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Effect of Rotation on MHD flow of visco-elastic (Rivlin-ericksen) fluid through porous medium with transpiration

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Abstract

The purpose of the present problem is to study the effect of Rotation on MHD flow of visco-elastic (Rivlin-Ericksen) fluid through porous medium with transpiration. The governing equations of motions are solved by a regular perturbation technique. The velocity of fluid, skin friction, temperature and concentration are discussed with the help of tables and graphs. The primary velocity of fluid increases with the increase in M (Hartman number), but it decreases with the increase in G_r (Grashof Number), K (Porosity parameter) and Ω (Rotation parameter). The secondary velocity increases with the increase in G_r, M and Ω, but it decreases with the increase in K.

Keywords : Rivlin-Ericksen Fluid, MHD Flow, Rotation, Heat transfer, Mass transfer, Porous medium, Transpiration.

Introduction

The flow of an incompressible viscous fluid past an impulsively started infinite horizontal plate, in its own plane, was first studied by Stoke's[13]. It is also known a Rayleigh's problem in the literature. Soundalgekar[11] first presented an exact solution to the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate. Soundalgekar[12] has studied mass transfer effects on flow past an impulsively started infinite vertical plate. The study of convection with heat and mass transfer is very useful in fields such as chemistry, agriculture and oceanography. A few representative fields of interest in which combined heat and mass transfer play an important role are the design of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agriculture fields and grooves of fruit trees, damage of crops due to freezing and pollution of the environment. This technique is used in the cooling processes of paper. Das. *et. al*.[4] considered the mass transfer effects on flow past an impulsively started infinite isothermal vertical plate with constant mass flux. Muthucummaraswamy *et al.*[9] have studied the heat and mass transfer effects on flow past an impulsively started vertical plate. Gebhart *et al.*[6] have discussed the nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass transfer and mass diffusion. Bathaih *et al.*[1] worked on the MHD unsteady hele-shaw of viscous fluid. Rathod and Shrikanth[10] have studied the unsteady MHD flow of Rivlin-Ericksen incompressible flow through an inclined channel with two parallel flat walls under the influence of magnetic field. Datta *et al.*[5] have studied the magnetohydrodyanamic unsteady flow of a visco-elastic liquid (Revilen-Ericksen) near a porous wall suddenly set in the motion with heat transfer including heat source generating sources or heat-absorbing sinks. Chakraborty and Borkakati[3] have investigated the laminar convection flow of an electrically conducting second order visco-elastic fluid in porous medium down an inclined parallel plate channel in the presence of uniform transverse magnetic field. Bodosa and Borkakati[2] have investigated MHD flow and heat transfer of Rivlin-Ericksen fluid through an inclined channel with heat source and sinks. Suvarna *et. al.*[14] has studied the hydromagnetic flow of Rivlin-Ericksen fluid down an inclined plate. Kundu and Sengupta[8] have worked on MHD flow of Reiner-Rivlin visco-elastic fluid between two coaxial circular cylinder with porous walls and rotating boundaries. Recently, Vir *et al.*[15] have analysed MHD flow of viscoelastic (Rivlin-Ericksen) fluid through porous medium. Gupta *et. al.*[7] have analysed effect of transpiration on MHD flow of visco-elastic (Rivlin-Ericksen) fluid through porous medium.

Present study is an extension of the work Gupta *et. al.*[7] with rotation. The aim of present study is to investigate the effect of rotation on the velocity of the fluid.

Formulation of the problem :

Consider the steady free convective flow with mass transfer of an electrically conducting viscous fluid past an infinite vertical porous plate at $z^* = 0$. Let the fluid and the plate be in a state of rigid rotation with constant angular velocity Ω about z*-axis, taken normal to the plate. A constant transverse magnetic field B_o is acting parallel to the axis of rotation. Taking the magnetic Reynolds number to be small, the induced magnetic field is neglected in comparison to the applied magnetic field B_o. Since the length of the plate is large, therefore, all the physical variables depend on z* only. In the present problem, magnetic field B_o is assumed to be constant throughout the motion. We further assume that the electric field is equal to zero. The governing equations of continuity, momentum, energy and diffusion for a free convective flow of an electrically conducting fluid along a hot, non conducting porous vertical plate in the presence of heat source and rotation are given as :

