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Heat transfer of Generelised Couette flow of two Immiscible liquids in a rotating system

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Abstract

When a straight channel formed by two parallel plates, through which two immiscible liquids occupying different heights are flowing under constant pressure gradients is rotated about an axis perpendicular to the plates, secondary motion is set up. The motion is caused due to the combined effects of the pressure gradient and motion of the upper plate. The heat transfer characteristics are evaluated assuming equal and different plate temperatures. The solutions are exact. The heat transfer coefficient at the upper and lower plate has been calculated and presented graphically.

Key words : Parallel plates, Temperature profiles, Heat Transfer Co-efficient.

Introduction

The study of flow and heat transfer in a rotating system has drawn considerable interest due to its wide applications in designing thermosiphon tube. In cooling turbine blades and have some is bearing in MHD Power generation.

If the channel is rotating about a vertical axis, there exists a secondary flow. The study of such secondary flows was presented in a rotating channel by Vidyanidhi and Nigam[8]. Jana and Datta[4] have considered the Couette flow and Heat transfer of a viscous incompressible fluid between two infinite parallel plates which rotate with a uniform angular velocity about an axis perpendicular to the plates. Ramana Rao & Narayana[5] extended the work of Jana & Datta[4] for the flow of two incompressible immiscible plates occupying equal heights between two parallel plates they studied the heat transfer characteristics, assuming equal and different plate temperatures. Ramana Rao & Narayana[5] extended the work of Vidyanidhi & Nigam[8] and studied the heat transfer characteristics assuming equal & different plates temperatures for poiseuille flow. Ch. Baby Rani and G. Koteswara Rao (2010) extended the work of

Ramana Rao and Narayana[6] for the Couette flow of two immiscible liquids occupying different heights between two parallel porous plates[7].

Baby Rani[3] extended the work of Ramana Rao & Narayana (1981) for the poiseuille flow and heat transfer of two immiscible liquids occupying different heights between two parallel non-porous plates in a rotating system. Baby Rani, Ramesh, Chalam[2] considered the heat transfer of Couette flow of two immiscible liquids occupying different heights between two parallel plates in a rotating system. In the present paper we consider the combined effect of the pressure gradient and motion of the upper plate *i.e.* generalized couette flow. We evaluated the heat transfer aspects for the generalized couette flow of two immiscible liquids occupying different heights between two parallel non-porous plates assuming equal and different plate temperatures. The Heat transfer coefficient for the upper and lower plate is determined. We take the constant pressure gradient Po is equal to 2 and constant velocity Co is equal to 1 for the purpose of calculations.

1. Basic equations and their solutions:

The equations of motion and continuity for the steady state in a rotating frame of reference $O'X'Y'Z'$ as in Squire (1956) for the two immiscible liquids as shown in the fig. (1) are

$$\left(\vec{U}_m^1 . \vec{\nabla}^1\right)\vec{U}_m^1 + 2\vec{\Omega}' \mathrm{x} \vec{U}_m^1 = -\rho_m^{-1}\vec{\nabla}^1 \pi_m^1 + \nu_m \vec{\nabla}^2 \vec{U}_m^1 \tag{1}$$

$$\vec{\nabla}' . \vec{U}_m' = 0 \tag{2}$$

where π_m' (modified pressures)

$$= P_m - \frac{1}{2}\rho_m \left| \vec{\Omega}' \mathrm{x}\, \vec{r}' \right|^2 \quad (m = 1,2) \tag{3}$$

Here the subscripts 1 and 2 refer to the zone-I (upper liquid) and zone-II (lower liquid) in the ranges $\in \leq Z' \leq L$ and $-L \leq Z' \leq \in$ respectively. Also $\vec{U}_1'$, $\vec{U}_2'$, $\vec{\Omega}'$ & $\vec{r}'$ are the velocities of upper and lower liquids, angular velocity and position vector respectively.

We choose a right handed cartesian system as shown in fig. (1) below such that Z' - axis is perpendicular to the motion of the liquids under the action of constant pressure gradient $P_m^1 = \left(-\frac{\partial \pi_m^1}{\partial x^1}\right)$ in the direction of X' axis between two parallel plates $Z' = \pm L$ (stationary relative to O'X'Y'Z'). The motion is caused when the upper plate moves with uniform velocity Uo along the X^1-axis.

$$-\pi_m' = \left(\frac{P_{m_1}' - P_{m_2}'}{D}\right)x' - P_{m_1}' \tag{4}$$

Where P_{m_1}' and P_{m_2}' stand for pressures on the planes x' = 0 and x' = D respectively.

