ISSN 2231-3478 (Print)
JUSPS-B Vol. 38(5), 59-68 (2026). Periodicity-Monthly
Section B
(Online) ISSN 2319-8052

JOURNAL OF ULTRA SCIENTIST OF PHYSICAL SCIENCES
An International Open Free Access Peer Reviewed Research Journal of Physical Sciences
website:- www.ultrascientist.org
Estd. 1989

Enhanced Product Type Estimators for Finite Population Mean in Two-Phase Sampling Scheme

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DOI : http://dx.doi.org/10.22147/jusps-B/380501

Acceptance Date 21st May 2026 Online Publication date 26th May 2026

Abstract

In survey sampling, generally the use of auxiliary information at the estimation stage enriches the efficiency of estimators. Product estimator suggested by Murthy (1964) is more efficient than mean per unit estimator when there exists a negative correlation between study variable and auxiliary variable. In surveys, whenever the population mean of the auxiliary variable is not known in advance, one can use two- phase sampling scheme or double sampling scheme. In this paper an attempt has been made to develop three product type estimators to estimate finite population mean using two phase sampling schemes suggested by Bose (1943) First estimator is constructed using known coefficient variation of study variable and other two estimators are constructed using estimated coefficient variation of study variable. The efficiencies are compared with mean per unit estimator, conventional two-phase product estimator and two-phase product type exponential estimator suggested by Singh and Vishwakarma (2007), both theoretically and empirically.

Key words : Two-phase sampling, Study variable, Coefficient of variation, Exponential product type estimators, Efficiency.
MSC: 62D05

1 Introduction

In the sampling survey theory of we generally use auxiliary information to improve the efficiency of the estimators. Most commonly used estimators in sampling literature are ratio estimator[1,2], regression estimator[3] and product estimator[4,5]. The ratio estimators (product estimators) are recommended when there exists a positive correlation (negative correlation) between auxiliary variable and study variable. Whenever the linear relationship between auxiliary variable and study variable is very strong, generally the ratio, product and regression estimators are suggested. But when the linear relationship is not very strong it is preferred to use exponential estimators. When the population mean of the auxiliary variable is not known in advance, then we use double sampling scheme/ two-phase sampling scheme[Bose (1943)]. Furthermore we assume that the auxiliary variable x and the study variable y are negatively correlated.

Bahl and Tuteja[6] developed a product type exponential estimator to estimate finite population mean. Following this paper Singh and Vishwakarma[7] suggested a product exponential estimator in two-phase sampling when the information of the population mean of auxiliary variable is unavailable.

In the present work following Searls[8], Srivastava[9] and Upadhyaya and Srivastava[11], we developed three number of product type exponential estimators in two-phase sampling. The above estimators are compared theoretically and numerically with the mean per unit estimator ($\bar{y}$), conventional two-phase product estimator ($t_{TP} = \frac{\bar{x}}{\bar{x}'}\,\bar{y}$) and two-phase product type exponential estimator suggested by Singh and Vishwakarma[7].

Let us consider U={1,2,3,.....,N} a finite population. Assuming x_j and y_j are the values on the j^{th} unit j={1,2,3,.....,N} of the two real variables x and y respectively (x be the auxiliary variable and y be the of study variable). Furthermore, assume that x and y are negatively correlated. Let us assume SRSWOR to draw samples for both phases in two-phase sampling set up. In the first phase sample s'($s' \subset U$) of size n' is drawn to observe the auxiliary variable 'x' only. In the second phase sample 's' of size 'n' is drawn to observe both x , y for given s' (n< n'). Following Bose (1943), let us considered. the second phase sample is directly selected from the population and is independent from the first-phase sample. The samples at both the phases are selected by the help of SRSWOR scheme.

