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Magnetohydrodynamic steady laminar generalized plane couette flow of viscous fluid through porous medium

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Abstract

The aim of the present paper is to study the magnetohydrodynamic steady laminar generalized plane couette flow of viscous fluid through uniform porous medium between two parallel plates. The viscous fluid is conducting and uniform magnetic field is applied perpendicularly to the flow of fluid. The skin friction, average, maximum and minimum velocities of the fluid, the volumetric flow and energy losses in the channel are also determined.

Nomenclature:

$u =$ Velocity components along x-axis
$v =$ Velocity components along y-axis
$w =$ Velocity components along z-axis
$p =$ The fluid pressure
$t =$ The time
$\rho =$ The density of the fluid
$\mu =$ Coefficient of viscosity
$\upsilon =$ Kinematic viscosity
$K =$ Permeability of porous medium
$\sigma =$ The electrical conductivity
$B_0 =$ The magnetic inductivity
$u_{avg} =$ Average velocity
$Q =$ The volumetric flow
$C_f =$ Drag coefficient at $y = 0$
$C_f^{'} =$ Drag coefficient at $y = 1$
$M =$ Hartmann number

Introduction

In classical viscous fluid we know that the fluid exerts viscosity effects when there is a tendency of shear flow of the fluid various types of basic problems of diversified nature have been solved in this branch.

The study of flow through porous medium is of considerable interest in the field of petroleum engineering concerned with the movement of oil and gas, ground water hydrology, heat transfer in cooling systems and chemical engineering for filtration process etc.

In hydromagnetic flow, we study of the flow of electrically conducting fluid in presence of Maxwell electromagnetic field. The flow of conducting fluid is effectively changed by the presence of the magnetic field where as the magnetic field is also perturbed due to the motion of the conducting fluid. This phenomena is called magnetohydrodynamics and in short written as MHD. There are wider application in engineering technology, cosmology, astrophysics and other applied sciences. On account of varied practical applications of MHD flow problems in channels through porous medium many researchers: Ram and Mishra[6], Gupta[2], Kuiry[3], Sengupta and Kumar[9], Avinash and Rao[1], Kumari and Varshney[5], Shakya and Johari[10], Kumar and Singh[4], Raveendranath and Rao[7] and Reddy and Verma[8] and others have paid their attentions in this direction.

In the present paper, the magnetohydrodynamic steady laminar generalized plane couette flow of viscous fluid through porous medium between two parallel plates under the influence of uniform magnetic field applied perpendicularly to the flow of fluid is studied. Some important cases related to the flow of fluid have also been derived.

Formulation of the Problem:

Consider two parallel flat plates separated at finite distance h. Let x-axis and z-axis lie at the first plate and y-axis perpendicular to both the axes *i.e.* perpendicular to both plates. Also let the flow of viscous fluid be along the x-axis. The porous medium contained between both the plate and uniform magnetic field is applied perpendicularly to the flow of fluid. If u, v, w be the components of the fluid velocity along the coordinate axes: x-axis, y-axis and z-axis respectively, then

$$v = 0, \quad w = 0 \quad and \quad \frac{\partial}{\partial z} = 0$$

Equation of continuity is

$$\frac{\partial u}{\partial x} = 0$$

which implies that u is independent of x, therefore u is the function of y only *i.e.* $u = u(y)$

It is assumed that the effect of induced magnetic field and electric field produced by the motion of electrically conducting fluid are negligible and no external field is applied.

The relevant equations of motion for viscous incompressible fluid are:

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} - \frac{\upsilon}{K}u \qquad (1)$$

and $$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad (2)$$

From (2) $\frac{\partial p}{\partial y} = 0 \Rightarrow p$ is independent of y so p also function of x only *i.e.*

$$p = p(x)$$

Boundary conditions for generalized plane couette flow are:

$$\left.\begin{array}{lll} u = 0, & at & y = 0 \\ u = constant, & at & y = h \end{array}\right\} \qquad (3)$$