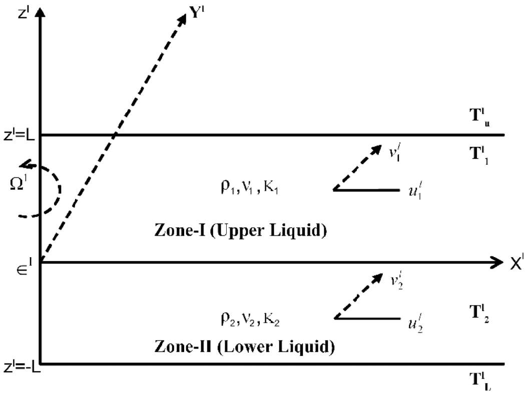


Fig. 1. Schematic diagram

The fluid region is divided into zone-I and zone-II. Zone-I contains the liquid in $\in \le Z \le L$ and zone-II contains the liquid region in $-L \le Z \le \in$

The velocities of the two liquids are then represented by

$$\vec{U}_1^1 = \left[u_1^1\left(z^1\right), v_1^1\left(z^1\right), -Wo\right], \quad \vec{U}_2^1 = \left[u_2^1\left(z^1\right), v_2^1\left(z^1\right), -Wo\right]$$

$$\vec{\Omega} = \left(0, 0, \vec{\Omega}^1\right), \left(-L \le \in^1 \le L\right) \tag{5}$$

Where u$_1$, v$_1$ are the velocity components in zone-I and u$_2$, v$_2$ are the velocity components in zone-II. $\Omega^1 = \frac{\alpha^2 v_2}{L^2}$ (Taylor number for the lower liquid)

Introducing the non-dimensional quantities

$$\mathrm{U}'_m = \frac{P'_m L^2}{2\rho_1 \vartheta_1} u_m \quad \mathrm{V}'_m = \frac{P'_m L^2}{2\rho_1 \vartheta_1} v_m \quad \vec{r}' = \vec{r}L$$

(Primary velocities of the upper and lower liquids for m = 1 and 2 respectively)

(Secondary velocities of the upper and lower liquids for m = 1 and 2 respectively)

$\alpha_m^2 = \frac{\Omega'_m L^2}{\upsilon_m}$ (Taylor number for the upper and lower liquids)

And $\lambda = \frac{\rho_2}{\rho_1}$ (Ratio of the densities of the lower and upper liquids)

$\mu^2 = \frac{\upsilon_2}{\upsilon_1}$ (Ratio of the viscosities of the lower and upper liquids) (6)

Let $q_1 = u_1 + iv_1$, $q_2 = u_2 + iv_2$, equation (1) reduces to

$$\frac{d^2 q_1}{dz^2} - 2i\mu^2\alpha^2 q_1 = -P_0$$

$$\frac{d^2 q_2}{dz^2} - 2i\alpha^2 q_2 = \frac{-P_0}{\lambda\mu^2} \tag{7}$$

Subject to Zone-I $q_1 = C_0$ at $z = 1$,

Interface condition $q_1 = q_2$ at $z = \in$, $(-1 < \in < 1)$

$$\frac{dq_1}{dz} = \lambda\mu^2 \frac{dq_2}{dz} \; at \; z = \in,$$

Zone-II $q_2 = 0$ at $z = -1$ (8)

Where P_0 is a constant pressure gradient and c_0 is the constant velocity with which the upper plate is moved.

Since, we consider two immiscible liquids with interface $z = \in$, the fluid region is divided into zone-I ($\in \le z \le L$) and zone-II ($-L \le z \le \in$).

This implies, $T_{z=\in} - T_2 \ne 0$ $T_1 - T_{z=\in} \ne 0$

Solving using boundary conditions, We get

$$q_1=A\,Sh\left(\frac{m_1}{2}+i\frac{n_1}{2}\right)z+B\,Ch\left(\frac{m_1}{2}+i\frac{n_1}{2}\right)z-\frac{ip_0}{2\mu^2\alpha^2} \quad (9)$$

$$q_2=C\,Sh\left(\frac{m_2}{2}+i\frac{n_2}{2}\right)z+D\,Ch\left(\frac{m_2}{2}+i\frac{n_2}{2}\right)z-\frac{ip_0}{2\lambda\mu^2\alpha^2} \quad (10)$$

where ‘Sh’ stands for sinh, ‘Ch’ stands for cosh.

Where $m_1=2\alpha\mu$, $n_1=2\alpha\mu$ $m_2=2\alpha$, $n_2=2\alpha$ (11)

$$A=\frac{H_2G_1-G_2H_1}{F_1G_2-F_2G_1}\frac{Ch\left(\frac{m_1}{2}+i\frac{n_1}{2}\right)}{Sh\left(\frac{m_1}{2}+i\frac{n_1}{2}\right)}+\frac{ip_0}{2\mu^2\alpha^2 Sh\left(\frac{m_1}{2}+i\frac{n_1}{2}\right)}+Co$$

$$B=\frac{G_2H_1-H_2G_1}{F_1G_2-F_2G_1}$$

$$C=\frac{Ch\left(\frac{m_2}{2}+i\frac{n_2}{2}\right)}{Sh\left(\frac{m_2}{2}+i\frac{n_2}{2}\right)}\frac{H_1F_2-H_2F_1}{G_1F_2-F_1G_2}-\frac{ip_0}{2\lambda\mu^2\alpha^2 Sh\left(\frac{m_2}{2}+i\frac{n_2}{2}\right)}$$

$$D=\frac{H_1F_2-H_2F_1}{G_1F_2-F_1G_2} \quad (12)$$

Further expressing

$$a_1=Sh\frac{m_1}{2}\cos\frac{n_1}{2} \quad a_2=Sh\frac{m_2}{2}\cos\frac{n_2}{2}$$

$$b_1=Ch\frac{m_1}{2}\sin\frac{n_1}{2} \quad b_2=Ch\frac{m_2}{2}\sin\frac{n_2}{2}$$

$$e_1=Ch\frac{m_1}{2}\cos\frac{n_1}{2} \quad e_2=Ch\frac{m_2}{2}\cos\frac{n_2}{2}$$

$$f_1=Sh\frac{m_1}{2}\sin\frac{n_1}{2} \quad f_2=Sh\frac{m_2}{2}\sin\frac{n_2}{2}$$

