

Magnetohydrodynamic unsteady flow of a dusty conducting fluid through porous medium in a rectangular channel with time dependent pressure gradient

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(Acceptance Date 7th March, 2013)

Abstract

The unsteady flow of a dusty conducting viscous incompressible fluid through porous medium in a long rectangular channel under the influence of uniform magnetic field and a time dependent pressure gradient has been studied. The particular cases when the pressure gradient is (i) an absolute constant, (ii) periodic function of time, (iii) an exponentially decreasing function of time and (iv) $Ct e^{-\lambda t}$, have been discussed in detail.

Introduction

Interest in problem of mechanics of systems with more than one phase has developed rapidly in recent years. The study of fluids having uniform distribution of solid spherical particles is of interest in a wide range of areas of technical importance. These areas include fluidization (flow through packed beds), flow in rocket tubes, where small carbon or metallic fuel particles are present, environmental pollution, the process by which rain drops are formed by the coalescence of small droplets, which might be considered as solid particles for the purpose of examining their movement prior to coalescence, combustion, and more recently,

blood flow in capillaries.

Saffman¹⁷ has expressed a model equation describing the influence of dust particles on the motion viscous fluids. Later on the large number of dusty viscous flow problems have been investigated by Marble¹³; Michael and Miller¹⁴; Michael and Norey¹⁵; Verma and Mathur (1973); Gupta⁹; Srivastava (2002); Sanyal and Dasgupta¹⁸; Gireesha and Bagewadi^{5,6}; Gireesha, Bagewadi and Venkatesh (2007); Gireesha, Venkatesh and Bagewadi⁷ and Elangovan and Ratchagar⁴ (2009) *etc.* through channels of various cross-sections under the influence of time dependent pressure

gradient. The basic theory of multiphase fluid flow has been given by Soo²¹.

Singh, Lal and Sharma¹⁹; Bhatnagar and Bhardwaj³; Mal and Sengupta¹²; Mishra and Bhola¹⁶; Varshney and Singh²²; Kumar, Jha and Shrivastava^{10,11}; Singh, Singh and Jha²⁰; Agrawal, Agrawal and Varshney¹ and Agrawal and Singh² *etc.* have discussed the unsteady flow of dusty fluid through porous medium in the different type of channels with time dependent pressure gradient¹⁴.

The present paper is concerned with the flow problem of a conducting viscous incompressible fluid with embedded non-conducting small identical spherical particles through porous medium in a long rectangular channel under the influence of uniform magnetic field applied perpendicularly to the flow of fluid and a time varying pressure gradient, taking the fluid and dust particles to be initially at rest. The expressions for velocities of conducting fluid and non-conducting particles are obtained by using Finite Fourier Cosine and Laplace transforms. The particular cases when (i) the pressure gradient is an absolute constant, (ii) the the pressure gradient is a periodic function of time, (iii) the pressure gradient is an exponentially decreasing function of time, and (iv) the pressure gradient is Ct , have also been discussed in detail

Equation of the problem :

Using the rectangular cartesian coordinate system, the walls of the channel are taken to be the planes $x = \pm a$ and $y = \pm b$. The fluid and dust particles velocities $u(x,y,t)$ and $v(x,y,t)$ respectively, are in z -direction *i.e.* along the

axis of rectangular channel. A uniform magnetic field is applied perpendicular to the planes $y = \pm b$ of rectangular channel. Taking the number density of non-conducting dust particles to be constant throughout the motion, the appropriate momentum equations of motion through porous medium, after introducing the electromagnetic force obtained by Soo²¹ (1968), are:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \frac{K_0 N_0}{\rho} (v - u) - \frac{\sigma B_0^2}{\rho} u - \frac{\nu}{K} u \quad (1)$$

$$\frac{\partial v}{\partial t} = \frac{K_0}{M} (u - v) - \frac{k'}{\rho} \frac{\partial p}{\partial z} \quad (2)$$

where u and v denote the velocities of fluid and dust particle respectively; p is the fluid pressure; M , the mass of a particle; K_0 , the Stokes resistance coefficient, which for spherical particle of radius r is $6\pi\mu r$, μ being the viscosity of the fluid; N_0 , the number density of particle; t , the time; ρ , ρ_p and $\bar{\rho}_p$ are density of fluid, mass density of particle and material density of the particle respectively; $\nu = \mu/\rho$, the kinematic viscosity of the fluid; B_0 , the magnetic inductivity; σ , the electric conductivity; K the permeability of porous medium and $k' = \rho/\rho_p$.

