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MHD Boundary Layer Stagnation Point Flow and Heat Transfer of a Micropolar Fluid over a Flat Plate with Uniform Suction/Injection

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Abstract

An analysis is presented for the problem of heat transfer in a steady laminar stagnation point flow of an incompressible electrically conducting micropolar fluid impinging on a permeable flat plate with uniform suction/injection. A uniform magnetic field is applied normal to the plate. The effect of viscous dissipation is taken into account. The governing partial differential equations are transformed into ordinary differential equations. A numerical solution is obtained for the ordinary differential equations using the computational software Matlab. Results are shown in graphical form. The effect of the uniform suction or injection, magnetic parameter, material parameter on the flow and heat transfer are presented and discussed.

Key words: Micropolar fluid, MHD, Heat transfer, Stagnation point, Suction and Injection.

Introduction

The theory of micropolar fluids can be used to explain the flow of colloidal fluids, liquid crystals, polymeric fluids *etc.* Ebert[1] presented a similarity solution for the boundary layer flow near a stagnation point. Guram and Smith[2] studied the stagnation flows of micropolar fluids with strong and weak interactions using the perturbation technique. Stagnation point flow of a micropolar fluid have been studied by Nazar *et al.*[3], Attia[4] and Ishak *et al.*[5].

Effect of suction/injection on the flow of a micropolar fluid past a continuously moving plate in the presence of radiation was studied by Arabay and Hassan[6]. Salem and Odda[7] studied the influence of thermal conductivity and variable viscosity on the flow of a micropolar fluid past a continuously moving plate with suction or injection. Attia[8] presented stagnation point flow and heat transfer of a micropolar fluid with uniform suction or blowing. Recently Chakraborty and Panja[9], Ishak and Nazar[10], Kishan and Deepa[11] have studied the micropolar fluid with suction or injection through different surfaces.

The hydromagnetic boundary layer stagnation point flows of micropolar fluid have attracted the researchers due to their interesting applications in engineering and industries. Mohammadien and Gorla[12] studied the effects of transverse magnetic field on a mixed convection in a micropolar fluid on a horizontal plate with vectored mass transfer. Flow of a magnetomicropolar fluid past a continuously moving plate was studied by Seddeek[13]. Ishaq *et al*.[14] presented MHD flow of a micropolar fluid towards a stagnation point on a vertical surface. Steady flow of a micropolar fluid under a transverse magnetic field with constant suction/injection was studied by Murthy and Bahali[15]. Patowary and Sut[16] presented the effects of variable viscosity and thermal conductivity of micropolar fluid past a continuously moving plate with suction/injection in the presence of magnetic field.

The objective of the present paper is to study the flow and heat transfer near the stagnation point of an electrically conducting micropolar fluid over a flat plate with suction/ injection in the presence of applied normal magnetic field.

Formulation of the Problem

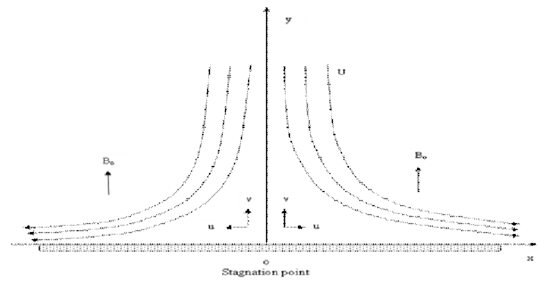

Figure 1. Physical model and coordinate system

Consider a steady two-dimensional flow of a viscous incompressible electrically conducting micropolar fluid impinging normally on a permeable horizontal flat plate placed at y=0, divides into two streams on the plate and leaves in both directions near a stagnation point. A uniform transverse magnetic field of strength B_o is applied normal to the plate (Figure 1). The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected as compared to the applied field. We assume that there is no polarization voltage, so the induced electric field is zero. The x-axis is chosen along the plate and y-axis is taken normal to it. Let u and v be the x- and y-components of velocity near to stagnation point respectively and N be the component of the micro-rotation vector normal to the xy-plane. A uniform suction/injection is applied at the plate with a transpiration velocity at the boundary of the plate given by $-v_0$, where $v_0 > 0$ for suction. The potential flow velocity external to the boundary layer be taken as U(x) = ax where the constant a (>0) is proportional to the free stream velocity far away from the plate. All the fluid properties are assumed to be constant throughout the motion. Under the usual boundary layer approximation including the viscous dissipation, the governing equations are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\frac{(\mu+h)}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{h}{\rho}\frac{\partial N}{\partial y}-\frac{1}{\rho}\sigma B_0^2 u \quad (2)$$

$$\rho\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=\frac{\gamma}{j}\frac{\partial^2 N}{\partial y^2}-\frac{h}{j}\left(2N+\frac{\partial u}{\partial y}\right) \quad (3)$$

