Ultra Scientist Vol. 25(3)B, 335-342 (2013).

Radiation effect on MHD free convection flow of stratified viscous fluid with heat and mass transfer

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(Acceptance Date 15th September, 2013)

Abstract

The objective of this paper is to study the effect of Radiation on MHD free convection flow of stratified viscous fluid past a vertical porous plate with heat and mass transfer taking Visco-elastic and Darcy resistance terms into account and the constant permeability of the medium numerically and neglecting induced magnetic field in comparison to applied magnetic field. The velocity, temperature and concentration distributions are derived and discussed graphically. It is observed that velocity increases with the increase in G_r (Grashof number), K (Permeability parameter) and N (Radiation parameter), but it decreases with the increase in M (Magnetic parameter).

Keywords: Heat and mass transfer, Free convection, MHD, Porous medium, Vertical plate, Radiation.

Introduction

The convection problem in a porous medium has important applications in geothermal reservoirs and geothermal extractions. The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear rector, chemical process industries and many engineering applications in which the fluid is the working medium. The wide range of technological and industrial applications has stimulated considerable amount of interest in the study of heat and mass transfer in convection flows. Free convective flow past a vertical plate has been studied extensively by Ostrach[9]. Siegel[12] investigated the transient free convection from a vertical flat plate. Cheng and Lau[4] and Cheng and Teckchandani[5] obtained numerical solutions for the convective flow in a porous medium bounded by two isothermal parallel plates in the presence of the withdrawal of the fluid. In all the above mentioned studies, the effect of porosity, permeability and the thermal resistance of the medium is ignored or treated as constant. However, porosity measurements by Benenati and Broselow[3] show that porosity is not constant but varies from the surface of the plate to its interior to which as a result permeability also varies. In case of unsteady free convective flow, Soundalgekar[14] studied the effects of viscous dissipation on the flow past an infinite vertical porous plate. The combined effect of buoyancy forces from thermal and mass diffusion on forced convection was studied by Chen *et al.*[6]. The free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient was studied by Ramanaiah and Malarvizhi[10]. Bejan and Khair[2] have investigated the vertical free convective boundary layer flow embedded in a porous medium resulting from the combined heat and mass transfer. Lin and Wu[7] analyzed the problem of simultaneous heat and mass transfer with the entire range of buoyancy ratio for most practical and chemical species in dilute and aqueous solutions. Rushi Kumar and Nagarajan[11] studied the mass transfer effects of MHD free convection flow of incompressible viscous dissipative fluid past an infinite vertical plate. Mass transfer effects on free convection flow of an incompressible viscous dissipative fluid have been studied by Manohar and Nagarajan[8]. Sivaiah *et al.*[13] studied heat and mass transfer effects on MHD free convective flow past a vertical porous plate. Recently, Agrawal *et al.*[1] have discussed the effect of stratified viscous fluid on MHD free convection flow with heat and mass transfer past a vertical porous plate.

In the present section we have considered the problem of Agrawal *et al.*[1] by the introducing Radiation under the same conditions taken by Agrawal *et al.*[1].

Mathematical Analysis:

We study the two-dimensional free convection and mass transfer flow of stratified viscous fluid past an infinite vertical porous plate under the following assumptions:

- The plate temperature is constant
- Viscous and Darcy's resistance terms are taken into account with constant permeability of the medium.
- Boussinesq's approximation is valid.
- The suction velocity normal to the plate is constant and can be written as,

$$v^1 = -U_0$$

A system of rectangular co-ordinates O (x^1, y^1, z^1) is taken, such that $y^1 = 0$ on the plate and z^1 axis is along its leading edge. All the fluid properties considered constant except that the influence of the density variation with temperature is considered. The influence of the density variation in other terms of the momentum and the energy equation and the variation of the expansion coefficient with temperature is considered negligible. The variations of density, viscosity, elasticity, Stefan-Boltzman constant and thermal conductivity are supposed to be of the form

$$\rho = \rho_o e^{-b^1 y^1},\quad \mu = \mu_o e^{-b^1 y^1},\quad \sigma = \sigma_o e^{-b^1 y^1},$$
$$\sigma^* = \sigma_o^* e^{-b^1 y^1},\qquad k_T = k_o e^{-b^1 y^1},$$

where ρ_o, μ_o, σ_o, σ_o^* and k_o are the coefficients of density, viscosity, elasticity, Stefan-Boltzman constant and thermal conductivity respectively at $y^1 = 0$, $b^1 > 0$ represents the stratification factor.