Let $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $\bar{x}' = \frac{1}{n'}\sum_{i=1}^{n'} x_i$

The traditional two-phase product type estimator and two-phase exponential product estimator proposed by Singh and Vishwakarma[7] are given by

$$t_{TP} = \frac{\bar{x}}{\bar{x}'}\,\bar{y} \tag{1.1}$$

$$t_{TEP1} = \bar{y}\,\mathrm{Exp}\left[\frac{\bar{x} - \bar{x}'}{\bar{x} + \bar{x}'}\right] \tag{1.2}$$

Now the mean square errors (MSEs) of t_{TP} and t_{TEP1} to $o\left(\frac{1}{n}\right)$ are given by

$$\mathrm{MSE}\,(t_{TP}) = \bar{Y}^2(\theta_1 - \theta_1')(C_{02} + C_{20} + 2C_{11}) + \theta_1'\bar{Y}^2 C_{02} \tag{1.3}$$

$$\text{MSE}\,(t_{TEP1}) = \bar{Y}^2\left[\theta_1\left(C_{02} + \frac{1}{4}C_{20} + C_{11}\right) + \frac{1}{4}\theta_1' C_{20}\right] \tag{1.4}$$

where, $\theta_1 = \left(\frac{1}{n} - \frac{1}{N}\right)$ and $\theta_1' = \left(\frac{1}{n'} - \frac{1}{N}\right)$

By comparing variance ($\bar{y}$), MSE (t_{TP}) and MSE (t_{TEP1}), we find t_{TEP1} performs better than the estimators ($\bar{y}$) and t_{TP} if

$$-\frac{3}{4}\frac{C_x}{C_y} < \rho < -\frac{1}{4}\frac{C_x}{C_y} \tag{1.5}$$

where, ρ is the population correlation coefficient between x and y.

Consider $C_x = C_y$ then the inequality (1.5) reduces to

$$-\frac{3}{4} < \rho < -\frac{1}{4} \tag{1.6}$$

From the above results, it indicates that for low negative correlation coefficient between x and y, t_{TEP1} performs better in terms of efficiency than two-phase product estimator and mean per unit estimator.

2 *Proposed Estimators :*

We propose the following modified exponential product type estimators to estimate population mean $\bar{Y}$ in two-phase (or double) sampling scheme.

$$t_{TEP2} = \frac{\bar{y}}{1 + \theta_1 C_y^2}\,\text{Exp}\left[\frac{\bar{x} - \bar{x}'}{\bar{x} + \bar{x}'}\right] \tag{2.1}$$

here, $C_y \left(= \frac{S_y}{\bar{Y}}\right)$, population C.V. of y and let us assume that it is known in advance.

$$t_{TEP3} = \frac{\bar{y}}{1 + \theta_1 \hat{C}_y^2}\,\text{Exp}\left[\frac{\bar{x} - \bar{x}'}{\bar{x} + \bar{x}'}\right] \tag{2.2}$$

here, $\hat{C}_y \left(= \frac{s_y}{\bar{y}}\right)$, sample C.V. of y.

$$t_{TEP4} = \bar{y}\left(1 + \theta_1 \hat{C}_y^2\right)\text{Exp}\left[\frac{\bar{x} - \bar{x}'}{\bar{x} + \bar{x}'}\right] \tag{2.3}$$

here, $S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2$ and $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$

3 *Bias and MSE of Different Estimators :*

The expansion due to Taylor's series is applicable to t_{TEP1}, t_{TEP2}, t_{TEP3} and t_{TEP4} and consider the expected value to $O\left(\frac{1}{n}\right)$, the bias of the different estimators are

$$B(t_{TEP1}) = E(t_{TEP1}) - \bar{Y} = \bar{Y}\left[\theta_1 \left(\frac{1}{2}C_{11} - \frac{1}{8}C_{20}\right) + \frac{3}{8}\theta_1' C_{20}\right] \quad (3.1)$$

$$B(t_{TEP2}) = E(t_{TEP2}) - \bar{Y} = \bar{Y}\left[\theta_1 \left(\frac{1}{2}C_{11} - \frac{1}{8}C_{20} - C_{02}\right) + \frac{3}{8}\theta_1' C_{20}\right] \quad (3.2)$$

