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On Tensor Product in Quantum Technology

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Abstract

The exponential scaling of Hilbert spaces presents a major computational bottleneck in simulating and controlling large scale multi qubit systems. This paper establishes the tensor product space not just as composite vector space, but fundamentally as a multi linear mapping that describes quantum operations across desperate subsystems. By modelling quantum gates and straight through this lense, we preserve the internal structural integrity of quantum operations while exponentially decreasing the computational overhead needed for simulation and control.

Key words : Tensor Product, Multilinear Mapping, Basis, Quantum Technology.

Introduction

The mathematics of quantum systems.

In quantum mechanics the state of an isolated quantum system is represented by a unit - norm vector in a complex, finite dimensional Hilbert spaces, **H** when dealing with composite quantum systems (such as a multi-qubit register), the state space is formed by the tensor product of the individual Hilbert spaces.

A subsystem A and B with corresponding Hilbert spaces $\mathbf{H_A}$ & $\mathbf{H_B}$, the composite system exists in the tensor product space $\mathbf{H_{AB}} = \mathbf{H_A} \otimes \mathbf{H_B}$

The Tensor product is fundamentally rooted in multi linear algebra, it acts as a bridge that reduces multi linear operations to linear ones. A tensor product maps a Cartesian product of n vector spaces to a single vector space that is linear in each argument. The Tensor product space $\mathbf{H_A} \otimes \mathbf{H_B}$ and the bilinear map $\otimes : \mathbf{H_A} \times \mathbf{H_B} \rightarrow \mathbf{H_A} \otimes \mathbf{H_B}$, possess the universal property : for any bilinear map $f : V \times W \rightarrow Z$, there exists an unique linear map $\underline{f}: V \otimes W \rightarrow Z$ such that $f = \underline{f} \circ \otimes$.

In quantum technology, this ensures that local operations on subsystem A and local operations on subsystem B cab be combined linearly in a globally coordinated framework, extending multi linearity natively.

Two qubit states and entanglement.

Consider two distinct qubits belonging to two parties, Alice and Bob.

- Alice's state space is $\mathbf{H_A} = C^2$, with a basis $\{|0>_A, |1>_A\}$.
- Bob's state space is $\mathbf{H_B} = C^2$ with a basis $\{|0>_B, |1>_B\}$.

The joint system lives in the tensor product space $\mathbf{H_A} \otimes \mathbf{H_B}$**,** which is $C^2 \otimes C^2 \sim C^4$.

The basis for this new space consists of all pairwise tensor products of the individual of basis vectors:

$|0>_A \otimes |0>_B = |00)$

$|0>_A \otimes |1>_B = |01)$

$|1>_A \otimes |0>_B = |10)$

$|1>_A \otimes |1>_B = |11)$

The argument - Separatibility Vs Entanglement :

A general state in this combined space is formed by taking linear combinations of these basis elements. A state is called separable if it can be written strictly as the tensor product of two Individual single-qubit states :

$|¥) = |p)_A \otimes |_q)_B$

For instance, |01) is a separable state.

The tensor product space is spanned by linear combinations of simple tensors, there exists vectors in the space that can't be factored into a simple tensor product. These are known as entangled states.

The Tensor product acts multilinearly, mapping the independent inputs (|p) , $|_q$)) to a joint global space, allowing it to capture correlations that defy classical probability and are the bedrock of quantum information protocols like quantum teleportations and dense coding.

1. Application in quantum technology :

networks Tensor The multi linear mapping property reveals it's true technological utility in the representation of many - body quantum systems. Because of the tensor product, the dimension of the state space of N qubits scales as 2^N. For 100 qubits storing a general quantum estate classically requires ~ 10^{30} complex numbers.

In quantum machine learning and quantum chemistry, researchers utilise tensor networks. By decomposing high rank tensors into multilinear maps of lower rank components. Such as metrics product states, algorithms can efficiently approximate ground states and unitary operations.

2. Future scope of present study :

The multi linear framework of the tensor product is currently evolving to address several critical frontiers in quantum technology :

1. Quantum Machine learning architecture : Recent advancements shows that the tensor product space can be used to compress generative AI and deep learning models, enabling energy-efficient deployment on quantum hardware,

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