Under these conditions, the problem is governed by the following system of Equations:

Equation of continuity:

$$\frac{\partial v^1}{\partial y^1} = 0 \tag{1}$$

Equation of Momentum:

$$\rho\left(\frac{\partial u^1}{\partial t^1} + v^1\frac{\partial u^1}{\partial y^1}\right) = \rho g\beta(T^1 - T^1_\infty) + \rho g\beta^1(C^1 - C^1_\infty) + \frac{\partial}{\partial y^1}\left(\mu\frac{\partial u^1}{\partial y^1}\right) - \left(\sigma B_0^2 + \frac{\mu}{K^1}\right)u^1 \tag{2}$$

Equation of Energy:

$$\frac{\partial T^1}{\partial t^1} + v^1\frac{\partial T^1}{\partial y^1} = \frac{1}{\rho C_p}\frac{\partial}{\partial y^1}\left(k_T\frac{\partial T^1}{\partial y^1}\right) - \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y} \tag{3}$$

Equation of Concentration:

$$\frac{\partial C^1}{\partial t^1} + v^1\frac{\partial C^1}{\partial y^1} = D\left(\frac{\partial^2 C^1}{\partial y^{1^2}}\right) \tag{4}$$

where u^1, v^1 are the velocity components. T^1, C^1 are the temperature and concentration components, ν is the kinematic viscosity. ρ is the density, σ is the electric conductivity, B_0 is the magnetic induction, k_T is the thermal conductivity and D is the concentration diffusivity, C_p is the specific heat at constant pressure.

The boundary conditions for the velocity, temperature and concentration fields are:

$$u^1 = 0, T^1 = T^1_w, C^1 = C^1_w \text{ at } y^1 = 0$$

$$u^1 = 0, T^1 = T_\infty, C^1 = C_\infty \text{ at } y^1 \to \infty \tag{5}$$

By using Rosseland approximation for the radiation, we take

$$q_r = \frac{-4\sigma^*}{3\lambda^*}\frac{\partial T^{1^4}}{\partial y^1} \tag{6}$$

where σ^* is the Stefan - Boltzman constant and λ^* is the mean absorption coefficient.

We assume that the temperature difference with in the flow is such that T^{1^4} may be expressed as a linear function of temperature

This is accomplished by expending T^{1^4} in a Taylor series about T^1_∞ and neglecting higher- order terms, thus

$$T^{1^4} \cong 4T_\infty^{1^3}T^1 - 3T_\infty^{1^4} \tag{7}$$

Let us introduce the non-dimensional variables

$$u = \frac{u^1}{U_0}, \quad t = \frac{t^1U_0^2}{\nu}, \quad y = \frac{y^1U_0}{\nu},$$

$$\theta = \frac{T^1 - T^1_\infty}{T^1_w - T^1_\infty}, \quad C = \frac{C^1 - C^1_\infty}{C^1_w - C^1_\infty}$$

$$K = \frac{K^1U_0^2}{\nu^2}, \quad P_r = \frac{\nu}{\alpha}, \quad S_c = \frac{\nu}{D},$$

$$M = \frac{\sigma B_0^2 \nu}{\rho U_0^2}, \quad N_0 = \frac{\beta^1 (C_w^1 - C_\infty^1)}{\beta (T_w^1 - T_\infty^1)},$$

$$G_r = \frac{\nu g \beta (T_w^1 - T_\infty^1)}{U_0^3}, \quad N = \frac{16\sigma * T_\infty^{1^3}}{3\lambda^* k_T}$$

$$b = \frac{b^1 \nu_o}{U_o}$$

where P_r is the Prandtl number, G_r is the Grashof number, N_0 is the buoyancy ratio, S_c is the Schmidt number, M is the magnetic parameter, K is the permeability parameter, β is the thermal expansion coefficient, β^1 is the concentration expansion coefficient and, b is the stratification parameter, N is the radiation parameter. Other physical variables have their usual meaning.

Introducing the non-dimensional quantities describes above, the governing equations reduce to

$$\frac{\partial u}{\partial t} - (1-b)\frac{\partial u}{\partial y} = G_r(\theta + N_0 C) + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{K}\right)u \quad (8)$$

$$P_r \frac{\partial \theta}{\partial t} - (P_r - b)\frac{\partial \theta}{\partial y} = (1+N)\frac{\partial^2 \theta}{\partial y^2} \quad (9)$$

$$\frac{\partial C}{\partial t} - \frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} \quad (10)$$

and the corresponding boundary conditions are

$$u = 0, \theta = 1, C = 1 \text{ at } y = 0$$

$$u = 0, \theta = 0, C = 0 \text{ at } y \to \infty \quad (11)$$

Method Of Solution:

We assume the solution of eq. (8), (9), (10) as

$$u(y,t) = u_0(y)e^{-nt},$$

$$\theta(y,t) = \theta_0(y)e^{-nt},$$

$$C(y,t) = C_0(y)e^{-nt} \quad (12)$$

Using eq.(12) in eq. (8), (9), (10) and we get

$$u_0'' + (1-b)u_0' - \left[\left(M + \frac{1}{K} - n\right)\right]u_0 = -G_r\theta_0 - G_r N_0 C_0 \quad (13)$$

$$(1+N)\theta_0'' + (P_r - b)\theta_0' + nP_r\theta_0 = 0 \quad (14)$$

$$C_0'' + S_c C_0' + S_c n C_0 = 0 \quad (15)$$

Now the corresponding boundary conditions are

$$u_0 = 0, \theta_0 = 1, C_0 = 1 \text{ at } y = 0$$

$$u_0 = 0, \theta_0 = 0, C_0 = 0 \text{ at } y \to \infty \quad (16)$$

Equations (13) to (15) are ordinary linear differential equations, now u_0, θ_0 and C_0 with boundary conditions (16) are

$$u_0 = (A_1 + A_2)e^{-m_3 y} - A_1 e^{-m_1 y} - A_2 e^{-m_2 y} \quad (17)$$

$$\theta_0 = e^{-m_1 y} \quad (18)$$

$$C_0 = e^{-m_2 y} \quad (19)$$

where

$$m_1 = \frac{(P_r - b) + \sqrt{(P_r - b)^2 - 4nP_r(1+N)}}{2(1+N)}$$

$$m_2 = \frac{S_c + \sqrt{S_c^2 - 4S_c n}}{2}$$

$$m_3 = \frac{(1-b) + \sqrt{(1-b)^2 + 4\left(M + \frac{1}{K} - n\right)}}{2}$$

$$A_1 = \frac{G_r}{\left[m_1^2 - (1-b)m_1 - \left(M + \frac{1}{K} - n\right)\right]}$$

$$A_2 = \frac{G_r N_0}{\left[m_2^2 - (1-b)m_2 - \left(M + \frac{1}{K} - n\right)\right]}$$

Hence, The equations for u, θ and C will be as follows

$$u(y,t) = \left[(A_1 + A_2)e^{-m_3 y} - A_1 e^{-m_1 y} - A_3 e^{-m_2 y}\right]e^{-nt} \quad (20)$$

$$\theta(y,t) = e^{-m_1 y} e^{-nt} \quad (21)$$

$$C(y,t) = e^{-m_2 y} e^{-nt} \quad (22)$$

Skin Friction:

The skin friction coefficient at y=0 is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left[-m_3(A_1 + A_2) + m_1 A_1 + m_2 A_2\right]e^{-nt} \quad (23)$$

Result and Discussion

Fluid velocity distribution of fluid flow is tabulated in Table 1 and plotted in Fig. 1 having six graphs at P_r=0.71, S_c = 0.4, n = 0.1, t = 0.1, N_0 = 1.5, b = 0.1 for following different value of G_r, M, K and N.

	G_r	M	K	N
For Graph-1	2	0.02	100	0
For Graph-2	2	0.02	100	0.05
For Graph-3	4	0.02	100	0.05
For Graph-4	2	0.04	100	0.05
For Graph-5	2	0.02	1000	0.05
For Graph-6	2	0.02	100	0.10

It is observed from Fig. 1 that all velocity graphs are increasing sharply up to y = 1.2 after that velocity in each graph begins to decrease and tends to zero with the increasing in y. It is also observed from Fig. 1 that velocity increases with the increase in G_r, K and N, but it decreases with the increase in M.

Table 1. Value of velocity u for Fig. 1 at P_r = 0.71, S_c = 0.4, n = 0.1, t =0.1, N_0 = 1.5, b = 0.1 and different values of G_r, M, K and N.

y	Graph 1	Graph 2	Graph 3	Graph 4	Graph 5	Graph 6
0	0	0	0	0	0	0
1	18.72249	19.05211	38.10423	15.71365	21.33853	19.43585
2	23.13596	23.55809	47.11619	19.17322	26.51654	24.05471
3	22.08269	22.48960	44.97920	18.10832	25.41845	22.97430
4	19.26483	19.61456	39.22913	15.66113	22.24324	20.03683
5	16.16468	16.44725	32.89451	13.04789	18.70109	16.79320

Table 2. Value of skin friction τ for Fig. 2 at P_r = 0.71, S_c = 0.4, n = 0.1, N_0 = 1.5, b = 0.1 and different values of G_r, M, K and N.

