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Effect of Hall currents on MHD free convection flow of stratified viscous fluid with heat and mass transfer past a vertical porous plate

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Abstract

A study of the effect of Hall Currents on MHD free convection flow of stratified viscous fluid past a vertical porous plate with heat and mass transfer taking viscous and Darcy resistance terms into account and the constant permeability of the medium numerically and neglecting induced magnetic field in comparison to applied strong magnetic field is investigated. The velocity, temperature and concentration distributions are derived and discussed numerically with the helps of graphs and tables. It is observed that velocity increases with the increase in G_r (Grashof number), K (Permeability parameter) and m (Hall currents parameter), but it decreases with the increase in M (Magnetic parameter).

Keywords : Heat and mass transfer, Free convection, MHD flow, Porous medium, Vertical plate, Stratified viscous fluid, Hall Currents.

Introduction

The convection problem in a porous medium has important applications in geothermal reservoirs and geothermal extractions. The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear rector, chemical process industries and many engineering applications in which the fluid is the working medium. The wide range of technological and industrial applications has stimulated considerable amount of interest in the study of heat and mass transfer in convection flows. Free convective flow past a vertical plate has been studied extensively by Ostrach[9]. Siegel[12] investigated the transient free convection from a vertical flat plate. Cheng and Lau[4] and Cheng and Teckchandani[5] obtained numerical solutions for the convective flow in a porous medium bounded by two isothermal parallel plates in the presence of the withdrawal of the fluid. In all the above mentioned studies, the effect of porosity, permeability and the thermal resistance of the medium is ignored or treated as constant. However, porosity measurements by Benenati and Broselow[3] show that porosity is not constant but varies from the surface of the plate to its interior to which as a result permeability also varies. In case of unsteady free convective flow, Soundalgekar[14] studied the effects of viscous dissipation on the flow past an infinite vertical porous plate. The combined effect of buoyancy forces from thermal and mass diffusion on forced convection was studied by Chen *et al.*[6]. The free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient was studied by Ramanaiah and Malarvizhi[10]. Bejan and Khair[2] have investigated the vertical free convective boundary layer flow embedded in a porous medium resulting from the combined heat and mass transfer. Lin and Wu[7] analyzed the problem of simultaneous heat and mass transfer with the entire range of buoyancy ratio for most practical and chemical species in dilute and aqueous solutions. Rushi Kumar and Nagarajan[11] studied the mass transfer effects of MHD free convection flow of incompressible viscous dissipative fluid past an infinite vertical plate. Mass transfer effects on free convection flow of an incompressible viscous dissipative fluid have been studied by Manohar and Nagarajan[8]. Sivaiah *et al.*[13] studied heat and mass transfer effects on MHD free convective flow past a vertical porous plate. Recently, Agrawal *et al.*[1] have discussed the effect of stratified viscous fluid on MHD free convection flow with heat and mass transfer past a vertical porous plate.

In the present section we have considered the problem of Agrawal *et al.*[1] with Hall Currents.

Mathematical analysis :

We study the two-dimensional free convection and mass transfer flow of stratified viscous fluid past an infinite vertical porous plate with Hall currents under the following assumptions:

- The plate temperature is constant
- Viscous and Darcy's resistance terms are taken into account with constant permeability of the medium.
- Boussinesq's approximation is valid.
- The suction velocity normal to the plate is constant and can be written as,

$$v^1 = -U_0$$

A system of rectangular co-ordinates O (x^1, y^1, z^1) is taken, such that $y^1 = 0$ on the plate and z^1 axis is along its leading edge. All the fluid properties considered constant except that the influence of the density variation with temperature is considered. The influence of the density variation in other terms of the momentum and the energy equation and the variation of the expansion coefficient with temperature is considered negligible. The variations of density, viscosity, elasticity and thermal conductivity are supposed to be of the form

$$\rho=\rho_o e^{-b^1 y^1},\ \ \mu=\mu_o e^{-b^1 y^1},\ \sigma=\sigma_o e^{-b^1 y^1},\ k_T= k_o e^{-b^1 y^1},$$

where ρ_o, μ_o, σ_o and k_o are the coefficients of density, viscosity, elasticity and thermal conductivity respectively at $y^1 = 0$, $b^1 > 0$ represents the stratification factor.