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\kappa\frac{\partial^2 T}{\partial y^2}+(\mu+h)\left(\frac{\partial u}{\partial y}\right)^2+\sigma B_0^2 u^2 \quad (4)$$

Subject to the boundary conditions:

$$y=0\ :\ u=0,\ v=-v_0,\ N=-m\frac{\partial u}{\partial y},\ T=T_w$$

$$y\to\infty\ :\ u=U(x)=ax, \qquad N\to 0, \qquad T=T_\infty \quad (5)$$

where T and T_w are the temperatures of the fluid and the plate, respectively whereas the temperature of the fluid far away from the plate is T_∞, μ is the viscosity, ρ is the density, σ is the electrical conductivity, κ is the thermal conductivity, c_p is the specific heat at constant pressure, j is the micro-inertia per unit mass, γ is the spin gradient viscosity, h is the vortex viscosity and m (0≤ m≤1) is the boundary parameter. Here γ, j and h are assumed to be constants and γ is assumed to be given by Nazar *et al.*[3]

$$\gamma=\left(\mu+\frac{h}{2}\right)j \quad (6)$$

We take $j=\frac{\upsilon}{a}$ as a reference length, where υ is the kinematic viscosity.

Analysis :

The equation of continuity (1) is identically satisfied by stream function Ψ(*x*,*y*) defined as

$$u=\frac{\partial\Psi}{\partial y}\ ,\ v=-\frac{\partial\Psi}{\partial x} \quad (7)$$

For the solution of momentum, micro-rotation (spin) and the energy equation (2) to (4), the following similarity transformations in order to convert the partial differential equations into the ordinary differential equations are defined:

$$\Psi(x,y)=x\sqrt{a\upsilon}\,f(\eta),\ N(x,y)=ax\sqrt{\frac{a}{\upsilon}}\,g(\eta),$$

$$\frac{T-T_\infty}{T_w-T_\infty}=\theta(\eta),\ \text{where}\ \eta=y\sqrt{\frac{a}{\upsilon}} \quad (8)$$

the equations (2) to (4) reduce to (after some simplifications):

$$(1+K)f'''+ff''-f'^2+Kg'+1-Mf'=0 \quad (9)$$

$$\left(1+\frac{K}{2}\right)g''+fg'-f'g-K\left(2g+f''\right)=0 \quad (10)$$

$$\theta''+\Pr f\theta'+(1+K)\,Pr\,Ec\,f''^2+M\,Ec\,Prf'^2=0 \quad (11)$$

The corresponding boundary conditions are:

$$\eta=0\ :\ f=S\ ,\ f'=0,\ g=-mf''\ ,\ \theta=1$$

$$\eta\to\infty:\ f'\to 1, \qquad g\to 0\ ,\ \theta\to 0 \quad (12)$$

where primes denote differentiation with respect to η, $K=\frac{h}{\mu}$ is the material parameter, $M=\frac{\sigma B_0^2}{\rho a}$ is the magnetic parameter, $Pr=\frac{\mu C_p}{\kappa}$ is the prandtl number, $S=\frac{v_0}{\sqrt{a\upsilon}}$ is the suction/ injection parameter and $Ec=\frac{U^2}{C_p(T_w-T_\infty)}$ is the Eckert number.

