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Determination of expected time to recruitment when backup resource of manpower exists

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Abstract

In any organization the exit of personnel is likely to occur especially when the policy decisions regarding pay, perquisites and targets are announced. Immediately after the exit of personnel the recruitment cannot be introduced because it is time consuming and costly. Frequent recruitments are also not desirable. So the recruitment is made only after the cumulative loss of manpower due to exit of personnel on successive occasion crosses a level called the threshold. The magnitude of the threshold is supplemented by a backup or the reserve of manpower resource. Assuming that the time intervals between successive decision epochs are random variables and the threshold level is also a random variable, the expected time to recruitment is attained as a tool for forecast of the likely time at which recruitment becomes necessary Numerical illustrations is also provided.

Notations :

X_i = a random variable denoting the amount of wastage in the i^{th} the epoch, i = 1, 2….k, and has p.d.f. g(.) and c.d.f G(.).

U_i = a random variable denoting the inte-rarrival times between decision epochs i = 1,2,…k and has p.d.f f(.) and c.d.f F(.).

Y = a random variable denoting the threshold level of wastage $Y \sim \exp(\theta_1)$

Z = the back up resource of manpower and $Z \sim \exp(\theta_2)$.

W = Z + Y

$F_k(.)$ = the k convolution of F(.)

$G_k(.)$ = the k convolution of G(.)

$g^*(.)$ = Laplace transform of g(.)

Introduction

The application of Stochastic processes to the studies in manpower planning has introduced a new area of study in Manpower planning. The greatest advantage of application of Stochastic processes is that any real life situation can be conceptualized as a mathematical model and the optimal solution can be derived by using standard techniques. It may be observed that if, the initial solution is not satisfactory, the model can be reformulated and an improved solution can be derived. This processes of iteration leads to the optimal solution. The formulation of suitable policies which would be profitable for the organizations is achieved by use of stochastic models in Manpower planning. For a detailed study refer to Bartholomew[2], Bartholomew and Forbes[4], McClean *et. al*. (1991), Grinold and Marshall[7]. In the earlier work relating to Stochastic models, were derived to determine the expression for E(T) and variance V(T) using the shock model approach. For a detailed study refer to Sathiyamoorthi and Elangovan[9]. In doing so it has been assumed that the threshold is a random variable and if the cumulative wastage crosses the threshold level then recruitment becomes made necessary. In this paper a stochastic model is derived to find the expression for E(T) and V(T) under different assumptions. Some interesting results relating to the current work can also be seen in Anantharaj *et. al*. (2008), Vijayasankar *et. al*.[11], Elangovan *et. al*.[6], Arivazhagan *et. al*.[1], Susiganeshkumar *et. al*.[10]. It is assumed the threshold comprises of two components namely the level of wastage which can be allowed and the manpower which is available from what is known as a backup resource. The threshold can be treated now as the total of the maximum allowable attrition and the maximum available back up resource[3,8]. The back up resource is similar to the manpower inventory on hand which can be utilised whenever it becomes necessary so it is assumed that the threshold comprises of two components. Under this assumption the expected time to recruitment and its variance are obtained using the shock model approach[5]. Numerical example is also presented.

Assumptions :

1) Wastage of manpower occurs at decision making epochs and the wastages are cumulative.
2) There is a threshold level for the level of wastage and also a resource back up available.
3) If the total wastage crosses the sum of the threshold and the back up resource available the break down occurs.
4) The process that generates the wastages and the threshold put to gether with the backup are linearly independent.

Results

Let Y ~ exp (θ_1) and Z ~ exp(θ_2).
Since W = Y + Z, the p.d.f. of W is the convolution ofY + Z and it is given by

$$q(w) = \int_0^w \theta_1 e^{-\theta_1 m} \theta_2 e^{-\theta_2 (w-m)} dm$$

$$= \frac{\theta_1\theta_2}{(\theta_1 - \theta_2)}\left[e^{-\theta_2 w} - e^{-\theta_1 w}\right] \text{ on simplification} \quad (1)$$