(13)

$$F_1=\frac{Ch\left(\frac{m_1}{2}+i\frac{n_1}{2}\right)}{Sh\left(\frac{m_1}{2}+i\frac{n_1}{2}\right)}Sh\left(\frac{m_1\in}{2}+i\frac{n_1\in}{2}\right)-Ch\left(\frac{m_1\in}{2}+i\frac{n_1\in}{2}\right)$$

$$G_1=\frac{Ch\left(\frac{m_2}{2}+i\frac{n_2}{2}\right)}{Sh\left(\frac{m_2}{2}+i\frac{n_2}{2}\right)}Sh\left(\frac{m_2\in}{2}+i\frac{n_2\in}{2}\right)+Ch\left(\frac{m_2\in}{2}+i\frac{n_2\in}{2}\right)$$

$$H_1=\frac{iP_o}{2\lambda\mu^2\alpha^2}\frac{Sh\left(\frac{m_2\in}{2}+i\frac{n_2\in}{2}\right)}{Sh\left(\frac{m_2}{2}+i\frac{n_2}{2}\right)}+\frac{iP_o}{2\lambda\mu^2\alpha^2}-\frac{iP_o}{2\mu^2\alpha^2}+\left[C_o+\frac{iP_o}{2\mu^2\alpha^2}\right]\frac{Sh\left(\frac{m_1\in}{2}+i\frac{n_1\in}{2}\right)}{Sh\left(\frac{m_1}{2}+i\frac{n_1}{2}\right)}$$

$$F_2=\frac{Ch\left(\frac{m_1}{2}+i\frac{n_1}{2}\right)}{Sh\left(\frac{m_1}{2}+i\frac{n_1}{2}\right)}\left\{\left(\frac{m_1}{2}+i\frac{n_1}{2}\right)Ch\left(\frac{m_1\in}{2}+i\frac{n_1\in}{2}\right)\right\}-\left(\frac{m_1}{2}+i\frac{n_1}{2}\right)Sh\left(\frac{m_1\in}{2}+i\frac{n_1\in}{2}\right)$$

$$G_2 = \lambda\mu^2\left[\frac{Ch\left(\frac{m_2}{2}+i\frac{n_2}{2}\right)}{Sh\left(\frac{m_2}{2}+i\frac{n_2}{2}\right)}\left\{\left(\frac{m_2}{2}+i\frac{n_2}{2}\right)Ch\left(\frac{m_2\in}{2}+i\frac{n_2\in}{2}\right)\right\} + \left(\frac{m_2}{2}+i\frac{n_2}{2}\right)Sh\left(\frac{m_2\in}{2}+i\frac{n_2\in}{2}\right)\right]$$

$$H_2 = \left(C_o + \frac{iP_o}{2\mu^2\alpha^2}\right)\frac{Ch\left(\frac{m_1\in}{2}+i\frac{n_1\in}{2}\right)}{Sh\left(\frac{m_1}{2}+i\frac{n_1}{2}\right)}\left(\frac{m_1}{2}+i\frac{n_1}{2}\right) + \lambda\mu^2\left[\frac{iP_o}{2\lambda\mu^2\alpha^2}\frac{Ch\left(\frac{m_2\in}{2}+i\frac{n_2\in}{2}\right)}{Sh\left(\frac{m_2}{2}+i\frac{n_2}{2}\right)}\left(\frac{m_2}{2}+i\frac{n_2}{2}\right)\right] \quad (14)$$

Heattransfer Characteristics :

The heat-transfer characteristics have been determined for the upper liquid (zone-I) and lower liquid (zone-II) occupying different heights under constant pressure gradients between two parallel plates in a rotating system, from the heat transferequations.

$$\frac{\nu_m}{c_p}\frac{dq^1_m}{dz^1}\frac{d\overline{q^1_m}}{dz^1} + K_m\frac{d^2T^1_m}{\left(dz^1\right)^2} = 0 \text{ for m=1and 2} \quad (15)$$

Here $q^1_m = u^1_m + iV^1_m \qquad \overline{q^1_m} = u^1_m - iV^1_m$

K_1 and K_2 are the thermal diffusivities of the upper and lower liquids respectively. Cp is the specific heat at constant pressure. The first term on the left hand side of the above equation is due to the viscous dissipation.

Case 1 : When Plates are maintained at equal temperatures.

It is assumed that the temperature of either plate is a constant T^1_L, as in Ramana Rao and Narayana[6]. Introducing the non-dimensional quantities equation [6] and in addition

$$\theta^1_1 = \frac{\left(T^1_1 - T^1_L\right)}{T^1_L}, \qquad \theta^1_2 = \frac{\left(T^1_2 - T^1_L\right)}{T^1_L}$$

P_1(Prandtl number for the upper liquid)=ν_1 / K_1

E_c(Eckert number for the upper liquid)=U^2_o / CpT^1_L,

$\eta = \frac{K_2}{K_1}$ (Ratio of the Thermal diffusivities of the lower and upper liquids)

From equation (15) $\frac{d^2\theta^1_1}{dz^2} = -P_1Ec\frac{dq_1}{dz}\frac{d\overline{q_1}}{dz}$

$$\frac{d^2\theta^1_2}{dz^2} = -\frac{\mu^2}{\eta}P_1Ec\frac{dq_2}{dz}\frac{d\overline{q_2}}{dz} \quad (16)$$

as the non-dimensional temperature equations for the upper and lower liquids respectively.

