2} \tag{30}$$

$$q=-1+\frac{3}{2m+1}\left\{\frac{\frac{a_4M}{\tau^{\frac{6m^2+2m-2}{m-1}}}+\frac{a_5\beta}{\tau^{2n+1}}+\frac{a_6}{\tau^{4m-2}}+\frac{a_7}{\tau^{2m}}}{\frac{M}{\tau^{\frac{6m^2+2m-2}{m-1}}}+\frac{a_1\beta}{\tau^{2m+1}}+\frac{a_2}{\tau^{4m-2}}+\frac{a_3}{\tau^{2m}}}\right\} \tag{31}$$

Here $a_4 = -\frac{(3m^2 + m - 1)}{(m - 1)}$, $a_5 = \frac{(2m + 1)}{(4m^2 + 3m - 1)}$, $a_6 = -\frac{5(2m - 1)}{4(m^2 + 4m - 2)}$

and $a_7 = \frac{2m}{(2m^2 + 2m - 1)}$

For this model

$$\Lambda = \frac{\beta}{\tau^{2m+1}} \tag{32}$$

and $\omega = 0$ (33)

We get following graphs for variations of different parameters:

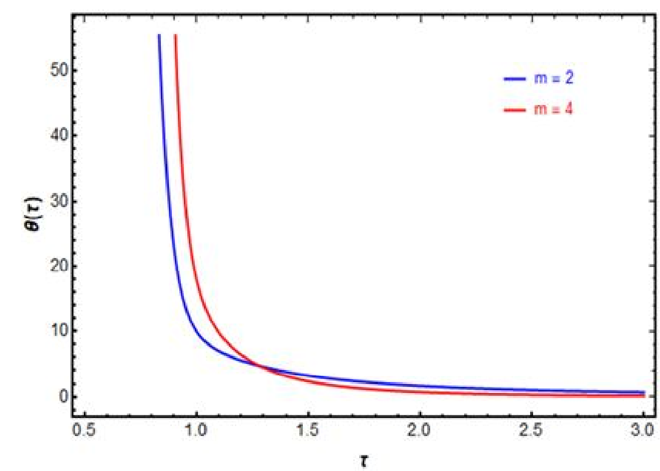

Figure 3.1: Variation of expansion scalar θ with cosmic time τ.

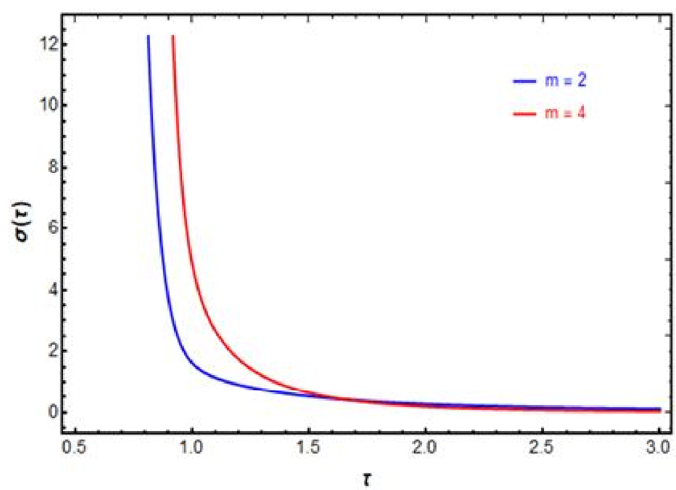

Figure 3.2: Variation of shear scalar σ with cosmic time τ.

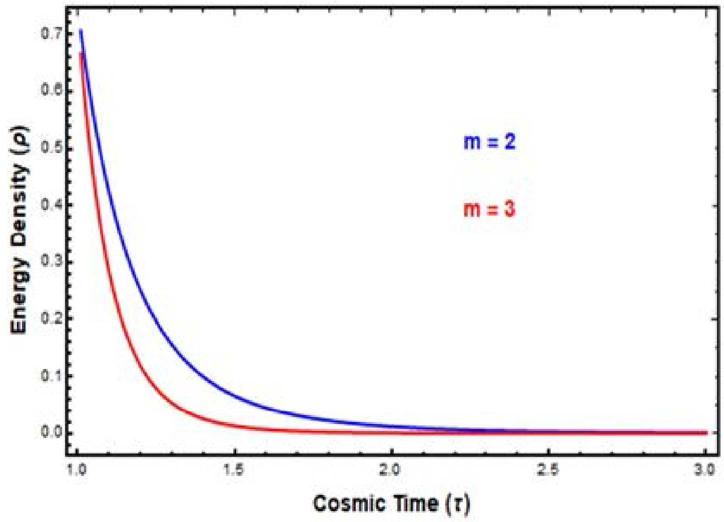

Figure 3.3: Variation of energy density ρ with cosmic time τ.

5. Conclusion

Model (22) begins to grow initially at the big bang. As time goes on, the expansion for θ slows down, gets closer to zero as $\tau \to \infty$, for $m > \frac{1}{2}$. Also model shows anisotropic behavior for $m \neq 1$, since for $\tau \to \infty$ the ratio of the shear σ and expansion θ tends to a finite value, i.e. $\frac{\sigma}{\theta} = \frac{\sqrt{2}(m-1)}{\sqrt{3}(2m+1)} \neq 0$. However this model isotropizes for $m = 1$. The parameters of these models, including the energy density ρ, pressure p, shear σ, Hubble parameter H, decrease with time τ and become closer to zero as $\tau \to \infty$ and $m > \frac{1}{2}$.

It is also observed that the cosmological constant decreases with cosmic time. A point type singularity is observed as $\tau \to 0, g_{11} \to 0, g_{22} \to 0, g_{33} \to 0$ for $m > \frac{1}{2}$. We conclude that deceleration parameter $q \to -1$ as $\tau \to \infty$ for this model, representing accelerating phase of the universe. Hence, in general, the present model represents expanding, shearing and non-rotating, anisotropic universe.

Scope and application : This model can be beneficial for the study of the combined effects of anisotropy and viscosity in the early universe and to explore extensions of theory involving more general dark energy models.

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