It is assumed that effect of the induced magnetic field and the electric field produced by motion of the electrically conducting field is negligible and no external field is applied.

The dust particles are non-conducting.

Introducing the non-dimensional quantities

$$x^* = \frac{x}{a}, \quad y^* = \frac{y}{a}, \quad z^* = \frac{z}{a},$$

$$p^* = \frac{a^2}{\rho v^2} p, \quad t^* = \frac{v}{a^2} t,$$

$$u^* = \frac{ua}{v}, \quad K^* = \frac{K}{a^2}, \quad v^* = \frac{va}{v}$$

Equations (1) and (2) become (dropping stars)

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial z} + \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \beta(v - u) - H^2 u - \frac{1}{K} u \quad (3)$$

$$\frac{\partial v}{\partial t} = \gamma'(u - v) - k' Q \frac{\partial p}{\partial z} \quad (4)$$

where

$$\beta = \frac{K'_0}{\gamma} = \frac{N_0 K_0 a^2}{\rho v}, \quad K'_0 = \frac{N_0 M}{\rho},$$

$$\gamma = \frac{Mv}{K_0 a^2} \partial, \quad \gamma' = \frac{1}{\gamma}, \quad Q = \frac{v}{a}$$

and

$$H = aB_0 \sqrt{\frac{\sigma}{\mu}} \quad (\text{Hartmann number})$$

Initially, the fluid and particles are rest. The flow takes place under the influence of time dependent pressure gradient with no-slip boundary conditions. From symmetric consideration, the flow in region $x \geq 0, y \geq 0$ is considered. Accordingly, the boundary conditions are:

$$\left. \begin{aligned} t > 0 \quad u(1, y, t) = 0 \\ v(1, y, t) = 0 \\ \frac{\partial u}{\partial x} = 0, \quad \frac{\partial v}{\partial x} = 0 \quad \text{at } x = 0 \end{aligned} \right\} \quad 0 \leq y \leq h \quad (5)$$

and

$$\left. \begin{aligned} u(x, h, t) = 0 \\ v(x, h, t) = 0 \\ \frac{\partial u}{\partial y} = 0, \quad \frac{\partial v}{\partial y} = 0 \quad \text{at } y = 0 \end{aligned} \right\} \quad 0 \leq x \leq 1 \quad (6)$$

where $h = b/a$

Solution of the problem :

For solving the problem, we choose the finite cosine transform defined as

$$\bar{u}(m, y, t) = \int_0^1 u(x, y, t) \cos q_m x \, dx \quad (7)$$

$$\bar{\bar{u}}(x, n, t) = \int_0^h u(x, y, t) \cos q_n y \, dy \quad (8)$$

where

$$q_m = \frac{2m+1}{2} \pi, \quad q_n = \frac{2n+1}{2h} \pi$$

It can be shown that the inversion formula for finite cosine transforms defined by (7) and (8) are given by

$$u(x, y, t) = 2 \sum_{m=0}^{\infty} \bar{u}(m, y, t) \cos q_m x \quad (9)$$

and

$$u(x, y, t) = \frac{2}{h} \sum_{n=0}^{\infty} \bar{\bar{u}}(x, n, t) \cos q_n y \quad (10)$$

Multiplying equations (3) and (4) and then integrating twice within the limits 0 to 1 and 0 to h and using the boundary conditions (5) and (6), we get

$$\frac{\partial U}{\partial t} = \frac{(-1)^{m+n}}{q_m q_n} f(t) - (q_m^2 + q_n^2)U + \beta(V - U) - \left(H^2 + \frac{1}{K}\right)U \quad (11)$$

$$\frac{\partial V}{\partial t} = \gamma'(U - V) - \frac{(-1)^{m+n} k' Q}{q_m q_n} f(t) \quad (12)$$

where

$$U = \int_0^1 \int_0^h u(x, y, t) \cos q_m x \cdot \cos q_n y \, dx \, dy$$

$$V = \int_0^1 \int_0^h v(x, y, t) \cos q_m x \cdot \cos q_n y \, dx \, dy$$

and

$$-\frac{\partial p}{\partial z} = f(t)$$

Again, applying Laplace transform to equations (11) and (12) under the transform initial condition

$U = 0, \quad V = 0, \quad \text{at } t = 0$
we get

$$s\bar{U} = \frac{(-1)^{m+n}}{q_m q_n} \bar{f}(s) - (q_m^2 + q_n^2)\bar{U} + \beta(\bar{V} - \bar{U}) - \left(H^2 + \frac{1}{K}\right)\bar{U} \quad (13)$$

$$s\bar{V} = \gamma'(\bar{U} - \bar{V}) - \frac{(-1)^{m+n} k' Q}{q_m q_n} \bar{f}(s) \quad (14)$$

where $\bar{U}, \bar{V}$ and $\bar{f}(s)$ are the Laplace transforms of the respective quantities.