Equation (10) is that obtained by Attia[8]

Table 1. Numerical values of $f''(0)$ for various values of parameters M and S when K=1

M	S= -1	S= -0.5	S=0	S=0.5	S=1
0	0.6766	0.8278	1.0063	1.2099	1.4352
0.1	0.6545	0.7952	0.9669	1.1606	1.3756
0.2	0.6339	0.7694	0.9299	1.1141	1.3191
0.5	0.5799	0.6925	0.8321	0.9902	1.1676
0.8	0.534	0.6347	0.744	0.8819	1.041

Table 2. Numerical values of $-\theta'(0)$ for various values of parameters M, S, Pr, Ec and K

Pr=0.05, S=0, K=1					Pr=0.05, S=1, K=1				
Ec	M=0	M=0.1	M=0.5	M=1	Ec	M=0	M=0.1	M=0.5	M=1
0	0.1994	0.1980	0.1931	0.1885	0	0.2315	0.2300	0.2244	0.2186
0.1	0.1944	0.1926	0.1867	0.1816	0.1	0.2235	0.2216	0.2153	0.2097
0.5	0.1723	0.1688	0.1593	0.1541	0.5	0.1916	0.1882	0.1789	0.1739
1	0.1446	0.1390	0.1251	0.1196	1	0.1518	0.1465	0.1335	0.1292
Pr=0.5, S=0, K=1					Pr=0.5, S=1, K=1				
Ec	M=0	M=0.1	M=0.5	M=1	Ec	M=0	M=0.1	M=0.5	M=1
0	0.4110	0.4038	0.3766	0.3471	0	0.7746	0.7677	0.7415	0.7128
0.1	0.3609	0.3515	0.3189	0.2882	0.1	0.6980	0.6879	0.6534	0.6212
0.5	0.1605	0.1422	0.0879	-0.0524	0.5	0.3913	0.3688	0.3009	0.2550
1	-0.0899	-0.1194	-0.2007	-0.2422	1	0.0080	-0.0301	-0.1396	-0.2027

for non-magnetic case and he obtained the solution as $g(\eta) = -\frac{1}{2} f''(\eta)$ whereas the equation (9) and (11) are differential equations with values prescribed at two boundaries which can be converted to initial value problem by known technique. These equations are solved numerically on the computer for different values of the various parameters.

The physical quantities of interest are the local skin-friction coefficient C_f and the local Nusselt number Nu which are defined, respectively, as

$$C_f = \frac{\tau_w}{\rho U^2/2} \quad \text{and} \quad Nu = \frac{x q_w}{\kappa(T_w - T_\infty)} \qquad (13)$$

where U(x) = ax is a characteristic velocity and τ_w is the wall shear stress which is given

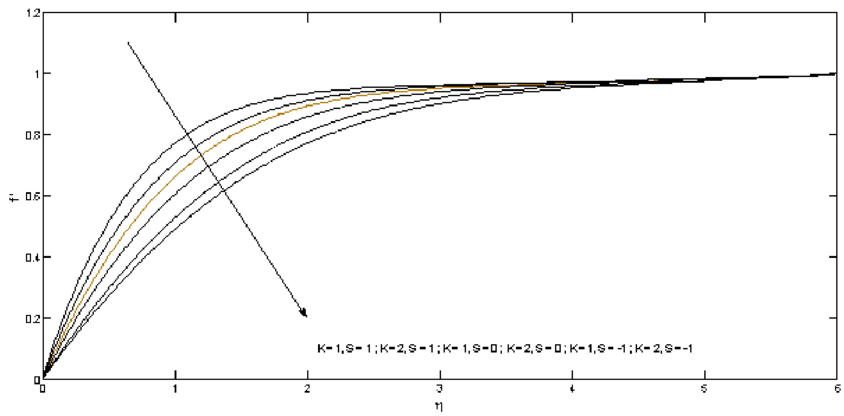

Figure 2. Velocity profile against η for various values of parameters K and S when M=0.1

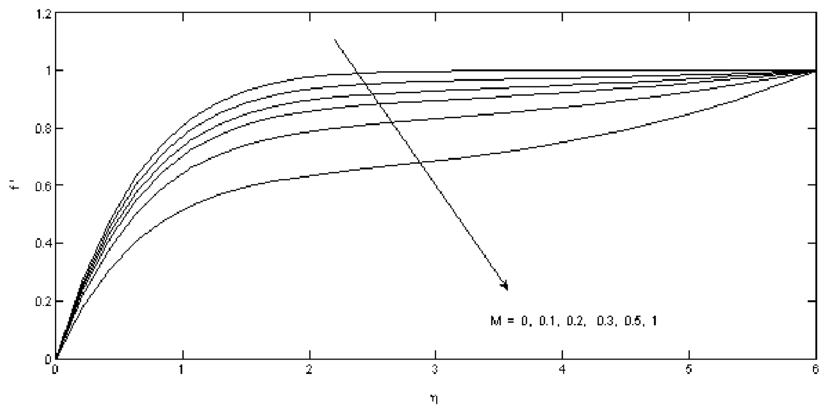

Figure 3. Velocity profile against η for various values of parameter M when S=1 and K=1