Solving (16) subject to the boundary conditions in the non-dimensional form.

$$\theta_1^1 = 0 \quad at \quad z = 1 \text{ (Zone-I)}$$

$$\theta_1^1 = \theta_2^1 \quad at \quad z = \in, (\text{-}1 < \in < 1)$$

$$\frac{d\theta_1^1}{dz} = \eta \frac{d\theta_2^1}{dz} \quad at \quad z = \in \text{(Interface condition)}$$

$$\theta_2^1 = 0 \quad at \quad z = -1 \text{ (Zone-II)} \tag{17}$$

We get $\theta_1^1 = A_1 + B_1 z - P_1 Ec\left(X_1 Chm_1 z + X_2 Shm_1 z + X_3 \cos n_1 z\right)$ (18)

$$\theta_2^1 = C_1 + D_1 z - \frac{\mu_o^2}{\eta} P_1 Ec\left(Y_1 Chm_2 z + Y_2 Shm_2 z + Y_3 \cos n_2 z\right) \tag{19}$$

$$A_1 = \frac{\eta P_1 Ec}{(\eta+1) + \in(1-\eta)} \begin{Bmatrix} (1-\in)S_1 + \mu_o^2 S_2(\in -1) + f(\in) - f(1) \\ + \dfrac{\mu_o^2}{\eta}\left(g(1) - g(\in)\right) \end{Bmatrix} - P_1 E_c\left(S_1 - \mu_o^2 S_2 - f(1)\right)$$

$$B_1 = \frac{-\eta P_1 Ec \begin{pmatrix} (1-\in)S_1 + \mu_o^2 S_2(\in -1) + f(\in) - f(1) + \\ \dfrac{\mu_o^2}{\eta}\left(g(1) - g(\in)\right) \end{pmatrix}}{(\eta+1) + \in(1-\eta)} + P_1 Ec\left(S_1 - \mu_o^2 S_2\right)$$

$$C_1 = \frac{-P_1 Ec \begin{pmatrix} (1-\in)S_1 + \mu_o^2 S_2(\in -1) + f(\in) - f(1) \\ + \dfrac{\mu_o^2}{\eta}\left(g(1) - g(\in)\right) \end{pmatrix}}{(\eta+1) + \in(1-\eta)} + \frac{\mu_o^2}{\eta} P_1 Ecg(1)$$

$$D_1 = \frac{-P_1 Ec \begin{pmatrix} (1-\in)S_1 + \mu_o^2 S_2(\in -1) + f(\in) - f(1) \\ + \dfrac{\mu_o^2}{\eta}\left(g(1) - g(\in)\right) \end{pmatrix}}{(\eta+1) + \in(1-\eta)} \tag{20}$$

Let $X_1 Chm_1 \in + X_2 Shm_1 \in + X_3 \cos n_1 \in = f(\in)$

Then $X_1 Chm_1 + X_2 Shm_1 + X_3 \cos n_1 = f(1)$

Let $Y_1 Chm_2 \in + Y_2 Shm_2 \in + Y_3 \cos n_2 \in = g(\in)$

Then $Y_1 Chm_2 + Y_2 Shm_2 + Y_3 \cos n_2 = g(1)$

Where $S_1 = X_1 m_1 Shm_1 \in + X_2 m_1 Chm_1 \in - X_3 n_1 Sin\, n_1 \in$

$$S_2 = Y_1 m_2 Shm_2 \in + Y_2 m_2 Chm_2 \in - Y_3 n_2 Sinn_2 \in \tag{21}$$

Here $X_1 = \dfrac{\left(A^2 + B^2\right)\left(m_1^2 + n_1^2\right)}{8m_1^2}$

$$X_2 = \frac{AB\left(m_1^2 + n_1^2\right)}{4m_1^2}$$

$$X_3 = -\frac{\left(A^2 - B^2\right)\left(m_1^2 + n_1^2\right)}{8n_1^2}$$

$$Y_1 = \frac{\left(C^2 + D^2\right)\left(m_2^2 + n_2^2\right)}{8m_2^2}$$

$$Y_2 = \frac{CD\left(m_2^2 + n_2^2\right)}{4m_2^2}$$

$$Y_3 = -\frac{\left(C^2 - D^2\right)\left(m_2^2 + n_2^2\right)}{8n_2^2}$$

$$\theta_1 = \frac{\theta_1^1}{P_1 Ec} = \frac{1}{P_1 Ec}\left(A_1 + B_1 z\right) - \left(X_1 Chm_1 z + X_2 Shm_1 z + X_3 \cos n_1 z\right) \tag{22}$$

$$\theta_2 = \frac{\theta_2^1}{P_1 Ec} = \frac{1}{P_1 Ec}\left(C_1 + D_1 z\right) - \frac{\mu_o^2}{\eta}\left(Y_1 Chm_2 z + Y_2 Shm_2 z + Y_3 \cos n_2 z\right) \tag{23}$$