Solving equations (13) and (14), we get

$$\bar{U} = \frac{(-1)^{m+n}}{q_m q_n} \frac{(s + \gamma' - k' Q \beta) \bar{f}(s)}{(s - \alpha_1)(s - \alpha_2)} \quad (15)$$

$$\bar{V} = \frac{(-1)^{m+n}}{q_m q_n} \left[\frac{\gamma'(s + \gamma' - k' Q \beta)}{(s + \gamma')(s - \alpha_1)(s - \alpha_2)} - \frac{k' Q}{(s + \gamma')} \right] \bar{f}(s) \quad (16)$$

where

$$\alpha_1 = -\frac{1}{2} \left[\left(\gamma' + \beta + H^2 + \frac{1}{K} + q_m^2 + q_n^2 \right) + \left\{ \left(\gamma' + \beta + H^2 + \frac{1}{K} + q_m^2 + q_n^2 \right)^2 \right. \right]$$

$$\left. -4\gamma' \left(q_m^2 + q_n^2 + H^2 + \frac{1}{K} \right) \right\}^{1/2} \right]$$

and

$$\alpha_2 = -\frac{1}{2} \left[\left(\gamma' + \beta + H^2 + \frac{1}{K} + q_m^2 + q_n^2 \right) - \left\{ \left(\gamma' + \beta + H^2 + \frac{1}{K} + q_m^2 + q_n^2 \right)^2 - 4\gamma' \left(q_m^2 + q_n^2 + H^2 + \frac{1}{K} \right) \right\}^{1/2} \right]$$

are the roots of the equation

$$s^2 + \left(\gamma' + \beta + H^2 + \frac{1}{K} + q_m^2 + q_n^2 \right) s + \gamma' \left(H^2 + \frac{1}{K} + q_m^2 + q_n^2 \right) = 0$$

Now to obtain u and v, we may invert the Laplace transform by convolution theorem and then applying the inversion formulae for the finite cosine transforms, we get

$$u = \frac{4}{h} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{q_m q_n} \left[\int_0^t f(t-\eta) \{A_1 e^{\alpha_1 \eta} + A_2 e^{\alpha_2 \eta}\} d\eta \right] \\ \times \cos q_m x \cdot \cos q_n y \quad (17)$$

and

$$v = \frac{4}{h} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{q_m q_n} \left[\int_0^t f(t-\eta) \{ \gamma' (B_1 e^{\alpha_1 \eta} + B_2 e^{\alpha_2 \eta}) + B_3 e^{-\gamma' \eta} \} d\eta \right] \\ \times \cos q_m x \cdot \cos q_n y \quad (18)$$

where

$$A_1 = \frac{(\alpha_1 + \gamma' - k' Q \beta)}{(\alpha_1 - \alpha_2)}, \quad A_2 = -\frac{(\alpha_2 + \gamma' - k' Q \beta)}{(\alpha_1 - \alpha_2)}, \\ B_1 = \frac{A_1}{(\alpha_1 + \gamma')}, \quad B_2 = \frac{A_2}{(\alpha_2 + \gamma')}, \quad B_3 = -\left\{ 1 + \frac{\gamma' \beta}{(\alpha_1 + \gamma')(\alpha_2 + \gamma')} \right\} k' Q$$

Particular cases :