t	Graph 1	Graph 2	Graph 3	Graph 4	Graph 5	Graph 6
0	31.54040	32.06260	64.12520	26.87100	35.70200	32.66588
0.2	30.91586	31.42772	62.85544	26.33892	34.99505	32.01905
0.4	30.30368	30.80541	61.61081	25.81737	34.30210	31.38503
0.6	29.70363	30.19542	60.39084	25.30615	33.62288	30.76357
0.8	29.11546	29.59751	59.19502	24.80506	32.95710	30.15441
1	28.53893	29.01144	58.02288	24.31389	32.30451	29.55731

Table 3. Value of temperature θ for Fig. 3 at P_r = 0.71, n = 0.1, t = 0.1, b = 0.1 and different values of N.

y	Graph 1	Graph 2	Graph 3
0	0.99005	0.99005	0.99005
1	0.63128	0.65051	0.67166
2	0.40252	0.42741	0.45566
3	0.25666	0.28083	0.30913
4	0.16365	0.18452	0.20972
5	0.10435	0.12124	0.14227

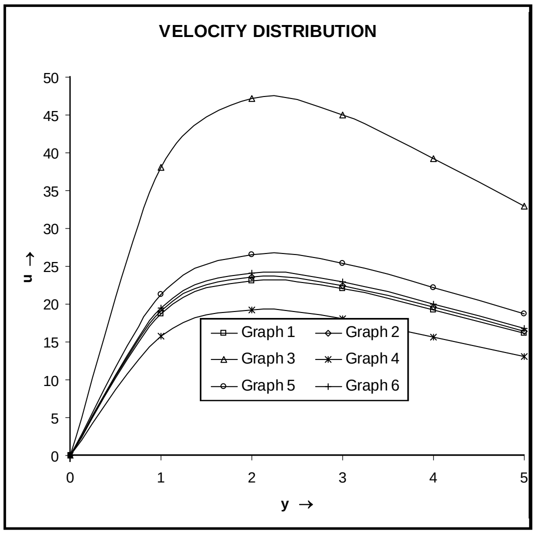

Fig. 1

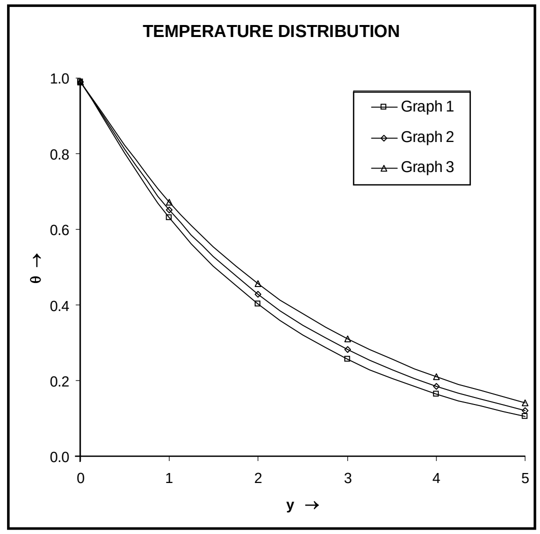

Fig. 3

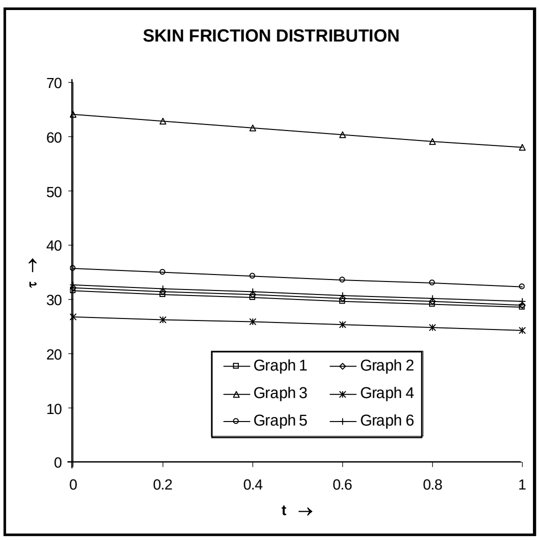

Fig. 2

The skin friction distribution is tabulated in Table 2 and plotted in Fig. 2 having six graphs. It is observed from Fig. 2 that skin friction increases with the increase in G_r, K and N, but it decreases with the increase in M.

The temperature distribution is tabulated in Table 3 and plotted in Fig. 3 having three graphs. It is observed from Fig. 3 that temperature increases with the increase in N. The concentration does not change with the change in above parameters taken for velocity.

Particular Case :

When N is equal to zero, this problem reduces to the problem of Agrawal *et al.*[1].

Conclusion

1. The velocity increases with the increase in N (Radiation parameter).
2. The skin friction increases with the increase in N.
3. The temperature also increases with the increase in N.

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