The probability that there are 'k' decision epochs in (0, t) and the cumulative wastage does not cross the combined threshold is given by

$$P\left[\sum_{i-1}^{k} X_i < (Y+Z)\right] = S(t) = \text{Survivor function}$$

$$= \sum_{k=0}^{\infty}\left[F_k(t) - F_{k+1}(t)\right]\int_0^{\infty} G_k(w) q(w) dw \quad (2)$$

$$= \sum_{k=0}^{\infty}\left[F_k(t) - F_{k+1}(t)\right]\frac{\theta_1\theta_2}{\theta_1 - \theta_2}\int_0^{\infty} G_k(w)\left[e^{-\theta_2 w} - e^{-\theta_1 w}\right] dw$$

$$= \frac{\theta_1\theta_2}{\theta_1-\theta_2} \sum_{k=0}^{\infty}[F_k(t)-F_{k+1}(t)] \; [G_k^*(\theta_2)-G_k^*(\theta_1)]$$

since $\int_0^\infty G_k(w)e^{-\theta_2 w}dw = G_k^*(\theta_2)$

and $\int_0^\infty G_k(w)e^{-\theta_1 w}dw = G_k^*(\theta_1)$

Since the random variables $X_1 X_2 \ldots X_k$ are i.i.d

$$G_k^*(\theta_2) = \frac{[g^*(\theta_2)]^k}{\theta_2} \text{ and } G_k^*(\theta_1) = \frac{[g^*(\theta_1)]^k}{\theta_1} \quad (3)$$

By virtue of the properties of the Laplace transforms.

Now we have

$$S(t) = \sum_{k=0}^{\infty}[F_k(t)-F_{k+1}(t)]\frac{\theta_1\theta_2}{\theta_1-\theta_2}\left[\frac{(g^*(\theta_2))^k}{\theta_2} - \frac{(g^*(\theta_1))^k}{\theta_1}\right]$$

$$= \sum_{k=0}^{\infty}[F_k(t)-F_{k+1}(t)] \left[\frac{\theta_1(g^*(\theta_2))^k - \theta_2(g^*(\theta_1))^k}{\theta_1-\theta_2}\right]$$

$$= 1 - \frac{\theta_2}{\theta_1-\theta_2}[1-g^*(\theta_2)] \sum_{k=1}^{\infty}\left[F_k(t)(g^*(\theta_2))^{k-1}\right]$$

$$+ \frac{\theta_2}{\theta_1-\theta_2}[1-g^*(\theta_2)] \sum_{k=1}^{\infty}\left[F_k(t)(g^*(\theta_1))^{k-1}\right] \text{ on simplification} \quad (4)$$

L(t) = 1-S(t)

and so,

$$\ell(t) = \frac{\theta_1}{(\theta_1-\theta_2)}[1-g^*(\theta_2)] \sum_{k=1}^{\infty} f_k(t)\,[(g^*(\theta_2))]^{k-1}$$

$$- \frac{\theta_2}{\theta_1-\theta_2}[1-g^*(\theta_1)] \sum_{k=1}^{\infty} f_k(t)[(g^*(\theta_1))]^{k-1}$$

$$\ell^*(s) = \frac{\theta_1 f^*(s) \; [1-g^*(\theta_2)]}{(\theta_1-\theta_2) \; [1-f^*(s)g^*(\theta_2)]} - \frac{\theta_2 f^*(s) \; [1-g^*(\theta_1)]}{(\theta_1-\theta_2) \; [1-f^*(s)g^*(\theta_1)]}$$

on simplification (5)

Let f(.) ~ exp (μ)

Hence $f^*(s) = \frac{\mu}{\mu+s}$ similarly if g(.) ~ exp (α)

$$g^*(\theta_1) = \frac{\alpha}{\alpha+\theta_1} \qquad g^*(\theta_2) = \frac{\alpha}{\alpha+\theta_2}$$

$$E(T) = -\left.\frac{d\ell^*(s)}{ds}\right|_{s=0}$$

Now substituting

f*(s), g*(θ_1) and g*(θ_2) in (5) we get

$$\ell^*(s) = \frac{\theta_1 \frac{\mu}{\mu+s}\left[1-\frac{\alpha}{\alpha+\theta_2}\right]}{(\theta_1-\theta_2)\left[1-\frac{\mu}{\mu+s}.\frac{\alpha}{\alpha+\theta_2}\right]} - \frac{\theta_2 \frac{\mu}{\mu+s}\left[1-\frac{\alpha}{\alpha+\theta_1}\right]}{(\theta_1-\theta_2)\left[1-\frac{\mu}{\mu+s}.\frac{\alpha}{\alpha+\theta_1}\right]}$$

$$= \frac{\mu\theta_1\theta_2}{(\theta_1-\theta_2)\ [s(\alpha+\theta_2)+\mu\theta_2\lambda]} - \frac{\mu\theta_1\theta_2}{(\theta_1-\theta_2)\ [s(\alpha+\theta_1)+\mu\theta_1]}$$