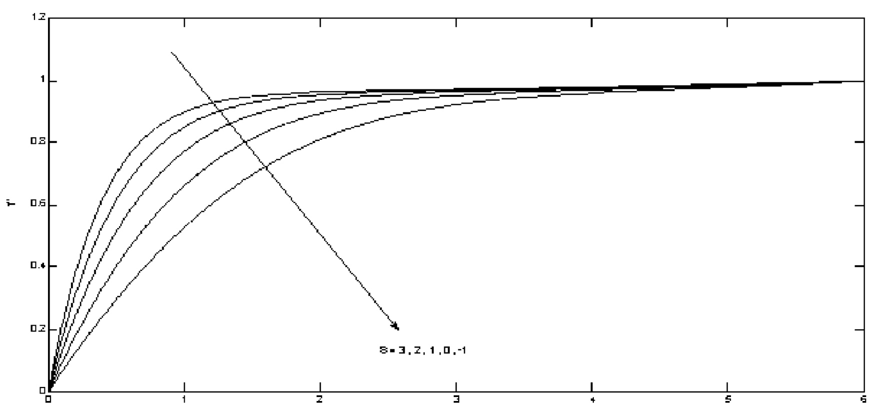

Figure 4. Velocity profile against η for various values of parameter S when M=0.1 and K=1

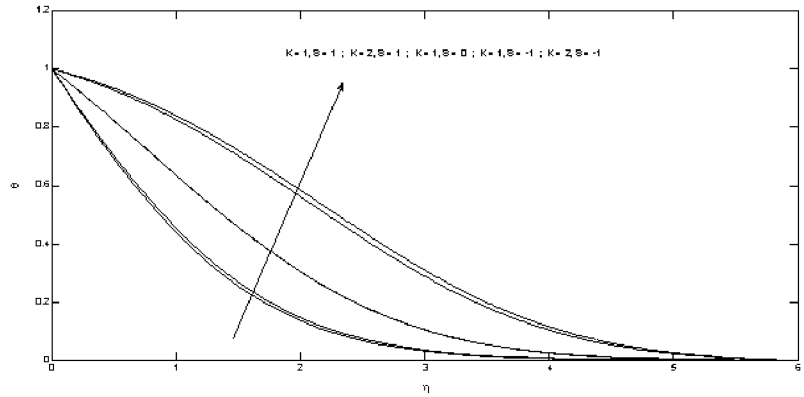

Figure 5. Temperature profile against η for various values of parameters K and S when M=0.1, Pr=0.5 and Ec=0.1

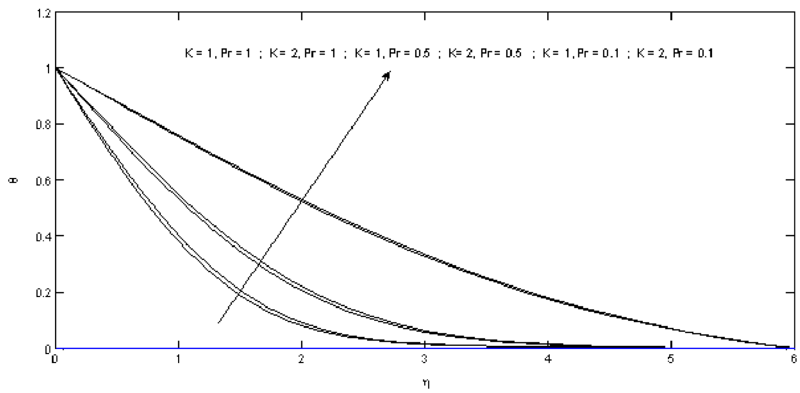

Figure 6. Temperature profile against η for various values of parameters K and Pr when M=0.1, S=0.5 and Ec=0.1