(i) *When the pressure gradient is constant :*

Substituting $f(t)=C$ (where C is an absolute constant) in the above equation and on simplifying, we get velocities of the fluid and dust particles

$$u = \frac{4C}{h} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{q_m q_n} \left[\frac{\gamma' - k' Q \beta}{\alpha_1 \alpha_2} + \frac{A_1}{\alpha_1} e^{\alpha_1 t} + \frac{A_2}{\alpha_2} e^{\alpha_2 t} \right] \cos q_m x. \cos q_n y \quad (19)$$

$$v = \frac{4C}{h} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{q_m q_n} \left[\left(\frac{\gamma' - k' Q \beta}{\alpha_1 \alpha_2} - \frac{k' Q}{\gamma'} \right) + \frac{\gamma' B_1}{\alpha_1} e^{\alpha_1 t} + \frac{\gamma' B_2}{\alpha_2} e^{\alpha_2 t} - \frac{k' Q B_3}{\gamma'} e^{-\gamma' t} \right] \cos q_m x. \cos q_n y \quad (20)$$

(ii) *When the Pressure Gradient is periodic Function of Time :*

Substituting $f(t)=C \sin \omega t$ (where C and ω are constants) in the above equations and on simplifying, we get velocities of the fluid and the dust particles.

$$u = \frac{4C}{h} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{q_m q_n} \left[\frac{\omega A_1}{(\alpha_1^2 + \omega^2)} e^{\alpha_1 t} + \frac{\omega A_2}{(\alpha_2^2 + \omega^2)} e^{\alpha_2 t} + \left\{ \frac{\omega^2 + (\gamma' - k' Q \beta)^2}{(\alpha_1^2 + \omega^2)(\alpha_2^2 + \omega^2)} \right\}^{1/2} \sin(\omega t - \psi_1) \right] \cos q_m x. \cos q_n y \quad (21)$$

and

$$v = \frac{4C}{h} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{q_m q_n} \left[\frac{\gamma' \omega B_1}{(\alpha_1^2 + \omega^2)} e^{\alpha_1 t} + \frac{\gamma' \omega B_2}{(\alpha_2^2 + \omega^2)} e^{\alpha_2 t} + \frac{\omega B_3}{(\gamma'^2 + \omega^2)} e^{-\gamma' t} + \gamma' \left\{ \frac{\omega^2 + (\gamma' - k' Q \beta)^2}{(\alpha_1^2 + \omega^2)(\alpha_2^2 + \omega^2)(\gamma'^2 + \omega^2)} \right\}^{1/2} \sin(\omega t - \psi_2) - \frac{k' Q}{(\gamma'^2 + \omega^2)^{1/2}} \sin(\omega t - \psi_3) \right] \cos q_m x. \cos q_n y \quad (22)$$

respectively,
where

$$\begin{aligned}\psi_1 &= \tan^{-1} \left(-\frac{\omega}{\alpha_1} \right) + \tan^{-1} \left(-\frac{\omega}{\alpha_2} \right) - \tan^{-1} \frac{\omega}{(\gamma' - k'Q\beta)} , \\ \psi_2 &= \tan^{-1} \left(-\frac{\omega}{\alpha_1} \right) + \tan^{-1} \left(-\frac{\omega}{\alpha_2} \right) + \tan^{-1} \left(\frac{\omega}{\gamma'} \right) - \tan^{-1} \frac{\omega}{(\gamma' - k'Q\beta)} , \\ \psi_3 &= \tan^{-1} \left(\frac{\omega}{\gamma'} \right)\end{aligned}$$

(iii) When the Pressure Gradient is Exponentially Decreasing Function Time :

Substituting $f(t) = Ce^{-\lambda t}$ (where C and λ are constants) in the above equations and on simplifying, we get velocities of fluid and dust particles

$$\begin{aligned}u &= \frac{4C}{h} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{q_m q_n} \left[\frac{\gamma' - k'Q\beta - \lambda}{(\alpha_1 + \lambda)(\alpha_2 + \lambda)} e^{-\lambda t} + \frac{A_1}{(\alpha_1 + \lambda)} e^{\alpha_1 t} \right. \\ &\quad \left. + \frac{A_2}{(\alpha_2 + \lambda)} e^{\alpha_2 t} \right] \cos q_m x \cdot \cos q_n y\end{aligned}\quad (23)$$

and

$$\begin{aligned}v &= \frac{4C}{h} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{q_m q_n} \left[\left\{ \frac{\gamma'(\gamma' - k'Q\beta - \lambda)}{(\gamma' - \lambda)(\alpha_1 + \lambda)(\alpha_2 + \lambda)} - \frac{k'Q}{(\gamma' - \lambda)} \right\} e^{-\lambda t} \right. \\ &\quad \left. + \frac{\gamma' B_1}{(\alpha_1 + \lambda)} e^{\alpha_1 t} + \frac{\gamma' B_2}{(\alpha_2 + \lambda)} e^{\alpha_2 t} - \frac{k'Q B_3}{(\gamma' - \lambda)} e^{-\gamma' t} \right] \cos q_m x \cdot \cos q_n y\end{aligned}\quad (24)$$

respectively.

