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Effect of Environmental Pollution on the Growth and Existence of Biological Populations and I-Function

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Abstract

The deleterious effect of environmental pollution on interacting biological populations depends upon the toxicity and the level of pollutant, the sort of damage it causes to the physiological and biochemical systems of the populations and their environment. To study this situation, in this paper, a mathematical model is presented by considering that the growth rate of species and the carrying capacity of its environment are directly affected by pollution and decrease as the concentration of the pollutant increases. After that a solution of this mathematical equation will also obtained with the help of I- function of two variables.

1. Introduction

Since long both various kinds of industrial discharges and wastes, causing damage to our ecosystems, are polluting our atmosphere and the aquatic environment.

The biological and ecological consequences of pollution in our environment may be considered in several ways depending upon the toxic level of pollutants (acute or chronic) and the ecotoxicological situations. One such situation is where the pollutants can adversely affect the natural resources, thereby influencing the growth of other biological populations, which may be depending upon these resources. The other such situation is where the pollutants can affect directly the species accompanied by rapid injury to the principal physiological and biochemical systems of the organism, and results in lethal toxication, elimination of individual species and populations or causes profound pathological alterations on the level of individual organisms, individual populations, and occasionally, on entire ecosystems which might change the carrying capacity of the environment[1-6]. Various investigations have been carried out in this direction, both experimentally and mathematically[1-6].

In view of the above, in this paper, we have studied the effect of environmental pollution on the growth and existence of two interacting biological populations in the situation where the pollutant causes injury to the principal physiological and biochemical systems of the populations and their environment. To study this situation, a mathematical model is presented here be considering that the growth rate of species and carrying capacity of its environment are directly affected by pollution and decrease as the concentration of the pollutant increases.

The I–function of two variables introduced by Sharma & Mishra[7], will be defined and represented as follows:

$$I\left[\begin{matrix}x\\y\end{matrix}\right] = I^{0,n:m_1,n_1:m_2,n_2}_{p_i,q_i:r:p_{i'},q_{i'}:r':p_{i''},q_{i''}:r''}\left[\begin{matrix}x\\y\end{matrix}\Big| \begin{matrix}[(a_j;\alpha_j,A_j)_{1,n}],[(a_{ji};\alpha_{ji},A_{ji})_{n+1,p_i}]\\ [(b_{ji};\beta_{ji},B_{ji})_{1,q_i}]\end{matrix}\right.$$

$$\left.\begin{matrix}:[(c_j;\gamma_j)_{1,n_1}],[(c_{ji'};\gamma_{ji'})_{n_1+1,p_{i'}}];[(e_j;E_j)_{1,n_2}],[(e_{ji''};E_{ji''})_{n_2+1,p_{i''}}]\\ :[(d_j;\delta_j)_{1,m_1}],[(d_{ji'};\delta_{ji'})_{m_1+1,q_{i'}}];[(f_j;F_j)_{1,m_2}],[(f_{ji''};F_{ji''})_{m_2+1,q_{i''}}]\end{matrix}\right]$$

$$= \frac{1}{(2\pi\omega)^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\,\theta_2(\xi)\theta_3(\eta)x^{\xi}y^{\eta}\,d\xi\,d\eta, \quad (1)$$

where

$$\phi_1(\xi,\eta)=\frac{\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\xi+A_j\eta)}{\sum_{i=1}^{r}\left[\prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}\xi-A_{ji}\eta)\prod_{j=1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}\xi+B_{ji}\eta)\right]}$$

$$\theta_2(\xi)=\frac{\prod_{j=1}^{m_1}\Gamma(d_j-\delta_j\xi)\prod_{j=1}^{n_1}\Gamma(1-c_j+\gamma_j\xi)}{\sum_{i'=1}^{r'}\left[\prod_{j=m_1+1}^{q_{i'}}\Gamma(1-d_{ji'}+\delta_{ji'}\xi)\prod_{j=n_1+1}^{p_{i'}}\Gamma(c_{ji'}-\gamma_{ji'}\xi)\right]}$$

$$\theta_3(\xi)=\frac{\prod_{j=1}^{m_1}\Gamma(f_j-F_j\eta)\prod\Gamma(1-e_j+E_j\eta)}{\sum_{i''=1}^{r''}\left[\prod_{j=m2+1}^{q_{i''}}\Gamma(1-f_{ji''}+F_{ji''}\eta)\prod_{j=n2+1}^{p_{i''}}\Gamma(e_{ji''}-E_{ji''}\eta)\right]}$$

x and y are not equal to zero, and an empty product is interpreted as unity $p_i, p_{i'}, p_{i''}, q_i, q_{i'}, q_{i''}$, n, n_1, n_2, n_j and m_k are non negative integers such that $p_i \geq n \geq 0$, $p_{i'} \geq n_1 \geq 0$, $p_{i''} \geq n_2 \geq 0$, $q_i > 0$, $q_{i'} > 0$, $q_{i''} > 0$, (i = 1, …, r; i´ = 1, …, r´; i´´ = 1, …, r´´; k = 1, 2) also all the A's, α's, B's, β's, γ's, δ's, E's and F's are assumed to be positive quantities for standardization purpose; the definition of I-function of two variables given above will however, have a meaning even if some of these quantities are zero. The contour L_1 is in the ξ–plane and runs from – ω∞ to + ω∞, with loops, if necessary, to ensure that the poles of $\Gamma(d_j-\delta_j\xi)$ (j = 1, ..., m_1) lie to the right, and the poles of $\Gamma(1-c_j+\gamma_j\xi)$ (j=1, ..., n_1), $\Gamma(1-a_j+\alpha_j\xi+A_j\eta)$ (j = 1, ..., n) to the left of the contour.