$$E(T) = \left.\frac{-d\ell^*(s)}{ds}\right|_{s=0} = -\frac{\mu\theta_1\theta_2}{(\theta_1-\theta_2)}\frac{(\alpha+\theta_1)}{[s(\alpha+\theta_2)+\mu\theta_2]^2} + \frac{\mu\theta_1\theta_2}{(\theta_1-\theta_2)}\frac{(\alpha+\theta_1)}{[s-(\alpha+\theta_1)+\mu\theta_1]^2}$$

$$+\frac{\mu\theta_1\theta_2}{(\theta_1-\theta_2)}\left[\frac{\alpha+\theta_2}{(\mu\theta_2)^2} - \frac{\alpha+\theta_1}{(\mu\theta_1)^2}\right]$$

$$= \frac{\mu\theta_1\theta_2}{(\theta_1-\theta_2)}\left[\frac{\theta_1^2(\alpha+\theta_2)-\theta_2^2(\alpha+\theta_1)}{\mu^2\theta_1^2\theta_2^2 \quad \mu\theta_1\theta_2}\right]$$

$$E(T) = \frac{1}{(\theta_1-\theta_2)}.\frac{1}{\mu\theta_1\theta_2}\left[\alpha\theta_1^2+\theta_1^2\theta_2-\alpha\theta_2^2-\theta_2^2\theta_1\right]$$

$$= \frac{1}{(\theta_1-\theta_2)}.\frac{1}{\mu\theta_1\theta_2}\left[\alpha(\theta_1^2-\theta_2^2)+\theta_1\theta_2(\theta_1-\theta_2)\right]$$

$$= \frac{1}{\mu\theta_1\theta_2}\left[\alpha(\theta_1+\theta_2)+(\theta_1\theta_2)\right]$$

$$E(T^2) = \left.\frac{d^2\ell^*(s)}{ds^2}\right|_{s=0}$$

$$= \frac{\mu\theta_1\theta_2}{(\theta_1-\theta_2)}\left\{\frac{d^2}{ds^2}\left[\frac{1}{s(\alpha+\theta_2)+\mu\theta_2}\right]-\frac{d^2}{ds^2}\left[\frac{1}{s(\alpha+\theta_1)+\mu\theta_1}\right]\right\}$$

$$= \frac{\mu\theta_1\theta_2}{(\theta_1-\theta_2)}\left[\frac{2(\alpha+\theta_2)^2}{(\mu\theta_2)^3}-\frac{2(\alpha+\theta_1)^2}{(\mu\theta_1)^3}\right]$$

$$V(T) = E(T^2)-[E(T)]^2 = \frac{2\mu\theta_1\theta_2}{(\theta_1-\theta_2)}\left[\frac{(\alpha+\theta_2)^2}{(\mu\theta_2)^3}-\frac{(\alpha+\theta_1)^2}{(\mu\theta_1)^3}\right]$$

$$-\frac{\left[\alpha^2(\theta_1+\theta_2)^2+\theta_1^2\theta_2^2+2\theta_1\theta_2\alpha(\theta_1+\theta_2)\right]}{\mu^2\theta_1^2\theta_2^2}$$

$$= \frac{2}{(\theta_1-\theta_2)}\left[\frac{\theta_1^3(\alpha^2+2\alpha\theta_2+\theta_2^2)-\theta_2^3(\alpha^2+2\alpha\theta_1+\theta_1^2)-\alpha^2\theta_1^2-\alpha^2\theta_2^2-2\alpha^2\theta_1\theta_2-2\alpha\theta_1^2\theta_2-2\alpha\theta_1\theta_2^2}{\mu^2\theta_1^2\theta_2^2}\right]$$

Numerical Illustration :

The values of E(T) and V(T) can be determined numerically using the above expressions when the values of the various parameters are given. The changes in E(T) and V(T) consequent to the changes in each of these parameters when others are kept fixed is also possible.

Table 1. Variation in E(T) and V(T) for Changes θ_1 $\theta_2 = 1.0$; $\alpha = 1.2$; $\mu = 1.8$