$$B(t_{TEP3}) = E(t_{TEP3}) - \bar{Y} = \bar{Y}\left[\theta_1 \left(\frac{1}{2}C_{11} - \frac{1}{8}C_{20} - C_{02}\right) + \frac{3}{8}\theta_1' C_{20}\right] \quad (3.3)$$

$$B(t_{TEP4}) = E(t_{TEP4}) - \bar{Y} = \bar{Y}\left[\theta_1 \left(\frac{1}{2}C_{11} - \frac{1}{8}C_{20} + C_{02}\right) + \frac{3}{8}\theta_1' C_{20}\right] \quad (3.4)$$

where, $C_{rs} = \dfrac{\mu_{rs}(x,y)}{\bar{X}^r\,\bar{Y}^s}$

$\mu_{rs}(x,y)$ being the $(r,s)^{th}$ bivariate moment of y and x.

The MSEs of different estimators to $O(\frac{1}{n^2})$ are derived as

$$MSE(t_{TEP1}) = \bar{Y}^2[\left\{\theta_1\left(\frac{1}{4}C_{20} + C_{02} + C_{11}\right) + \frac{1}{4}\theta_1' C_{20}\right\} + \left\{\left(\theta_2 - \frac{3\theta_1}{N}\right)\left(\frac{1}{4}C_{21} - \frac{1}{8}C_{30} + C_{12}\right) - \frac{3}{8}\left(\theta_2' - \frac{3}{N}\theta_1'\right)C_{30}\right\} + \{\theta_1^2\left(\frac{7}{64}C_{20}^2 - \frac{5}{8}C_{11}C_{20}\right) + \frac{79}{64}\theta_1'^2 C_{20}^2\}] \quad (3.5)$$

where, $\theta_2 = (\frac{1}{n^2} - \frac{1}{N^2})$, $\theta_2' = (\frac{1}{n'^2} - \frac{1}{N^2})$

$$MSE(t_{TEP2}) = MSE(t_{TEP1}) - \bar{Y}^2[\theta_1^2(3C_{11}C_{02} + \frac{1}{4}C_{02}C_{20} + C_{02}^2 + \frac{5}{4}\theta_1\theta_1' C_{02}C_{20}] \quad (3.6)$$

$$MSE(t_{TEP3}) = MSE(t_{TEP1}) - \bar{Y}^2[\theta_1^2(C_{11}C_{02} + \frac{1}{4}C_{02}C_{20} + C_{12} + 2C_{03} + C_{02}^2) + \frac{5}{4}\theta_1\theta_1' C_{02}C_{20}] \quad (3.7)$$

$$MSE(t_{TEP4}) = MSE(t_{TEP1}) - \bar{Y}^2\left[\theta_1^2(C_{02}^2 - C_{11}C_{02} - \frac{1}{4}C_{02}C_{20} - C_{12} - 2C_{03}\right) - \frac{5}{4}\theta_1\theta_1' C_{02}C_{20}] \quad (3.8)$$

4 *Comparison Efficiency :*

From (3.5), (3.6), (3.7) and (3.8) it is seen that the mean square errors of t_{TEP1}, t_{TEP2}, t_{TEP3} and t_{TEP4} to $O\left(\frac{1}{n}\right)$ are same. So

the mean square error of the estimators are considered up to $O(\frac{1}{n^2})$ for comparison of efficiency.

i. Estimator t_{TEP2} is more efficient than t_{TEP1} if

$$C_{11} > -\frac{1}{12\theta_1}[5\theta_1' C_{20} + + 4\theta_1 C_{02} + \theta_1 C_{20}] \quad (4.1)$$