Under these conditions, the problem is governed by the following system of Equations:

Equation of continuity:

$$\frac{\partial v^1}{\partial y^1} = 0 \tag{1}$$

Equation of Momentum:

$$\rho\left(\frac{\partial u^1}{\partial t^1} + v^1\frac{\partial u^1}{\partial y^1}\right) = \rho g\beta(T^1 - T^1_\infty) + \rho g\beta^1(C^1 - C^1_\infty) + \frac{\partial}{\partial y^1}\left(\mu\frac{\partial u^1}{\partial y^1}\right) - \left(\frac{\sigma B_0^2}{(1+m^2)} + \frac{\mu}{K^1}\right)u^1 \tag{2}$$

Equation of Energy:

$$\frac{\partial T^1}{\partial t^1} + v^1\frac{\partial T^1}{\partial y^1} = \frac{1}{\rho C_p}\frac{\partial}{\partial y^1}\left(k_T\frac{\partial T^1}{\partial y^1}\right) \tag{3}$$

Equation of Concentration:

$$\frac{\partial C^1}{\partial t^1} + v^1\frac{\partial C^1}{\partial y^1} = D\left(\frac{\partial^2 C^1}{\partial {y^1}^2}\right) \tag{4}$$

where u^1, v^1 are the velocity components. T^1, C^1 are the temperature and concentration components, ν is the kinematic viscosity, ρ is the density, σ is the electric conductivity, B_0 is the magnetic induction, K_T is the thermal conductivity and D is the concentration diffusivity, C_P is the specific heat at constant pressure, m is Hall Currents parameter.

The boundary conditions for the velocity, temperature and concentration fields are:

$$u^1 = 0, T^1 = T^1_w, C^1 = C^1_w \text{ at } y^1 = 0 \tag{5}$$

$$u^1 = 0, T^1 = T_\infty, C^1 = C_\infty \text{ at } y^1 \to \infty$$

Let us introduce the non-dimensional variables

$$u = \frac{u^1}{U_0},\ t = \frac{t^1 U_0^2}{\nu_o},\ y = \frac{y^1 U_0}{\nu_o},\ \theta = \frac{T^1 - T^1_\infty}{T^1_w - T^1_\infty},$$

$$C = \frac{C^1 - C^1_\infty}{C^1_w - C^1_\infty} \quad K = \frac{K^1 U_0^2}{{\nu_o}^2},\ P_r = \frac{\mu_o C_p}{k_o},$$

$$S_c = \frac{\nu_o}{D},\ M = \frac{\sigma_o B_0^2 \nu_o}{\rho_o U_0^2},\ b = \frac{b^1 \nu_o}{U_o}$$

$$N_0 = \frac{\beta^1(C^1_w - C^1_\infty)}{\beta(T^1_w - T^1_\infty)},$$

$$G_r = \frac{\nu_o g\beta(T^1_w - T^1_\infty)}{U_0^3},$$

where P_r is the Prandtl number, G_r is the Grashof number, N_0 is the buoyancy ratio, S_c is the Schmidt number, M is the magnetic parameter, K is the permeability parameter, b is the stratification parameter. Other physical variables have their usual meaning.