Table 1a

θ_1	E(T)
0.5	2.55
1.0	1.88
1.5	1.66
2.0	1.55
2.5	1.48

Table 1b

θ_1	V(T)
1.5	5.35
2.0	3.87
2.5	3.46
3.0	3.28
3.5	3.19
4.0	3.13

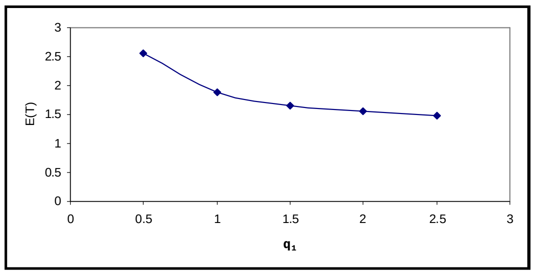

Fig. 1a. Variation in E(T) for Changes θ_1

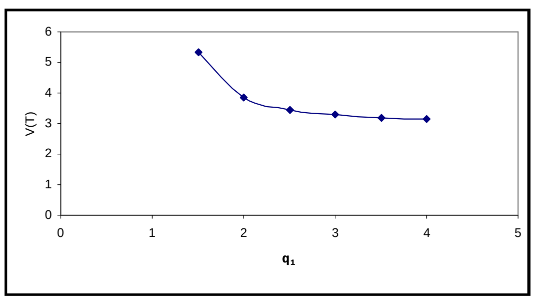

Fig. 1b. Variation in V(T) for Changes θ_1

Table 2. Variation in E(T) and V(T) for Changes θ_2, $\theta_1 = 1.5$; $\alpha = 1.2$; $\mu = 1.8$

Table 2a

θ_2	E(T)
0.5	2.33
1.0	1.66
1.5	1.44
2.0	1.33
2.5	1.26

Table 2b

θ_2	V (T)
1.0	5.35
2.0	1.23
3.0	2.06
4.0	2.07
5.0	2.08

Table 3. Variation in E(T) and V(T) for Changes á, $\theta_1 = 1.0$; $\theta_2 = 1.5$; $\mu = 1.8$

α	**E(T)**	**V(T)**
0.5	1.01	1.80
1.0	1.48	4.11
1.5	1.94	7.56
2.0	2.40	12.13
2.5	2.87	17.84

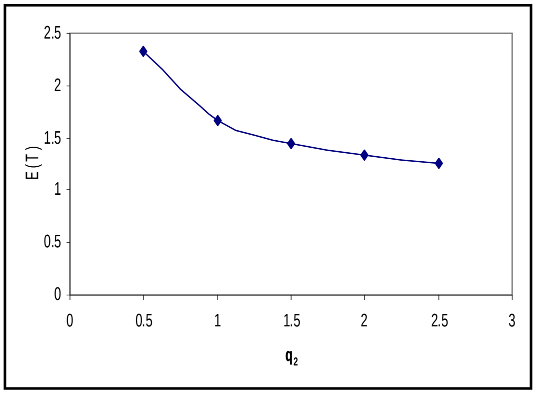

Fig. 2a. Variation in E(T) for Changes θ_2

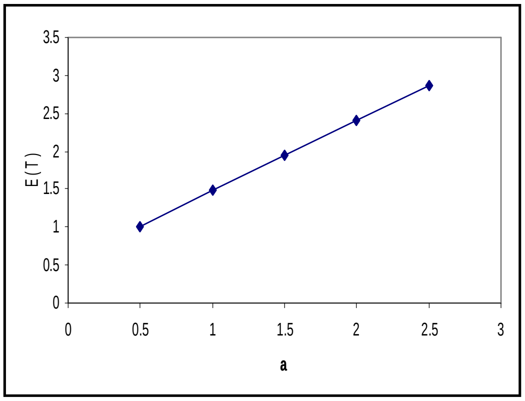

Fig. 3a. Variation in E(T) for Changes α

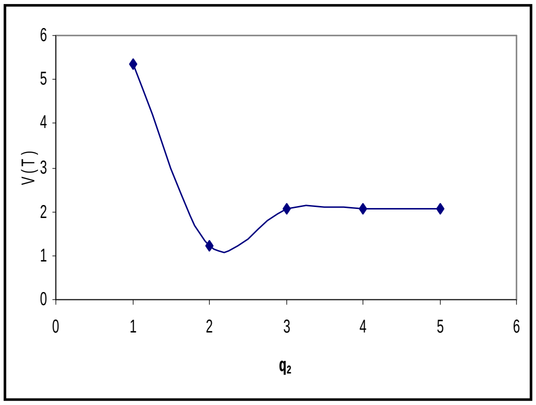

Fig. 2b. Variation in V(T) for Changes θ_2

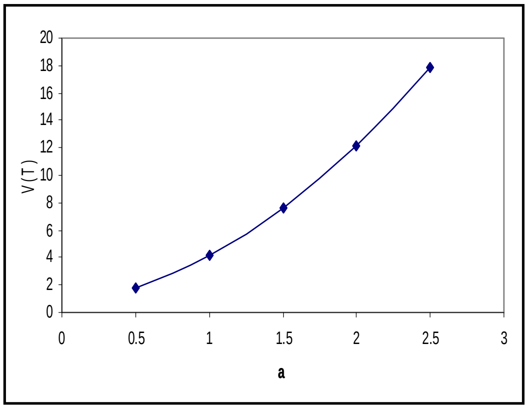

Fig. 3b. Variation in V(T) for Changes α

Table 4. Variation in E(T) and V(T) for Changes μ $\theta_1 = 1.0$; $\theta_2 = 1.5$; $\alpha = 1.2$;