The heat transfer coefficients at the upper and lower plates are respectively given by

$$H_U = \frac{-d\theta_1^1}{dz}\bigg|_{z=1} = -B_1 + P_1 Ec\begin{bmatrix} m_1 X_1 Shm_1 \\ + m_1 X_2 Chm_1 - n_1 X_3 \sin n_1 \end{bmatrix} \tag{24}$$

$$H_L = \frac{d\theta_2^1}{dz}\bigg|_{z=-1} = D_1 - \frac{\mu_o^2}{\eta} P_1 Ec\begin{bmatrix} -m_2 Y_1 Shm_2 \\ + m_2 Y_2 Chm_2 + n_2 Y_3 \sin n_2 \end{bmatrix} \tag{25}$$

Results & Discussion

Figures show the temperature distribution for plates maintained at equal temperatures for various values of $\alpha, \mu, \lambda, \eta, P_1 = 0.72$ and Ec = 0.02 for $\in = -0.4, 0$ *and* 0.4. We follow the convention that the non-dimensional temperature θ is positive or negative according as the actual temperature T_1 or T_2 is greater / less than T_L. The temperature distribution depreciates with increase of α, for $\in = -0.4, 0$ *and* 0.4 while μ, λ *and* η being fixed. As the position of the interface increases the temperature depreciates with respect to α for fixed μ, λ and η. As μ increases, the temperature distribution decreases for $\in = -0.4, 0$ *and* 0.4 while α, λ *and* η being fixed. As the position of the interface increase from -0.4 to 0.4 the temperature distribution depreciates. As λ increases the temperature distribution gradually decreases for $\in = -0.4$, 0 *and* 0.4, while α, μ *and* η being fixed. For λ= 1, 1.5 and 2 at ∈=0 the distribution is maximum when compared to ∈= - 0.4 and 0.4 while α, μ *and* η being fixed.

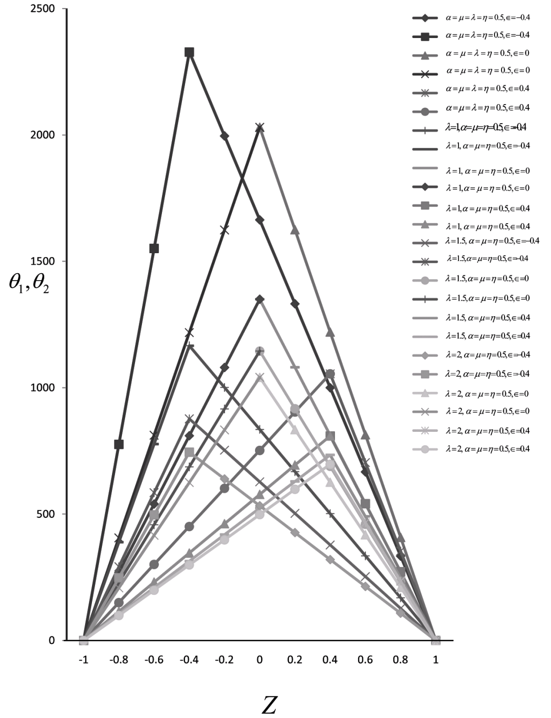
As η increases the temperature distribution gradually decreases for $\in = -0.4, 0$ *and* 0.4, but when η=1, 1.5 and 2 the distribution is maximum at ∈=0 when compared to ∈= - 0.4 and 0.4 while α, μ *and* λ being fixed. For large values of α, μ *and* η the temperature distributions are very low. As ∈ value increases the temperature distribution decreases for small values of α, μ λ *and* η. When λ and η increases the temperature distribution is maximum at ∈=0 (*i.e.* at the centre of the channel). With increase of α, the heat transfer co-efficient at the upper plate decreases for ∈=-0.4, 0 and 0.4 while μ, λ and η being fixed. As μ increases heat transfer co-efficient at the upper plate decreases for ∈=-0.4, 0 and 0.4 while α, λ and η being fixed.

As λ increases heat transfer co-efficient at the upper plate increases for ∈=-0.4, 0 and 0.4 while α, μ and η being fixed. As η increases heat transfer co-efficient at the upper plate increases for ∈=-0.4 and 0, increases and then slightly decreases for ∈= 0.4, while α, μ and λ being fixed. With increase of ∈ value the heat transfer co-efficient at the upper plate decreases for small values of α, μ, η and λ and increases for α = λ = 1, 1.5 and 2. As ∈ value increases the heat transfer co-efficient at the upper plate decreases for λ = η = 1, 1.5 and 2.