Introducing the non-dimensional quantities describes above, the governing equations reduce to

$$\frac{\partial u}{\partial t}-(1-b)\frac{\partial u}{\partial y}=G_r(\theta+N_0C)+\frac{\partial^2 u}{\partial y^2}-\left(\frac{M}{(1+m^2)}+\frac{1}{K}\right)u \quad (6)$$

$$P_r\frac{\partial \theta}{\partial t}-(P_r-b)\frac{\partial \theta}{\partial y}=\frac{\partial^2 \theta}{\partial y^2} \quad (7)$$

$$\frac{\partial C}{\partial t}-\frac{\partial C}{\partial y}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} \quad (8)$$

and the corresponding boundary conditions are

$$u=0,\theta=1,C=1 \text{ at } y=0 \quad (9)$$
$$u=0,\theta=0,C=0 \text{ at } y\to\infty$$

Method of solution :

We assume the solution of eq. (6), (7), (8) as

$$u(y,t)=u_0(y)e^{-nt},$$
$$\theta(y,t)=\theta_0(y)e^{-nt},$$
$$C(y,t)=C_0(y)e^{-nt} \quad (10)$$

Using eq.(10) in eq. (6), (7), (8) and we get

$$u_0''+(1-b)u_0'-\left(\frac{M}{(1+m^2)}+\frac{1}{K}-n\right)u_0=-G_r\theta_0-G_rN_0C_0 \quad (11)$$

$$\theta_0''+(P_r-b)\theta_0'+nP_r\theta_0=0 \quad (12)$$

$$C_0''+S_cC_0'+S_cnC_0=0 \quad (13)$$

Now the corresponding boundary conditions are

$$u_0=0,\theta_0=1,C_0=1 \text{ at } y=0$$
$$u_0=0,\theta_0=0,C_0=0 \quad y\to\infty \quad (14)$$

Equations (11) to (13) are ordinary linear differential equations, now u_0, θ_0 and C_0 with boundary conditions (14) are

$$u_0=(A_1+A_2)e^{-m_3y}-A_1e^{-m_1y}-A_2e^{-m_2y} \quad (15)$$

$$\theta_0=e^{-m_1y} \quad (16)$$

$$C_0=e^{-m_2y} \quad (17)$$

where

$$m_1=\frac{(P_r-b)+\sqrt{(P_r-b)^2-4nP_r}}{2}$$

$$m_2=\frac{S_c+\sqrt{S_c^2-4S_cn}}{2}$$

$$m_3=\frac{(1-b)+\sqrt{(1-b)^2+4\left(\frac{M}{(1+m^2)}+\frac{1}{K}-n\right)}}{2}$$

$$A_1=\frac{G_r}{\left[m_1^2-(1-b)m_1-\left(\frac{M}{(1+m^2)}+\frac{1}{K}-n\right)\right]}$$

$$A_2=\frac{G_rN_0}{\left[m_2^2-(1-b)m_2-\left(\frac{M}{(1+m^2)}+\frac{1}{K}-n\right)\right]}$$

Hence, The equations for u, θ and C will be as follows

$$u(y,t) = \left[(A_1 + A_2)e^{-m_3 y} - A_1 e^{-m_1 y} - A_3 e^{-m_2 y}\right]e^{-nt} \quad (18)$$

$$\theta(y,t) = e^{-m_1 y}e^{-nt} \quad (19)$$

$$C(y,t) = e^{-m_2 y}e^{-nt} \quad (20)$$

Skin Friction:

The skin friction coefficient at y = 0 is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left[-m_3(A_1 + A_2) + m_1 A_1 + m_2 A_2\right]e^{-nt} \quad (21)$$

Result and Discussion

Fluid velocity distribution of fluid flow is tabulated in Table 1 and plotted in Fig. 1 having six graphs at P_r=0.71, S_c = 0.4, n = 0.1, t = 0.1, N_0 = 1.5, b = 0.1 for following different value of G_r, M, K and m.

	G_r	M	K	m
For Graph-1	2	0.02	100	0
For Graph-2	2	0.02	100	0.5
For Graph-3	4	0.02	100	0.5
For Graph-4	2	0.04	100	0.5
For Graph-5	2	0.02	1000	0.5
For Graph-6	2	0.02	100	1.0

It is observed from Fig. 1 that all velocity graphs are increasing sharply up to y = 1.2 after that velocity in each graph begins to decrease and tends to zero with the increasing in y. It is also observed from Fig. 1 that that velocity increases with the increase in G_m, K and m, but it decreases with the increase in M.