μ	E(T)	V(T)
0.5	6.00	69.44
1.0	3.00	17.36
1.5	2.00	7.71
2.0	1.50	4.34
2.5	1.20	2.27

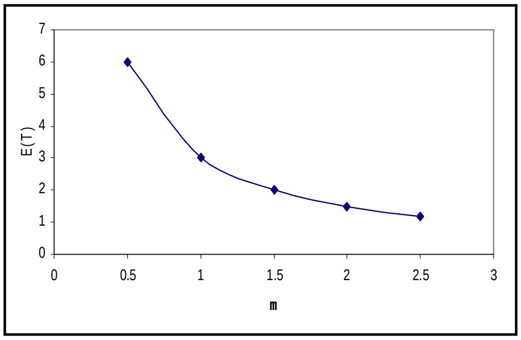

Fig. 4a. Variation in E(T) for Changes μ

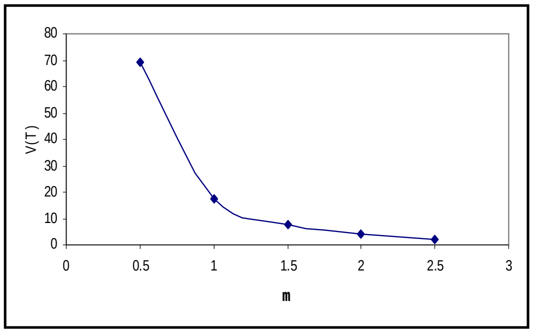

Fig. 4b. Variation in V(T) for Changes μ

Conclusion

The following conclusions can be drawn on the basis of numerical illustration provided.

Case (i): If the value of the parameter θ_1 increases, there is a decrease in the values of E(T) as indicated, in Table 1a and Table 1b with Fig. 1a and Fig. 1b. This is due to fact that θ_1 the parameter of the threshold Y increases $E(Y) = 1/\theta_1$ decreases and so the expected time to the cross the threshold decreases. A similar behaviour is noted the values of V(T).

Case (ii): If the value of parameter θ_2 increases E(T) decreases. This is due to the fact that θ_2 is the parameter of the exponential distribution of the backup resource random variable Z. Hence $E(Z)=1/\theta_2$ decreases and there is a reduction in the backup resource and so the overall threshold namely Y + Z becomes smaller. Hence E(T) decreases a similar behaviour is notated in the values of V(T) also, as indicated in Table 2a and Table 1b with Fig. 2a and Fig. 2b.

Case (iii): When the value of α which is the parameter of the exponential distribution followed by X_i increases E(T) also increases. This is due to the fact that X_i is the magnitude of wastage at every epochs and since X-i ~ $E(\alpha_i)=1/\alpha$. Therefore α increases the amount of wastage of each epoch decreases and so it would take longer time to cross the threshold. Hence E(T) increases the variance V(T) also increases. It is given in Table 3 with Fig. 3a and Fig. 3b.

Case (iv): If the value of μ which is parameter of the exponential distribution followed by the random variable, U namely the interarrival times between the wastage epochs increases and E(T) decreases. This is due to the fact that $E(\mu) = 1/\mu$ so the average interarrival time between wastages becomes smaller and so the wastage are more frequent and higher. Hence E(T) decreases V(T) also decreases in this case. This is explained in Table 4 with Fig. 4a and Fig. 4b.

Discussion

The findings of any research work should be such that they have the viability for field applications. This is very much essential in the case of Stochastic models which are conceptualized and solved based on ground realities. In the case of man power planning models the usefulness of the models is very much emphasized since proper planning of human resources is essential in any organization. These are many areas of an organisation or industry in which the application of Stochastic models is quite necessary. It would be very much useful in every sector of human activity. First of all it is imperative to identify those areas of human activity where the demand for manpower and supply are at disequilibrium. Especially in the area of specialist skill, it becomes necessary to identify where the disequilibrium exists and also were there is interruption in the work schedule due to shortage of manpower. The identification of such areas, the type of problems involved and the conversion of a real life situation into a mathematical model are essential to develop Human Resource Management which will yield profits not only to the management but also to the society itself.

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