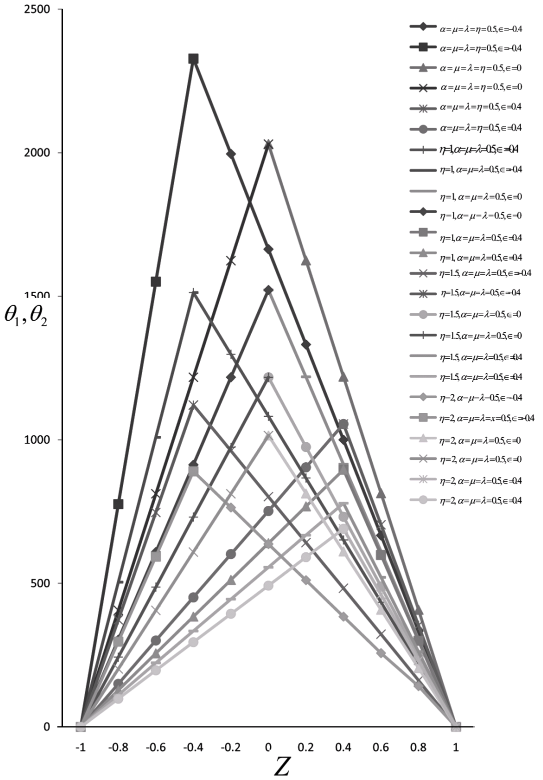
As α increases the heat transfer co-efficient at the lower plate increases and then decreases for ∈= -0.4 and 0, it decreases and then increases for ∈= 0.4, while μ, λ and η being fixed. As μ increases the heat transfer co-efficient at the lower plate increases for ∈ = -0.4 and 0, it decreases for ∈= 0.4 while α, λ and η being fixed. As λ increases the heat transfer co-efficient at the lower plate decreases for ∈ = -0.4, 0 and 0.4, while α, μ and η being fixed. As η increases the heat transfer co-efficient at the lower plate decreases for ∈= -0.4, 0 and 0.4, while α, μ and λ being fixed. As ∈ increases from -0.4 to 0.4 the heat transfer co-efficient at the lower plate increases for μ, increases and then decreases for α. With the increase in position of the interface the heat transfer co-efficient at the lower plate decreases for λ and η.

Conclusion

The temperature distribution of the upper liquid (zone-I) decreases to zero, from its value at *z*=∈, where as that of the lower liquid (zone-II) increases from zero, to its value at *z*=∈, both the values at *z*=∈ being the same. As the position of the interface increases from lower to the upper plate the temperature depreciates with respect to α, μ, λ and η.



Temperature profiles when plates maintained at equal temperatures



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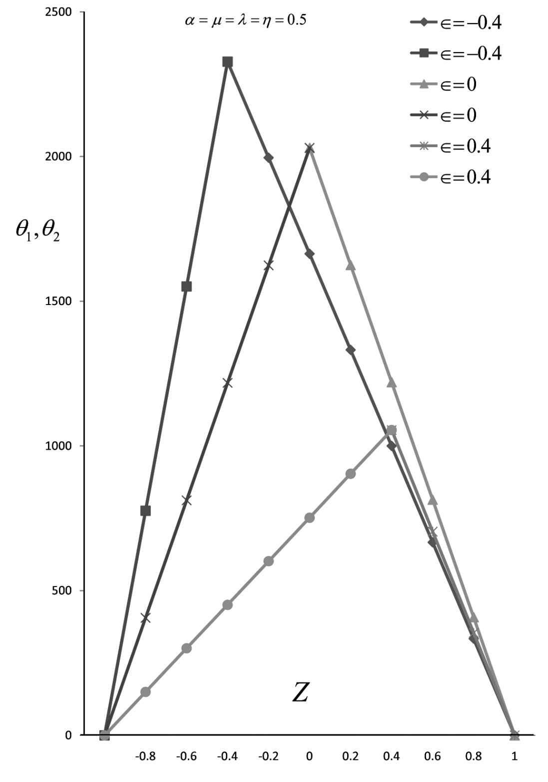

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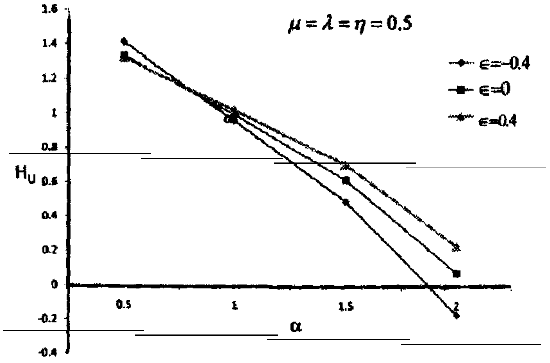
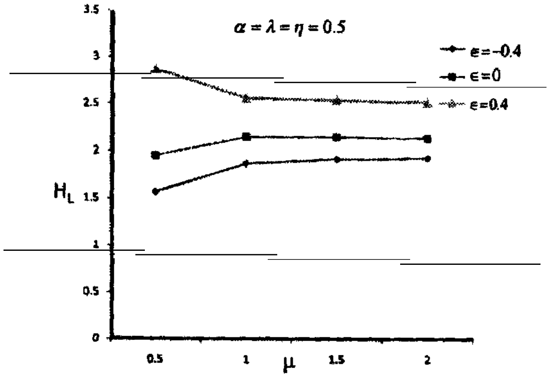


Heat Transfer Co-efficient at the upper plate



Heat Transfer Co-efficient at the lower plate

Case 2 : Plates maintained at different temperatures :

It is assumed that the temperature of the upper plate T_U^1 is greater than the temperature of the lower plate T_L^1, in terms of non-dimensional quantities.

$$\theta_1^1=\frac{T_1^1-T_L^1}{T_U^1-T_L^1},\qquad \theta_2^1=\frac{T_2^1-T_L^1}{T_U^1-T_L^1},$$

$$Ec=\frac{U_o^2}{Cp\left(T_U^1-T_L^1\right)} \tag{26}$$

In case of different plate temperatures $T_U\neq T_L \Rightarrow T_U-T_L\neq 0$ therefore we define the non-dimensional temperatures in zone-I and zone-II are

$$\theta_1=\frac{T_1-T_L}{T_U-T_L},\ \theta_2=\frac{T_2-T_L}{T_U-T_L}$$

Here T_1 and T_2 are the liquid temperatures in zone-I and zone-II.