$$\frac{\partial u^*}{\partial t^*} - 2\Omega^* v^* = \nu\left(1 + E'\frac{\partial}{\partial t^*}\right)\frac{\partial^2 u^*}{\partial z^{*2}} + g\beta(T^* - T_\infty) + g\beta'(C^* - C_\infty) - \frac{\sigma B_o^2}{\rho}u^* - \frac{\nu}{K^*}u^* \quad (1)$$

$$\frac{\partial v^*}{\partial t^*} + 2\Omega^* u^* = \nu\left(1 + E'\frac{\partial}{\partial t^*}\right)\frac{\partial^2 v^*}{\partial z^{*2}} - \frac{\sigma B_o^2}{\rho}v^* - \frac{\nu}{K^*}v^* \quad (2)$$

$$\frac{\partial T^*}{\partial t^*}=\frac{\kappa}{\rho C_P}\frac{\partial^2 T^*}{\partial z^{*2}} \quad (3)$$

$$\frac{\partial C^*}{\partial t^*}=D\frac{\partial^2 C^*}{\partial z^{*2}}+D_1\frac{\partial^2 T^*}{\partial z^{*2}} \quad (4)$$

where ρ is the density, g is the acceleration due to gravity, β is the coefficient of volume expansion, β′ is the coefficient of concentration expansion, ν is the Kinematic viscosity, E′ is the coefficient of visco-elastic, T_∞ is the temperature of the fluid in the free stream, σ is the electric conductivity, B_0 is the magnetic induction, D is the chemical molecular diffusivity, κ is the thermal conductivity, C_∞ is the concentration at infinity, C_p is the specific heat at constant pressure, D is the thermal diffusivity, Ω^* is the angular velocity.

The boundary conditions at the wall and in the free stream are:

$$\left.\begin{array}{llll} u^*=u_0, & \dfrac{\partial T^*}{\partial z^*}=-\dfrac{q}{\kappa}, & \dfrac{\partial C^*}{\partial z^*}=-\dfrac{j''}{D}, & \text{at } z^*=0,\ t^*=0 \\ u^*=0 & T^*\to T^*_\infty & C^*\to C^*_\infty, & \text{at } z^*\to\infty,\ t^*>0 \end{array}\right\} \quad (5)$$

On introducing the following non dimensional quantities

$$\left.\begin{array}{lll} u=\dfrac{u^*}{u_o}, & v=\dfrac{v^*}{u_o}, & z=\dfrac{z^* u_o}{\nu} \\ \theta=\dfrac{\left(T^*\text{-}T^*_\infty\right)}{\left(\dfrac{q\nu}{\kappa u_0}\right)} & & \phi=\dfrac{\left(C^*\text{-}C^*_\infty\right)}{\left(\dfrac{j''\nu}{Du_0}\right)} \end{array}\right\}$$

in equations (1) to (4), we get

$$\frac{\partial f}{\partial t}=G_r\theta+G_c\phi+(1+E\frac{\partial}{\partial t})\frac{\partial^2 f}{\partial z^2}-\left(M_2+2i\Omega\right)f \quad (6)$$

$$\frac{\partial\theta}{\partial t}=\frac{1}{P_r}\frac{\partial^2\theta}{\partial z^2} \quad (7)$$

$$\frac{\partial\phi}{\partial t}=\frac{1}{S_c}\frac{\partial^2\phi}{\partial z^2}+A\frac{\partial^2\theta}{\partial z^2} \quad (8)$$

The boundary condition (5) becomes:

$$\left.\begin{array}{llll} f=1, & \dfrac{\partial\theta}{\partial z}=-1, & \dfrac{\partial\phi}{\partial z}=-1, & \text{at } z=0,\ t=0 \\ f=0 & \theta\to\infty & \phi\to 0, & \text{at } z\to\infty, \end{array}\right\} \quad (9)$$

where

$$G_r=\frac{\nu g\beta\left(\dfrac{q\nu}{\kappa u_0}\right)}{u_0^3}, \text{ (Grashof number)}$$

$$G_c=\frac{\nu g\beta^*\left(\dfrac{j''\nu}{Du_0}\right)}{u_0^3}, \text{ (Modified Grashof number)}$$