Assuming symmetrical bivariate distribution of (x, y) the inequality (4.1) reduces to

$$\rho > -\frac{1}{12Z\,\theta_1}[5\theta_1' Z^2 + 4\theta_1 + \theta_1 Z^2] \quad (4.2)$$

where, $Z = \left(\frac{C_{20}}{C_{02}}\right)^{\frac{1}{2}}$

ii. Estimator t_{TEP3} is more efficient than t_{TEP1} if

$$C_{11} > \frac{-1}{4\theta_1 C_{02}}[5\theta_1' C_{02C_{20}} + \theta_1(4C_{02}^2 + C_{20}C_{02} + 4C_{12} + 8C_{03})] \quad (4.3)$$

Assuming symmetrical bivariate distribution of (x, y) the inequality (4.3) reduces to

$$\rho > \frac{-1}{4\theta_1 Z}\left[\, 5\theta_1' Z^2 \; + \theta_1(4 + Z^2)\,\right] \tag{4.4}$$

iii. Estimator t_{TEP4} is more efficient than t_{TEP1} if

$$C_{11} < \frac{1}{4\theta_1 C_{02}}\left[\, 5\theta_1' C_{02C_{20}} + \theta_1(C_{20}C_{02} + 4C_{12} + 8C_{03} - 4C_{02}^2)\right] \tag{4.5}$$

Assuming symmetrical bivariate distribution of (x, y) the inequality (4.5) reduces to

$$\rho < \frac{-1}{4\theta_1 Z}\left[5\theta_1 Z^2 + \theta_1(Z^2 - 4)\right] \tag{4.6}$$

iv. Estimator t_{TEP3} is more efficient than t_{TEP2} if

$$C_{11} > \frac{1}{3C_{02}}\,(C_{12} + 2C_{03}) \tag{4.7}$$

Assuming symmetrical bivariate distribution of (x, y) the inequality (4.7) reduces to $\rho > 0$ (4.8)

v. Estimator t_{TEP4} is more efficient than t_{TEP2} if

$$C_{11} < \frac{-1}{8\theta_1 C_{02}}\left[\, 5\theta_1' C_{02C_{20}} + \theta_1(C_{20}C_{02} + 2C_{12} + 4C_{03})\right] \tag{4.9}$$

Assuming symmetrical bivariate distribution of (x, y) the inequality (4.9) reduces to

$$\rho - \frac{Z}{8\theta_1}(\, 5\theta_1' \theta_1) \tag{4.10}$$

vi. Estimator t_{TEP4} is more efficient than t_{TEP3} if

$$C_{11} < \frac{-1}{4\theta_1 C_{02}}\left[\, 5\theta_1' C_{02C_{20}} + \theta_1(C_{20}C_{02} + 4C_{12} + 8C_{03})\right] \tag{4.11}$$

Assuming symmetrical bivariate distribution of (x, y) the inequality (4.11) reduces to