On z = 1, $T_1=T_U \Rightarrow \theta_1=1$ On z = -1, $T_2=T_L$ $\Rightarrow \theta_2=0$

We have to solve equation (16) subject

to the boundary conditions of (17) except when $\theta_1^1 = 1 \ at \ \ z = 1$, as in Ramana Rao & Narayana[6].

$$\theta_1 = \frac{\theta_1^1}{P_1 E_c} = \frac{A_2 + B_2 z}{P_1 E_c} - \begin{pmatrix} X_1 Chm_1 z + \\ X_2 Shm_1 z + X_3 \cos n_1 z \end{pmatrix} \tag{27}$$

$$\theta_2 = \frac{\theta_2^1}{P_1 E_c} = \frac{C_2 + D_2 z}{P_1 E_c} - \frac{\mu_o^2}{\eta} P_1 Ec \begin{pmatrix} Y_1 Chm_2 z \\ + Y_2 Shm_2 z + Y_3 \cos n_2 z \end{pmatrix} \tag{28}$$

$$A_2 = \frac{1 + \in(1-\eta) + \eta P_1 E_c \begin{Bmatrix} (1-\in)S_1 + \mu_o^2 S_2(\in -1) + \\ f(\in) - f(1) + \dfrac{\mu_o^2}{\eta}(g(1) - g(\in)) \end{Bmatrix}}{(\eta+1) + \in(1-\eta)}$$
$$- P_1 Ec\left(S_1 - \mu_o^2 S_2 - f(1)\right)$$

$$B_2 = \frac{\eta\left(1 - P_1 Ec\begin{pmatrix} (1-\in)S_1 + \mu_o^2 S_2(\in -1) + f(\in) \\ - f(1) + \dfrac{\mu_o^2}{\eta}(g(1) - g(\in)) \end{pmatrix}\right)}{(\eta+1) + \in(1-\eta)}$$
$$- P_1 Ec\left(S_1 - \mu_o^2 S_2\right)$$

$$C_2 = \frac{1 - P_1 Ec\begin{pmatrix} (1-\in)S_1 + \mu_o^2 S_2(\in -1) + f(\in) - f(1) \\ + \dfrac{\mu_o^2}{\eta}(g(1) - g(\in)) \end{pmatrix}}{(\eta+1) + \in(1-\eta)}$$
$$+ \frac{\mu_o^2}{\eta} P_1 Ec g(1)$$

$$D_2 = \frac{1 - P_1 Ec\begin{pmatrix} (1-\in)S_1 + \mu_o^2 S_2(\in -1) + f(\in) - f(1) \\ + \dfrac{\mu_o^2}{\eta}(g(1) - g(\in)) \end{pmatrix}}{(\eta+1) + \in(1-\eta)} \tag{29}$$

The Heat Transfer coefficients at the upper and lower plates are respectively given by

$$H_U = \frac{-d\theta_1^1}{dz}\bigg|_{z=1} = -B_2 + P_1 Ec\begin{bmatrix} m_1 X_1 Shm_1 \\ + m_1 X_2 Chm_1 - n_1 X_3 \sin n_1 \end{bmatrix} \tag{30}$$

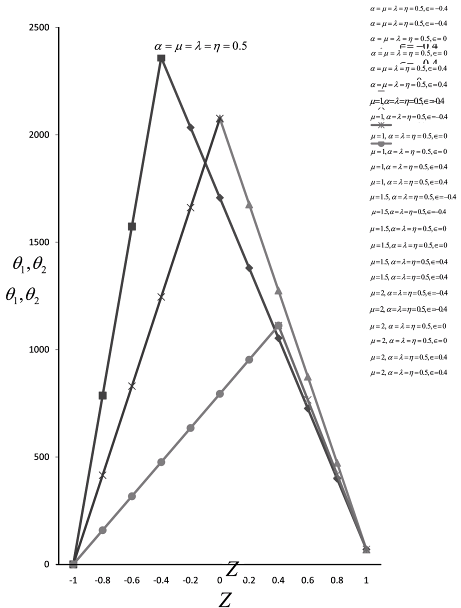
$$H_L = \frac{d\theta_2^1}{dz}\bigg|_{z=-1} = D_2 - \frac{\mu_o^2}{\eta} P_1 Ec\begin{bmatrix} -m_2 Y_1 Shm_2 \\ + m_2 Y_2 Chm_2 + n_2 Y_3 \sin n_2 \end{bmatrix} \tag{31}$$