$$P_r=\frac{\mu Cp}{\kappa}, \text{ (Prandtl number)}$$

$$S_c=\frac{\nu}{D}, \text{ (Schmidt number)}$$

$$M=\frac{\sigma B_0^2\nu}{\rho u_0^2}, \text{ (Hartmann number)}$$

$$K=\frac{K' u_0^2}{\nu^2}, \text{ (Porosity parameter)}$$

$$E=\frac{E' u_0^2}{\nu^2}, \text{ (Visco-elastic parameter)}$$

$$A=\frac{DD_1 q}{\nu\rho\kappa}, \text{ (Thermal diffusion parameter)}$$

$$\Omega = \frac{\nu\Omega^*}{u_o^2} \text{ (Rotation parameter)}$$

f = u + iv, M1 = M2 + 2iΩ, M2 = M + 1/K

We assume the following solutions:

$$\left.\begin{aligned} f(z,t) &= f_0(z).e^{-nt} \\ \theta(z,t) &= \theta_0(z).e^{-nt} \\ \phi(z,t) &= \phi_0(z).e^{-nt} \end{aligned}\right\} \quad (10)$$

Substituting equation (9) into equations (6) to (8), we find

$$f_o'' - (M_1 - n)A_1 f_o = -G_r A_1 \theta_o - G_c A_1 \phi_o \quad (11)$$

$$\theta_o'' + n\,P_r\,\theta_0 = 0 \quad (12)$$

$$\phi_o'' + nS_c\phi_o = -AS_c\theta_o'' \quad (13)$$

with corresponding boundary conditions:

$$\left.\begin{aligned} &f_0 = 1, \quad \frac{\partial\theta_0}{\partial y} = -1, \quad \frac{\partial\phi_0}{\partial y} = -1, \quad \text{at } y = 0, \\ &f_0 = 0 \quad \theta_0 \to 0 \quad \phi_0 \to 0, \quad \text{as } y \to \infty, \end{aligned}\right\} \quad (14)$$

Solving equations (11) to (13) under boundary condition (14), we get

$$f_o = e^{-\sqrt{A_2}z} - G_r A_7 \sin A_3 z - G_c A_8 \sin A_4 z + G_c AA_1 S_c A_9 \left(\frac{A_7 \sin A_3 z}{A_5} + \frac{A_8 A_{10} \sin A_4 z}{A_6}\right) \quad (15)$$

$$\theta_o = -\frac{\sin A_3 z}{A_3} \quad (16)$$

$$\phi_o = -\frac{\sin A_4 z}{A_4} + AS_c A_9 (A_{10} \sin A_4 z - \sin A_3 z) \quad (17)$$

where

$$A_1 = \frac{1}{(1 - nE)}, \qquad A_2 = A_1(M_1 - n)$$

$$A_3 = \sqrt{nP_r}, \qquad A_4 = \sqrt{nS_c}$$

$$A_5 = \frac{A_1}{A_3} \qquad A_6 = \frac{A_1}{A_4}$$

$$A_7 = \frac{A_5}{A_3^2 + A_2} \qquad A_8 = \frac{A_6}{A_4^2 + A_2}$$

$$A_9 = \frac{A_3}{A_3^2 - nS_c} \qquad A_{10} = \frac{A_3}{A_4}$$

The primary velocity u (real part of f) and secondary velocity v (imaginary part of f) from equation (12) are given as

$$u = [e^{-az}\cos(bz) - G_r A_{7r} \sin A_3 z - G_c A_{8r} \sin A_4 z + G_c AA_1 S_c A_9 \left(\frac{A_{7r} \sin A_3 y}{A_5} + \frac{A_{8r} A_{10} \sin A_4 z}{A_6}\right)].e^{-nt} \quad (18)$$

$$v = [-e^{-az}\sin(bz) - G_r A_{7i} \sin A_3 z - G_c A_{8i} \sin A_4 z + G_c AA_1 S_c A_9 \left(\frac{A_{7i} \sin A_3 y}{A_5} + \frac{A_{8i} A_{10} \sin A_4 z}{A_6}\right)].e^{-nt} \quad (19)$$

where

$$a = \left[\frac{\{(A_1(M_2 - n))^2 + 4\Omega^2 A_1^2\}^{1/2} + A_1(M_2 - n)}{2} \right]^{1/2}$$