$$\rho < \frac{-z}{4\,\theta_1}(\, 5\theta_1' + \theta_1) \tag{4.12}$$

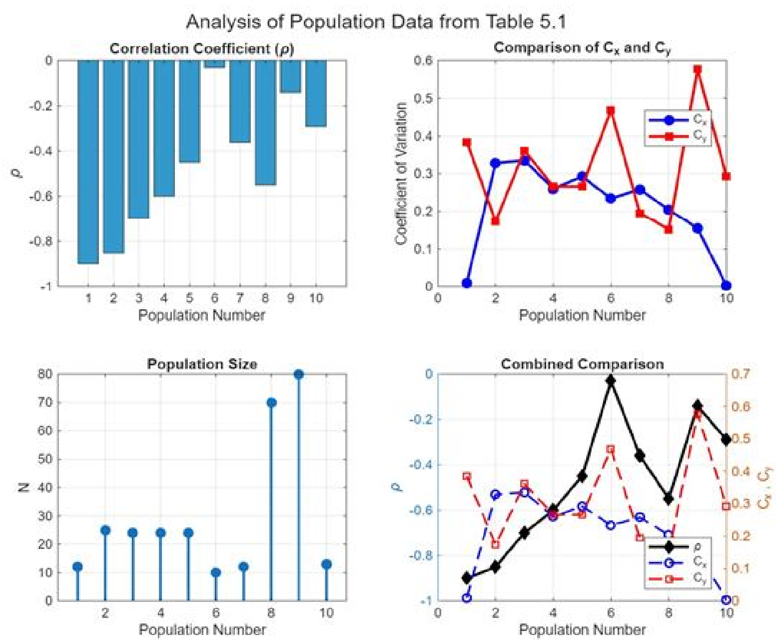
5 *Empirical Study :*

Let us consider 10 natural populations from various textbooks to study the efficiency of different estimators. The comparisons are based on exact mean square errors. The exact mean square errors are calculated from given populations. The descriptions of populations with population size N, population correlation coefficient ρ and population coefficient of variation C_x and C_y are given in Table 5.1. The exact MSE of different estimators *i.e.* mean per unit estimator $t_0(=\bar{y})$, $t_{TP} = \frac{\bar{x}}{\bar{x}'}\,\bar{y}$, t_{TEP1}, t_{TEP2}, t_{TEP3} and t_{TEP4} are given in Table 5.2.

Table 5.1. Description of the Populations

Pop^n No.	Source	X	Y	N	ρ	C_x	C_y
1	Black (2009) p.476	Year	Number of Farms (in millions)	12	-0.9	0.009	0.385
2	Draper & Smith	Average	Amount of	25	-0.85	0.328	0.173

	(1966) p.352	Atmospheric Temp.	Steam Used per Month				
3	Härdle & Hlávka (2007) p.335	Marks for Car Safety	Marks for Car Price	24	-0.7	0.335	0.361
4	Härdle & Hlávka (2007) p.335	Marks for Car Design	Marks for Car Economy	24	-0.6	0.26	0.266
5	Härdle & Hlávka (2007) p.335	Marks for Car Sportiness	Marks for Car Economy	24	-0.45	0.292	0.266
6	Härdle & Hlávka (2007) p.337	Wheat Yeild with Fertiliser B	Wheat Yeild with Fertiliser C	10	-0.03	0.234	0.469
7	Härdle & Hlávka (2007) p.339	Expenditure on Vegetables	Expenditure on Wine	12	-0.36	0.258	0.195
8	Jobson (1991) p.112	Loads of Garbage	Cost of Garbage Disposal	70	-0.55	0.204	0.151
9	Jobson (1992) p.674	Total Mortality Rate	Largest Biweekly Particulate Reading	80	-0.14	0.154	0.577
10	Maddala (1988) p.109	Year	Unemployment Rate	13	-0.29	0.002	0.292



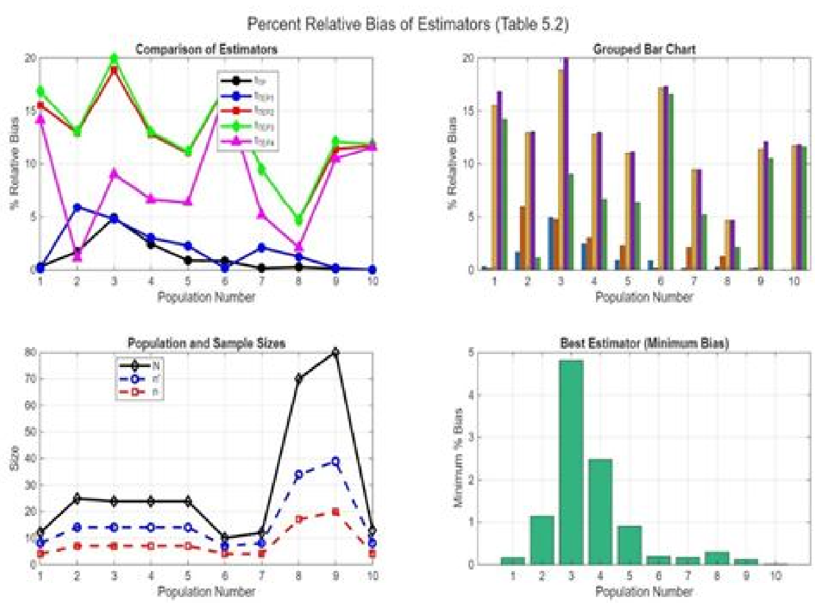
Observation :