The contour L_2 is in theη–plane and runs from – ω∞ to + ω∞, with loops, if necessary, to ensure that the poles of$\Gamma(f_j-F_j\eta)$ (j=1,....., n_2) lie to the right, and the poles of

$\Gamma(1-e_j+E_j\eta)$ (j=1, ..., m_2), $\Gamma(1-a_j+\alpha_j\xi+A_j\eta)$ (j = 1, ..., n) to the left of the contour. Also contour. Also

$$R=\sum_{j=1}^{p_i}\alpha_{ji}+\sum_{j=1}^{p_{i''}}\gamma_{ji'}-\sum_{j=1}^{q_i}\beta_{ji}-\sum^{q_{i''}}\delta_{ji'}<0,$$

$$S=\sum_{j=1}^{p_i}A_{ji}+\sum_{j=1}^{p_{i''}}E_{ji''}-\sum_{j=1}^{q_i}B_{ji}-\sum^{q_{i''}}F_{ji''}<0,$$

$$U=\sum_{j=n+1}^{p_i}\alpha_{ji}-\sum_{j=1}^{q_i}\beta_{ji}+\sum_{j=1}^{m_1}\delta_j-\sum_{j=m_1+1}^{q_{i'}}\delta_{ji'}+\sum_{j=1}^{n_1}\gamma_j-\sum_{j=n_1+1}^{p_{i'}}\gamma_{ji'}>0, \quad (2)$$

$$V=-\sum_{j=n+1}^{p_i}A_{ji}-\sum_{j=1}^{q_i}B_{ji}-\sum_{j=1}^{m_2}F_j-\sum_{j=m_2+1}^{q_{i''}}F_{ji''}+\sum_{j=1}^{n_2}E_j-\sum_{j=n_2+1}^{p_{i''}}E_{ji''}>0, \quad (3)$$

and $|\arg x| < \frac{1}{2}U\pi$, $|\arg y| < \frac{1}{2}V\pi$. (4)

2. Mathematical Model:

Consider the growth of interacting and dispersing biological species of density $N_i(x, t)$, (i = 1, 2) in a one dimensional linear habitat 0 5 x ≤ L, whose growth rate and the carrying capacity of the environment is decreasing due to the environmental pollution present in the habitat. The dynamical equations governing the growth of the species are assumed to be given by the following system of non- linear partial differential equations

$$\partial N_i/\partial t = N_iF_i(N_1, N_2, r_i(C), K_i(C)) + D_i(\partial^2 N_i/\partial t^2),\ i = 1, 2 \quad (5)$$

where, $F_i(N_1, N_2, r_i(C), K_i(C))$ determines the interaction function of the species. $r_i(C)$ and $K_i(C)$ are the intrinsic growth rate and the carrying capacity of the environment respectively which are affected by the concentration C (x, t) of pollutant. The positive constant D_i (i = 1, 2) is the dispersion coefficient of the species. The dynamics of the concentration C(x, t) of the pollutant is considered to be given by the following equation[8]

$$\partial C/\partial t = Q_0 - \alpha C + D_c(\partial^2 C/\partial x^2) \quad (6)$$

where, $Q_0 > 0$ is the constant determining the exogenous rate of input of pollutant into the habitat, $\alpha > 0$ represents the first order decay constant as a result of biological (including consumption by the species), chemical or geological processes. $D_c > 0$ is the diffusion coefficient of the pollutant. In the formulation of the model it has been assumed that the organismal uptake of the pollutant is proportional to the concentration of the pollutant present in the environment of the population. On the basis of the mathematical model (6) the solution of this mathematical equation will be obtained with the help of I-function of two variables in the subsequent part of this section.

3. Result in terms of I-Function of two variables:

Choose concentration C (x, t) in terms of I-function of two variable as

$$C(x,t) = I^{0,n_1;m_2,n_2;m_3,n_3}_{p_i,q_i:r;p_i',q_i':r';p_i'',q_i'':r''}\left[\begin{matrix} z_1x^{\sigma}t^{\mu} \\ z_2 \end{matrix}\right], \quad (7)$$

provided that $\sigma > 0$, $\mu > 0$, $U' > 0, V' > 0$, $|\arg z_2| < \frac{1}{2}U'\pi$, where U' and V' are given in (1.3.4) and (1.3.5).