$$b = \left[\frac{\{(A_1(M_2 - n))^2 + 4\Omega^2 A_1^2\}^{1/2} - A_1(M_2 - n)}{2} \right]^{1/2}$$

$$A_{7r} = \frac{A_5[A_3^2 + A_1(M_2 - n)]}{[A_3^2 + A_1(M_2 - n)]^2 + 4A_1^2\Omega^2}$$

$$A_{7i} = \frac{-2A_5 A_1 \Omega}{[A_3^2 + A_1(M_2 - n)]^2 + 4A_1^2\Omega^2}$$

$$A_{8r} = \frac{A_6[A_4^2 + A_1(M_2 - n)]}{[A_4^2 + A_1(M_2 - n)]^2 + 4A_1^2\Omega^2}$$

$$A_{8i} = \frac{-2A_6 A_1 \Omega}{[A_4^2 + A_1(M_2 - n)]^2 + 4A_1^2\Omega^2}$$

Results and Discussion

The primary and secondary velocity distributions are tabulated in Table 1 & Table 2 and plotted in Fig. 1 & 2 having six graphs at n = 0.1, P_r = 0.71, S_c = 0.4, t = 1, E = 0.2, G_c = 3, A = 4 and following different values of G_r, M, K and Ω.

	G_r	M	K	Ω
For Graph-1	5	0.2	2	0
For Graph-2	5	0.2	2	0.5
For Graph-3	10	0.2	2	0.5
For Graph-4	5	0.4	2	0.5
For Graph-5	5	0.2	3	0.5
For Graph-6	5	0.2	2	1

From Graphs - I to VI of Fig. 1, it is found that the primary velocity u increases with the increase in z. On comparing Graphs –II to VI with Graph-I it is observed that primary velocity increases with the increase in M, but it decreases with the increase in G_r, K and Ω.

Table 1. Value of primary velocity u for Fig. 1 at n = 0.1, P_r = 0.71, S_c = 0.4, t = 1, E = 0.2, G_c = 3, A = 4 and different values of G_r, M, K and Ω.

z	Graph-I	Graph-II	Graph-III	Graph-IV	Graph-V	Graph-VI
0	0.90484	0.90484	0.90484	0.90484	0.90484	0.90484
1	19.57884	6.01595	3.95847	6.45602	5.21739	2.02725
2	37.51167	11.18679	7.21656	12.07854	9.60249	3.57322
3	53.61016	15.93856	10.33484	17.22618	13.65922	5.11685
4	66.96994	19.90918	13.06614	21.51804	17.06055	6.41446
5	76.88019	22.84991	15.24888	24.69523	19.58063	7.36656

Table 2. Value of secondary velocity v for Fig-2 at n = 0.1, P_r = 0.71, S_c = 0.4, t = 1, E = 0.2, G_c = 3, A = 4 and different values of G_r, M, K and Ω.

z	Graph-I	Graph-II	Graph-III	Graph-IV	Graph-V	Graph-VI
0	0	0	0	0	0	0
1	0	-8.94788	-5.86761	-7.37595	-10.30192	-5.85625
2	0	-17.19696	-11.25309	-14.16758	-19.80685	-11.07278
3	0	-24.54031	-16.15093	-20.22103	-28.25962	-15.76478
4	0	-30.63224	-20.38745	-25.24511	-35.26904	-19.68620
5	0	-35.15211	-23.77254	-28.97294	-40.46908	-22.59794