1. Negative Correlation in All Populations:
   All populations have negative correlation coefficients (ρ), indicating inverse relationships between X and Y. The value of correlation coefficient ρ is ranging from -0.03 to -0.9.
2. Highest Negative Correlation:
   Population 1 has the highest negative correlation (ρ =–0.90), followed by Population 2 (–0.85).

3. Lowest Negative Correlation:
   Population 6 has lowest negative correlation (ρ =–0.03).
4. Coefficient of Variation of X:
   The value of coefficient of variation of X i.e., C_x is ranging from 0.002 to 0.335.
5. Coefficient of Variation of Y:
   The value of coefficient of variation of Y i.e., C_y is ranging from 0.151 to 0.577.
6. Population Size N:
   The value of population size N is ranging from 10 to 80. Largest population size is 80 (Population 9), while smallest is 10 (Population 6).

Table 5.2. Percent Relative Bias of Estimators
$(\mathbf{t_{TP}}, \mathbf{t_{TEP1}}, \mathbf{t_{TEP2}}, \mathbf{t_{TEP3}}, \mathbf{t_{TEP4}})$

				Estimators				
Pop^n No.	N	n'	n	t_{TP}	t_{TEP1}	t_{TEP2}	t_{TEP3}	t_{TEP4}
1	12	8	4	0.3239	0.1615	15.543	16.862	14.206
2	25	14	7	1.714	5.9478	12.985	13.084	1.1411
3	24	14	7	4.9369	4.8053	18.861	19.989	9.0547
4	24	14	7	2.4783	3.0328	12.813	13.045	6.7111
5	24	14	7	0.9007	2.3122	11.078	11.204	6.4126
6	10	7	4	0.8671	0.1886	17.191	17.343	16.612
7	12	8	4	0.166	2.1342	9.4685	9.478	5.2005
8	70	34	17	0.291	1.2671	4.6796	4.6996	2.1389
9	80	39	20	0.1224	0.1961	11.447	12.124	10.504
10	13	8	4	0.0232	0.0117	11.774	11.867	11.66



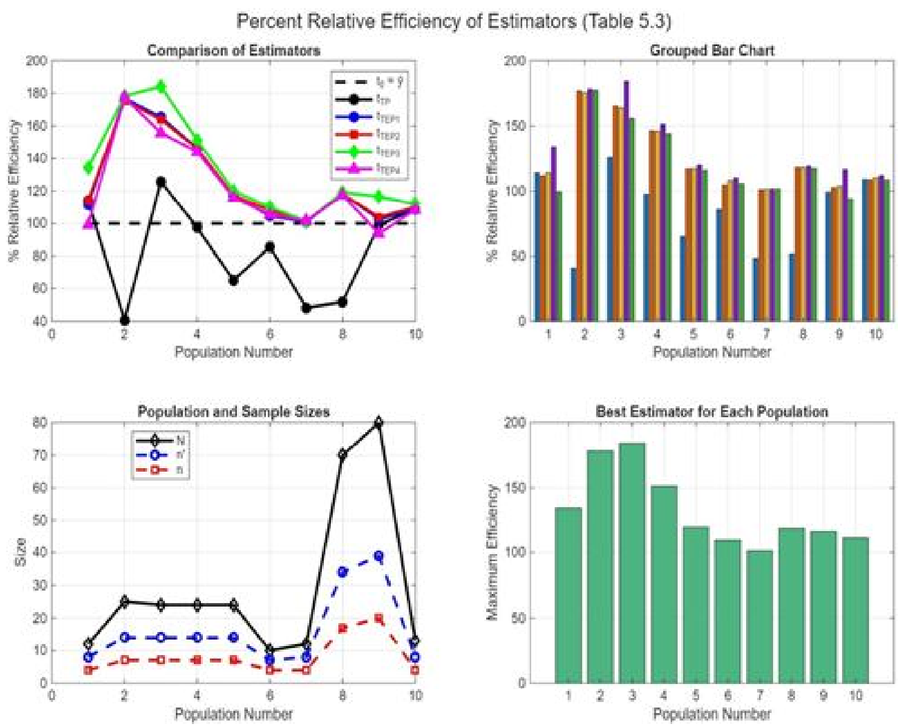
Observation :