Results & Discussion

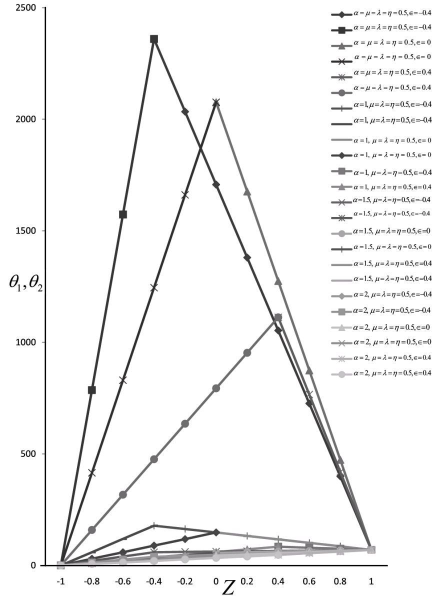
The temperature distribution when plates are maintained at different temperatures for various values of α, μ, λ, η, P_1=0.72, E_c=0.02, P_0=2, C_0=1 for $\in$ = -0.4, 0 and 0.4.

The temperature distribution decreases with increase of α, while μ, λ *and* η being fixed for ∈= -0.4, 0 and 0.4, but for large values of α=1, 1.5 and 2 the temperature distribution is very low and also near the boundary of the plates. With the increase of the interface the temperature distribution depreciates. As μ increases the temperature distribution decreases, for ∈= -0.4, 0 and 0.4, while α, λ and η being fixed, but for μ=1, 1.5 and 2 the distribution is very low and also near the boundary of the plates. As the position of the interface increases the temperature distribution depreciates.

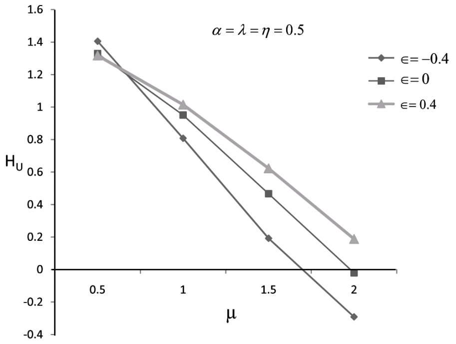
As λ increases the temperature



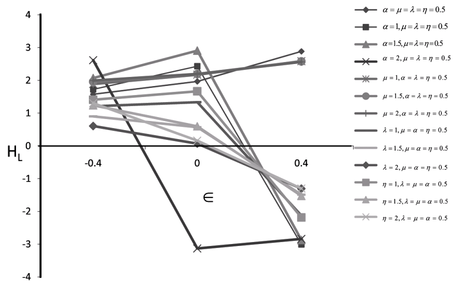
Temperature profiles when plates maintained at different temperatures



Temperature profiles when plates maintained at different temperatures



Heat Transfer Co-efficient at the upper plate



Heat Transfer Co-efficient at the lower plate

distribution decreases for ∈= -0.4, 0 and 0.4 while α, μ and η being fixed. When both the liquids occupying equal heights (∈=0) the temperature distribution is high when compared to ∈=-0.4 and 0.4, for λ = 1, 1.5 and 2, while α, μ and η being fixed. As the position of the interface increases from lower to the upper plate the temperature distribution depreciates. As η increases the temperature distribution gradually decreases, for ∈= -0.4, 0 and 0.4, while α, μ and λ being fixed, when η = 1, 1.5 and 2 the distributions are high for ∈= 0. The position of the interface increases from -0.4 to 0.4, the temperature depreciates.

For large values of α, μ, λ and η the temperature distribution decreases for ∈=-0.4, 0 and 0.4. As value increases the temperature distribution decreases for small values of α, μ, λ and η. Figures show the heat transfer co-efficient at the upper and lower plates respectively. As α increases the heat transfer co-efficient at the upper plate decreases for ∈=-0.4, 0 and 0.4, while μ, λ and η being fixed. As μ increases the heat transfer co-efficient at the upper plate decreases for ∈= -0.4, 0 and 0.4 while α, λ and η being fixed.

As λ increases the heat transfer co-efficient at the upper plate increases for ∈= -0.4, 0 and 0.4 while α, μ and η being fixed. As η increases the heat transfer co-efficient at the upper plate increases for ∈= -0.4, 0 and 0.4 while α, μ and λ being fixed. As value increases the heat transfer co-efficient at the upper plate increases for α and μ and decreases for λ and η. As α increases the heat transfer co-efficient at the lower plate increases and then decreases for ∈= -0.4 and 0, it decreases and then increases for ∈= 0.4, while μ, λ and η being fixed. As μ increases the heat transfer co-efficient at the lower plate increases for ∈ = -0.4 and 0, it decreases for ∈= 0.4 while α, λ and η being fixed. As λ increases the heat transfer co-efficient at the lower plate decreases for ∈= -0.4, 0 and 0.4, while α, μ and η being fixed. As η increases the heat transfer co-efficient at the lower plate decreases for ∈= -0.4, 0 and 0.4, while α, μ and being fixed. As increases from -0.4 to 0.4 the heat transfer co-efficient at the lower plate increases for μ, increases and then decreases for α. With the increase in position of the interface the heat transfer co-efficient at the lower plate decreases for λ and η.

Conclusion

As the position of the interface increases from lower to the upper plate the temperature distribution depreciates with respect to α, μ, λ and η. It is found that the conclusions of the temperature distribution of the previous case *i.e.* when the plate temperatures are equal are also valid in this case.

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