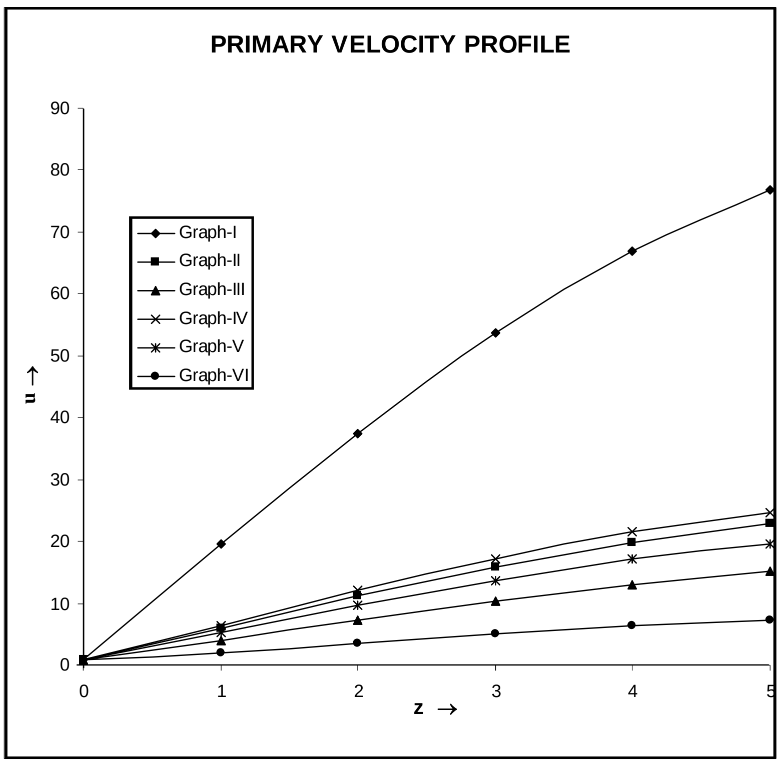

Fig. 1.

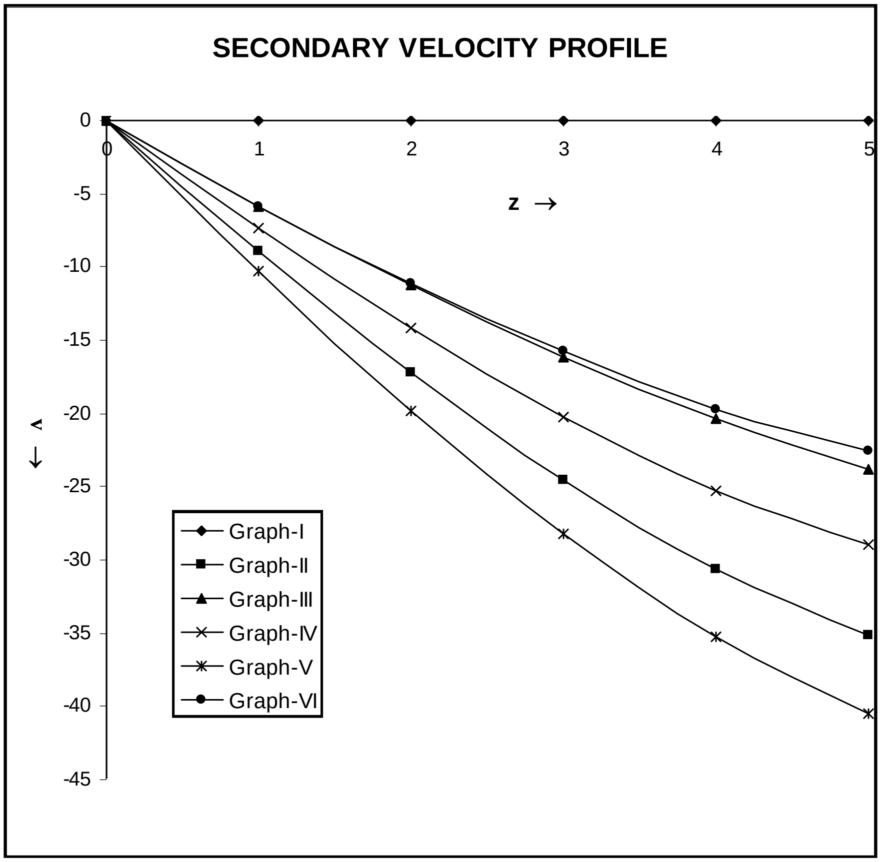

Fig. 2.

From Graph-I of Fig. 2 it is noticed that secondary velocity is zero in the absence of rotation velocity. From Graphs - II to VI of Fig. 2, it is found that the secondary velocity v decreases with the increase in z. On comparing Graphs-III to VI with Graph-II it is observed that secondary velocity increases with the increase in G_r, M and Ω, but it decreases with the increase in K.

The temperature and concentration distribution don't change with the change in parameters taken for velocity.

Particular case :

When Ω is equal to zero, this problem reduces to the problem of Gupta *et. al*.[7].

Conclusion

The primary velocity decreases with the increase in Ω (Rotation parameter) and secondary velocities increases with the increase in Ω .

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