1. For population 2 percent relative bias of t_{EP4} is the least.
2. For populations 1, 3, 6 and 10 percent relative bias of t_{EP1} is the least.
3. For population 4,5,7,8 and 9 percent relative bias of t_{TP} is the least.

Table 5.3. Percent Relative Efficiency of Estimators with respect to $\bar{y}$

$(\mathbf{t_{TP}}, \mathbf{t_{TEP1}}, \mathbf{t_{TEP2}}, \mathbf{t_{TEP3}}, \mathbf{t_{TEP4}})$

				Estimators					
Pop^n No.	N	n'	N	$t_0 = \bar{y}$	t_{TP}	t_{TEP1}	t_{TEP2}	t_{TEP3}	t_{TEP4}
1	12	8	4	100	113.96	111.5	113.97	134.13	99.333
2	25	14	7	100	40.777	176.99	175.63	178.32	177.45
3	24	14	7	100	125.66	165.42	164.01	184.21	155.76
4	24	14	7	100	97.375	146.26	146.01	151.33	144.2
5	24	14	7	100	65.115	116.91	117.15	119.84	115.85
6	10	7	4	100	85.721	104.57	107.93	109.83	105.43
7	12	8	4	100	48.231	100.87	101.18	101.39	101.45
8	70	34	17	100	51.869	117.98	117.98	118.99	117.26
9	80	39	20	100	98.987	102.58	103.8	116.45	93.759
10	13	8	4	100	108.77	108.55	110.04	111.8	108.37



Observation :

1. Baseline Estimator:
   The estimator $t_0 = \bar{y}$ has constant efficiency 100 for all populations.
2. Most Efficient Estimator:
   Estimator t_{TEP3} gives the highest efficiency in most of the populations i.e., population no. 1, 2, 3,4, 5, 6, 8, 9 and 10.
   Estimator t_{TEP4} gives the highest efficiency in population no.7.
3. Weak Performance of t_{TP}:
   Estimator t_{TP} performs poorly in Populations 2, 4, 5, 6, 7, 8 and 9, where efficiency is below 100.
4. Performance of t_{TEP2} :
   For all of the populations, the estimator t_{TEP2} is more efficient than the mean per unit estimator $t_0 = \bar{y}$ and t_{TP} .
   For population 1, 5, 6, 8, 9 and 10 the estimator t_{TEP2} is more efficient than t_{TEP1},
5. Performance of t_{TEP4}:
   For all populations (except Population 1), the estimator t_{TEP4} is more efficient than the mean per unit estimator
   $t_0 = \bar{y}$. In population 1, the percent relative efficiency of the estimator t_{TEP4} is 99.333, slightly lower than $\bar{y}$.
   For population 2,3,4,5,6,7 and 8 the estimator t_{TEP4} is more efficient than the estimator t_{TP}.

6 Conclusion

For two-phase sampling set up (the second phase sample is directly selected from the population i.e., second phase sample is independent from the first-phase sample and at both the phases, the samples are selected by using SRSWOR scheme) the estimator is more efficient and followed by for different estimators considered here.

Scope of future work :

Future studies aim to integrate the coefficient of variation into two-phase sampling frameworks where non-respondents are handled using Hansen-Hurwitz technique. Another way of future study may be performed by developing generalized weighting schemes and deriving their optimal values when different phases contain differing auxiliary data.

Declaration

All authors declare that they have no conflicts of interest.

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