Introducing the following non-dimensional quantities:

$$u^* = \frac{h}{\upsilon}u, \quad x^* = \frac{x}{h}, \quad y^* = \frac{y}{h}, \quad p^* = \frac{h^2}{\rho\upsilon^2}p,$$

$$t^* = \frac{\upsilon}{h^2}t, \quad K^* = \frac{K}{h^2}$$

in eqn. (1), we get (after dropping the stars)

$$0 = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - \left(M^2 + \frac{1}{K}\right)u$$

or $\frac{d^2u}{dy^2} - \left(M^2 + \frac{1}{K}\right)u = \frac{dp}{dx} \quad \because p = p(x), \quad u = u(x)$

or
$$\frac{d^2u}{dy^2} - \left(M^2 + \frac{1}{K}\right)u = P \qquad (4)$$

where $B_0 h\sqrt{\frac{\sigma}{\mu}}$ (Hartmann number) and

$$\frac{dp}{dx} = constant = P$$

Boundary conditions (3) become:

$$\left.\begin{aligned} u &= 0, & at \quad y &= 0 \\ u &= U(constant), & at \quad y &= 1 \end{aligned}\right\} \qquad (5)$$

Solution of the Problem:

Solving eqn. (4) and applying boundary conditions (5), the velocity of viscous fluid is given by

$$u = \frac{P}{\sqrt{M^2 + \frac{1}{K}}}\left(\cosh\sqrt{M^2 + \frac{1}{K}}\, y - 1\right) + \left\{\frac{U}{\sinh\sqrt{M^2 + \frac{1}{K}}} - \frac{P}{\sqrt{M^2 + \frac{1}{K}}}\left(\frac{\cosh\sqrt{M^2 + \frac{1}{K}}}{\sinh\sqrt{M^2 + \frac{1}{K}}} - 1\right)\right\} \times \sinh\sqrt{M^2 + \frac{1}{K}}\, y$$

or

$$u = \frac{P}{A}(\cos h\, Ay - 1) + B\sin h\, Ay \qquad (6)$$

where $A = \sqrt{M^2 + \frac{1}{K}}$, $\mathbf{B} = \left\{\frac{U}{\sinh A} - \frac{P}{A}\left(\frac{\cosh A}{\sinh A} - 1\right)\right\}$ (7)

1. The average velocity :

$$u_{avg} = \int_0^1 u\,dy$$

$$u_{avg} = \int_0^1 \left\{\frac{P}{A}(\cosh Ay - 1) + B\sinh Ay\right\}dy$$

$$= \frac{P}{A^2}(\sinh A - A) + \frac{B}{A}(\cosh A - 1) \qquad (8)$$

2. Maximum and Minimum velocities:

Necessary condition $\frac{du}{dy} = 0$

$$\Rightarrow P\sinh Ay + AB\cosh Ay = 0$$

$$\Rightarrow \tanh Ay = -\frac{AB}{P} \qquad (9)$$

Sufficient condition

$$\frac{d^2u}{dy^2} = A\left(P\cos h\,A\,y + AB\sin h\,A\,y\right) = 0$$

$$= \frac{A}{P}\left(P^2 - AB\right)\cos hAy$$

$$= -ve, \quad if \;\; P < AB \qquad \therefore \max imum \quad (10)$$

$$= +ve, \quad if \;\; P > AB \qquad \therefore \min imum \quad (11)$$

3. The volumetric flow per unit time:

$$Q = u_{avg}$$

$$= \frac{P}{A^2}\left(\sinh A - A\right) + \frac{B}{A}\left(\cosh A - 1\right) \quad (12)$$

4. Energy loses in channel :

At lower plate

$$C_f = \frac{\left(\mu \dfrac{du}{dy}\right)_{y=0}}{\dfrac{1}{2}\rho u_{avg}^2}$$

$$= \frac{2\mu AB}{\rho\left\{\dfrac{P}{A^2}\left(\sinh A - A\right) + \dfrac{B}{A}\left(\cosh A - 1\right)\right\}^2}$$

$$= \frac{2\mu A^5 B}{\rho\left\{P\left(\sinh A - A\right) + AB\left(\cosh A - 1\right)\right\}^2} \quad (13)$$

At upper plate

$$C_f^{'} = \frac{\left(\mu \dfrac{du}{dy}\right)_{y=1}}{\dfrac{1}{2}\rho u_{avg}^2}$$

$$= \frac{2\mu A^4\left(P\sinh A + AB\cosh A\right)}{\rho\left\{P\left(\sinh A - A\right) + AB\left(\cosh A - 1\right)\right\}^2} \quad (14)$$

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