The skin friction distribution is tabulated in Table 2 and plotted in Fig. 2 having six graphs. It is observed from Fig. 2 that skin friction increases with the increase in G_m, K and m, but it decreases with the increase in M.

The temperature and concentration don't change with the change in above parameters taken for velocity.

Particular case

When m is equal to zero, this problem reduces to the problem of Agrawal *et al.* (2012).

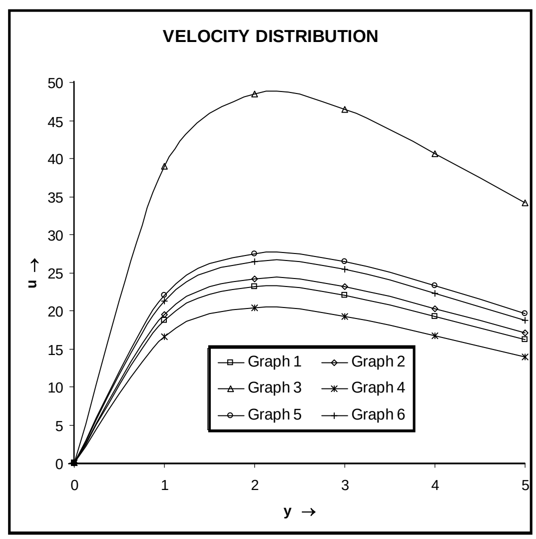

Fig. 1

Table 1. Value of velocity u for Fig. 1 at P_r = 0.71, S_c = 0.4, n = 0.1, t =0.1, N_0 = 1.5, b = 0.1 and different values of G_r, M, K and m

Y	Graph 1	Graph 2	Graph 3	Graph 4	Graph 5	Graph 6
0	0	0	0	0	0	0
1	18.72249	19.52304	39.04607	16.62129	22.05697	21.32877
2	23.13596	24.23104	48.46207	20.35584	27.51786	26.49516
3	22.08269	23.20931	46.41862	19.28923	26.47118	25.39970
4	19.26483	20.30310	40.60620	16.73319	23.23744	22.23729
5	16.16468	17.07113	34.14226	13.98041	19.59241	18.71134

Table 2. Value of skin friction τ for Fig. 2 at P_r = 0.71, S_c = 0.4, n = 0.1, N_0 = 1.5, b = 0.1 and different values of G_r, M, K and m.

t	Graph 1	Graph 2	Graph 3	Graph 4	Graph 5	Graph 6
0	31.54040	32.71600	65.43200	28.30460	36.73990	35.71200
0.2	30.91586	32.06818	64.13636	27.74413	36.01240	35.00486
0.4	30.30368	31.43319	62.86637	27.19476	35.29931	34.31171
0.6	29.70363	30.81077	61.62154	26.65627	34.60033	33.63230
0.8	29.11546	30.20067	60.40135	26.12844	33.91520	32.96633
1	28.53893	29.60266	59.20532	25.61106	33.24364	32.31355

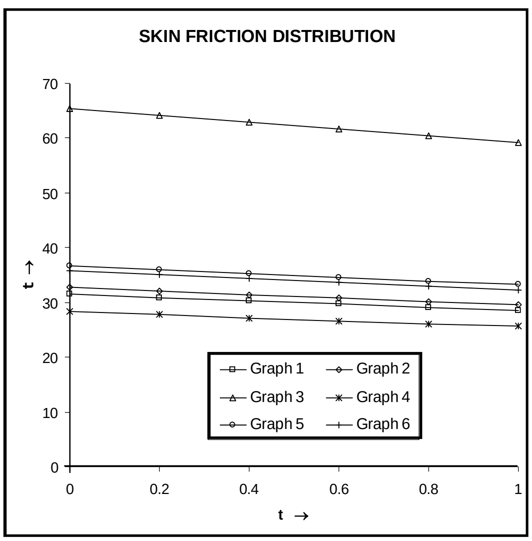

Fig. 2

Conclusion

1. The velocity increases with the increase in m (Hall currents parameter).
2. The skin friction increases with the increase in m.

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