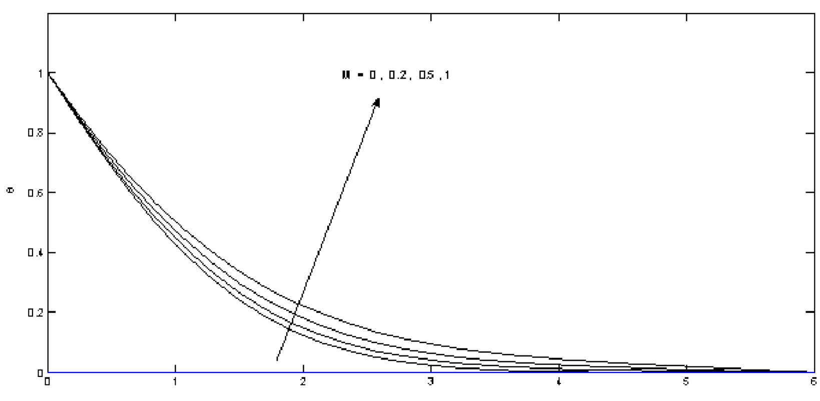

Figure 7. Temperature profile against η for various values of parameter M when S=1, K=1, Pr=0.5 and Ec=0.1

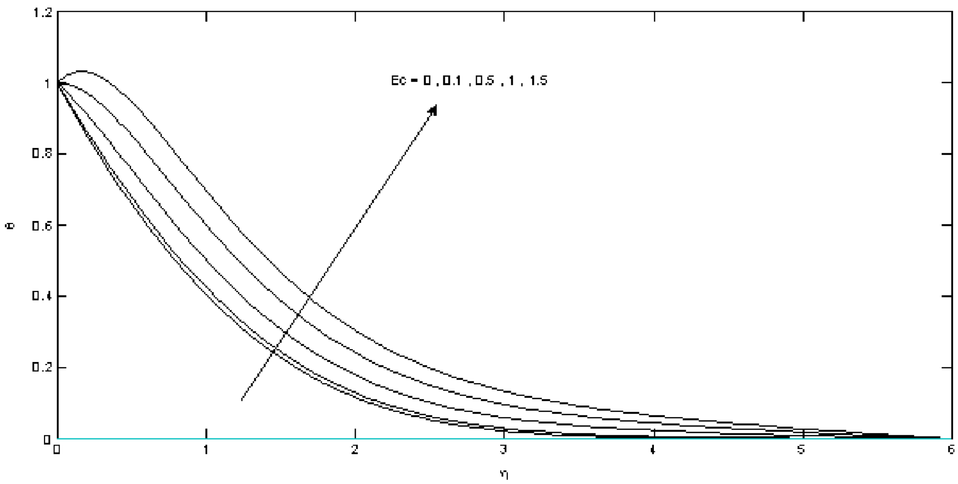

Figure 8. Temperature profile against η for various values of parameter Ec when M=0.1, Pr=0.5, K=1 and S=1

by

$$\tau_w = \left\{(\mu + h)\frac{\partial u}{\partial y} + hN\right\}_{y=0}$$

and $q_w = -\kappa\left\{\frac{\partial T}{\partial y}\right\}_{y=0}$ is the heat transfer from the plate.

Then, we get

$$C_f = \frac{2\left\{1+\frac{K}{2}\right\}f''(0)}{\sqrt{Re}} \text{ and } Nu = -\sqrt{Re}\ \theta'(0) \quad (14)$$

where $Re = \frac{xU}{\upsilon}$ is the local Reynolds number.

Numerical values of the skin friction coefficient and Nusselt number which are proportional to $f''(0)$ and $-\theta'(0)$ respectively are presented in tables 1 and 2 respectively. It is observed from the tables that both the skin friction coefficient and Nusselt number increases with the increasing values of the parameter S for all M and decreases with the increasing values of the parameter M for all S.

Results and Discussion

The non linear ordinary differential equations (9) and (11) subject to the boundary conditions (12) were solved numerically. The computations were done by a programme which uses a symbolic and computational computer language Matlab. Figures 2 to 4 present the velocity profile for various values of different parameters. It is observed from these graphs that the velocity increases with the increasing values of the parameter S while it decreases with the increase of the parameters K and M. Figures 5 to 8 show the profile of temperature distribution for various parameters. It is evident from these graphs that the temperature distribution shows reverse phenomenon with the parameters S and Pr i.e. the temperature decreases with the increasing values of these parameters while it increases with the increasing values of parameters K, M and Ec.

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