(iv) When $f(t) = Cte^{-\lambda t}$

The velocities of the fluid and the dust particles are

$$u = \frac{4C}{h} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{q_m q_n} \left[\frac{A_1}{(\alpha_1 + \lambda)^2} e^{\alpha_1 t} + \frac{A_2}{(\alpha_2 + \lambda)^2} e^{\alpha_2 t} \right]$$

$$\begin{aligned}
& + \left\{ \frac{\gamma' - k'Q\beta - \lambda}{(\alpha_1 + \lambda)(\alpha_2 + \lambda)} t \right. \\
& \left. + \frac{(2\lambda + \alpha_1 + \alpha_2)(\gamma' - k'Q\beta) + (\alpha_1\alpha_2 - \lambda^2)}{(\alpha_1 + \lambda)^2(\alpha_2 + \lambda)^2} \right\} e^{-\lambda t} \Big] \\
& \times \cos q_m x. \cos q_n y
\end{aligned} \tag{25}$$

and

$$\begin{aligned}
v = & \frac{4C}{h} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{q_m q_n} \left[\frac{\gamma' B_1}{(\alpha_1 + \lambda)^2} e^{\alpha_1 t} + \frac{\gamma' B_2}{(\alpha_2 + \lambda)^2} e^{\alpha_2 t} + \frac{B_3}{(\gamma' - \lambda)^2} e^{-\gamma' t} \right. \\
& + \left\{ \frac{(\gamma' - k'Q\beta - \lambda)\gamma' t}{(\gamma' - \lambda)(\alpha_1 + \lambda)(\alpha_2 + \lambda)} + \frac{(\gamma' - k'Q\beta)D}{\alpha_1 \alpha_2 \lambda} \right. \\
& \left. \left. - \frac{k'Q\{(\gamma' - \lambda)t - 1\}}{(\gamma' - \lambda)} \right\} e^{-\lambda t} \right] \cos q_m x. \cos q_n y
\end{aligned} \tag{26}$$

respectively,

where

$$\begin{aligned}
D = & [\{(\alpha_1 - \alpha_2)(\alpha_1 + \gamma')(\alpha_2 + \gamma')(\alpha_1 + \lambda)^2(\alpha_2 + \lambda)^2(\gamma' - \lambda)^2(\gamma' - k'Q\beta) \\
& + \alpha_2 \gamma' \lambda^2(\gamma' - k'Q\beta + \alpha_1)(\alpha_2 + \gamma')(\alpha_2 + \lambda)^2(\gamma' - \lambda)^2 - \alpha_1 \gamma' \lambda^2 \\
& \times (\gamma' - k'Q\beta + \alpha_2)(\alpha_1 + \gamma')(\alpha_1 + \lambda)^2(\gamma' - \lambda)^2 + \alpha_1 \alpha_2 \lambda^2 k'Q\beta \\
& \times (\alpha_1 - \alpha_2)(\alpha_1 + \lambda)^2(\alpha_2 + \lambda)^2 + \alpha_1 \alpha_2 \gamma'(k'Q\beta - \gamma' + \lambda)(\alpha_1 - \alpha_2) \\
& \times (\alpha_1 + \gamma')(\alpha_2 + \gamma')(\alpha_1 + \lambda)(\alpha_2 + \lambda)(\gamma' - \lambda)\} / \{(\gamma' - k'Q\beta) \\
& \times (\alpha_1 - \alpha_2)(\alpha_1 + \gamma')(\alpha_2 + \gamma')(\alpha_1 + \lambda)^2(\alpha_2 + \lambda)^2(\gamma' - \lambda)^2\}].
\end{aligned}$$

Discussion

magnetic field.

Evidently the velocities of fluid and the dust particles will be slower due to the presence of porous medium and the applied uniform

In the particular case (iv), if we put $\lambda=0$, the velocities of the fluid and the dust particles can be obtained for a linearly time

dependent pressure gradient.

If $H=0$, all the velocities expressions for fluid and dust particles can be obtained in the absence of magnetic field under the influence of various pressure gradient and also if $K'=0$, the results are in agreement with those of Gupta and Gupta⁸ (1976).

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