Now differentiate it with respect to x and t partially, we get

$$\partial C/\partial t = (1/t)\ I^{0,n_1+1;m_2,n_2;m_3,n_3}_{p_i+1,q_i+1:r;p_i',q_i':r';p_i'',q_i'':r''}\left[\begin{matrix} z_1 x^{\sigma} t^{\mu} \\ z_2 \end{matrix}\middle|\begin{matrix}(0,\mu)......,......:.....,........,:......,....... \\ ......,......(1,\mu):.....,.........,:......,.......\end{matrix}\right] \quad (8)$$

and

$$(\partial^2 C/\partial x^2) = (1/x^2)\ I^{0,n_1+1;m_2,n_2;m_3,n_3}_{p_i+1,q_i+1:r;p_i',q_i':r';p_i'',q_i'':r''}\left[\begin{matrix} z_1 x^{\sigma} t^{\mu} \\ z_2 \end{matrix}\middle|\begin{matrix}(0,\sigma)......,......:.....,...:......,... \\ ......,......(2,\sigma):.....,...,:......,....\end{matrix}\right]. \quad (9)$$

Now after using (7), (8) and (9) in (6), we get following result

$$(1/t)\ I^{0,n_1+1;m_2,n_2;m_3,n_3}_{p_i+1,q_i+1:r;p_i',q_i':r';p_i'',q_i'':r''}\left[\begin{matrix} z_1 x^{\sigma} t^{\mu} \\ z_2 \end{matrix}\middle|\begin{matrix}(0,\mu)......,......:.....,........,:......,....... \\ ......,......(1,\mu):.....,.........,:......,.......\end{matrix}\right]$$

$$= Q_0 - \alpha\ \ I^{0,n_1;m_2,n_2;m_3,n_3}_{p_i,q_i:r;p_i',q_i':r';p_i'',q_i'':r''}\left[\begin{matrix} z_1 x^{\sigma} t^{\mu} \\ z_2 \end{matrix}\right]$$

$$+ D_C\,(1/x^2)\ I^{0,n_1+1;m_2,n_2;m_3,n_3}_{p_i+1,q_i+1:r;p_i',q_i':r';p_i'',q_i'':r''}\left[\begin{matrix} z_1 x^{\sigma} t^{\mu} \\ z_2 \end{matrix}\middle|\begin{matrix}(0,\sigma)......,......:.....,...:......,... \\ ......,......(2,\sigma):.....,...,:......,....\end{matrix}\right]. \quad (10)$$

provided that $\sigma > 0$, $\mu > 0$, $U' > 0$, $V' > 0$, $|\arg z_2| < \frac{1}{2} U'\pi$, where U' and V' are given in (2) and (3).

Special Cases:

On choosing $r' = 1$ and $r'' = 1$ and $m_2 = 0$, $n_2 = 0$, $m_3=0$, $n_3=0$, $p_i' = 0$, $q_i' = 0$, $p_i'' = 0$, $q_i'' = 0$ in (10), we get following result in terms of I-function of one variable, which are the results given by Tiwari[7]:

$$(1/t)\ I^{m,\,n+1}_{p_i+1,\,q_i+1:\,r}\left[z\, x^{\sigma}\, t^{\mu}\ \middle|\ \begin{matrix}(0,\mu),\ ...... \\ ......,\ (1,\mu)\end{matrix}\right]$$

$$= Q_0 - \alpha\quad I^{m,\,n}_{p_i,\,q_i:\,r}\,[z\, x^{\sigma}\, t^{\mu}]$$

$$+ D_C\,(1/x^2)\ I^{m,\,n+1}_{p_i+1,\,q_i+1:\,r}\left[z\, x^{\sigma}\, t^{\mu}\middle|\begin{matrix}(0,\sigma),\ ...... \\ ......,\ (2,\sigma)\end{matrix}\right], \quad (11)$$

On taking r = 1 in (11), we get the following result in terms of H–function, which is a result given by Kushwaha [8, p. 106]:

$$(1/t)\ H^{m,\,n+1}_{p+1,\,q+1}\left[z\, x^{\sigma}\, t^{\mu}\ \middle|\ \begin{matrix}(0,\mu),\ (a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q},\ (1,\mu)\end{matrix}\right]$$

$$= Q_0 - \alpha\ H^{m,\,n}_{p,\,q}\left[z\, x^{\sigma}\, t^{\mu}\ \middle|\ \begin{matrix}(a_j,\alpha_j)_{1,p} \\ (b_i,\beta_i)_{1,q}\end{matrix}\right]$$

$$+ D_C\,(1/x^2) H^{m,\,n+1}_{p+1,\,q+1}\left[z\, x^{\sigma}\, t^{\mu}\ \middle|\ \begin{matrix}(0,\sigma),\ (a_j,\alpha_j)_{1,p} \\ (b_j,\beta_j)_{1,q},\ (2,\sigma)\end{matrix}\right], \quad (12)$$

On specializing the parameters, I-function may be reduced to G-function, Lauricella's functions Legendre functions, Bessel functions, hypergeometric functions, Appell's functions, Kampe de Feriet's functions and several other higher transcendental functions. Therefore the result (10) is of general nature and may reduced to be in different forms, which will be useful in the literature on applied Mathematics and other branches.

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