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Effect of Hall current and chemical reaction on MHD flow along a porous flat plate with internal heat absorption/generation

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Abstract

Effect of Hall current on the unsteady free convection flow of a viscous incompressible and electrically conducting fluid in the presence of foreign gasses (such as H_2, CO_2, H_2O, NH_3) along a porous flat plate subject to a transpiration velocity inversely proportional to the square root of time is investigated in the presence of a uniform transverse magnetic field and internal heat absorption/generation. Further, the present study accounts for the 1^{st} order chemical reaction affecting the flow characteristics. The governing equations are solved in closed form applying $Hh_n(x)$ function. The effects of pertinent parameters characterizing the flow field are discussed with the help of graphs and tables.

Keywords : Hall current; Porous plate; Heat absorption/ generation; Chemical reaction.

Introduction

The influence of magnetohydrodynamics heat and mass transfer processes over a moving surface are of interest in engineering and geophysical applications such as thermal insulation, enhanced oil recovery and cooling of nuclear reactor. The effect of Hall current on the unsteady free convection magnetohydrodynamic flow along a vertical flat plate has gained much importance because of its application in magnetohydrodynamic power generators, space flight and nuclear fusion research. When the magnetic field is very strong, one cannot neglect the effect of Hall current. Due to Hall effect of the effect of chemical and thermal diffusion on unsteady hydromagnetic flow near an infinite vertical porous plate has been

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studied by Acharya *et al.*[1]. Dash and Biswal[2] have studied the free convection effect on the flow of an elastico-viscous fluid past an exponentially accelerated vertical plate.

Study of stellar structure at least on the solar surface is connected with mass transfer phenomena. Mass transfer certainly occurs within the mantles and cores of planets of the size of or larger than the Earth. It is therefore, interesting to investigate this phenomenon and study the effect of mass transfer on the free convection flow. Sattar[3] investigated the free convection and mass transfer flow through a porous medium past an infinite vertical porous plate with time dependent temperature and concentration. Dash and Rath[4] have studied the problem of laminar flow and heat transfer of viscous incompressible and electrically conducting fluid between two parallel porous walls experiencing a discontinuous change of wall temperature by explicit finite difference schemes. Flow and heat transfer of an electrically conducting visco-elastic fluid between two horizontal squeezing/stretching plates were reported by Rath *et al.*[5]. In many chemical engineering processes, chemical reaction take place between a foreign mass and the working fluid which moves due to the stretching of a surface. A chemical reaction is said to be of first order and homogeneous if its rate of reaction is directly proportional to the concentration and it occurs as a single phase volume reaction. Laminar natural convection flow over a semi-infinite vertical plate at constant species concentration with chemical reaction was studied by Rahman and Mulolani[6]. Muthucumarswami[7] studied the effects of a homogeneous first-order chemical reaction on a moving isothermal vertical infinitely long surface with suction. This problem is extended through the inclusion of magnetic field and heat generation/ absorption by Chamkha[8]. Afify[9] studied the effects of a chemical reaction on the free convective flow and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching surface. An analytical study of boundary layer flows on a continuous stretching surface were reported by Bararnia *et al.*[10]. MHD flow through a porous medium past a stretched vertical permeable surface in the presence of heat source/sink and a chemical reaction have been studied by Dash *et al.*[11]. Free convective MHD flow through porous media of a rotating oldroyd fluid past an infinite vertical porous plate with heat and mass transfer was studied by Dash *et al.*[12]. Dash and Rath[13] have studied the effect of Hall current on hydrodynamic free convection flow near an exponentially accelerated porous plate with mass transfer. Rath *et al.*[14] have studied the effect of Hall current and chemical reaction on MHD flow along an exponentially accelerated porous flat plate with interval heat absorption/generation.

In the present study, we propose to investigate the effect of Hall current on the unsteady free convection flow of a viscous incompressible and electrically conducting fluid with internal heat absorption/generation and mass transfer in the presence of chemical reaction along a vertical porous flat plate.

Mathematical formulation :

Let us consider an unsteady free convection flow in the presence of a heat source/ sink of a viscous, incompressible and electrically conducting fluid with mass transfer past an infinite vertical porous flat plate subject to a time dependent suction or blowing velocity. The flow is assumed to be along x-axis which is taken along the vertical upward direction while the y-axis is normal to it. It is assumed that the plate is stationary. At time $t > 0$, the temperature and species concentration at the plate are raised to $T_w\ (\neq T_\infty)$ and $C_w\ (\neq C_\infty)$. It is further assumed that the level of species concentration is very low and hence the species thermal diffusion and the diffusion thermal energy effects may be neglected. Also it is assumed that the induced magnetic field is negligible and hence $B=(0, B_0, 0)$. This assumption is justifiable only when the magnetic Reynolds Number is very small. The equation of conservation of electric charge $\nabla.\ J = 0$ gives J_y = constant, where $\vec{J} = (J_x, J_y, J_z)$. Now at the plate since the plate is non-conducting, $J_y = 0$ and hence zero at every where. Assuming the magnetic field strength to be very large, the generalized Ohm's law in absence of electric field takes the form:

$$\vec{J}+\frac{w_e\tau_e}{B_0}\left(\vec{J}\times\vec{B}\right)=\sigma\left(\mu_e\vec{V}\times\vec{B}+\frac{\nabla P_e}{en_e}\right) \qquad (1)$$

where $\vec{V}$ is the velocity, w_e is the electron frequency, τ_e is the electron collision time, μ_e is the magnetic permeability, e is the electron charge, n_e is the number density of the electron, P_e is the electron pressure and σ is the electric conductivity. By assuming the electron pressure for weakly ionized gas, the thermoelectric pressure and ion-slip are negligible, equation (1) gives :

$$J_x = \frac{\sigma\mu_e B_0}{1+m^2}(mu-w)$$

and $$J_z = \frac{\sigma\mu_e B_0}{1+m^2}(u+mw) \qquad (2)$$

where u and w are the x and z components of $\vec{V}$ and $m = w_e\,\tau_e$ is the Hall current parameter.

The equations governing the flow under the Boussinesq's approximation are as follows:

$$\frac{\partial v}{\partial y}=0 \qquad (3)$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\mu_e^2B_0^2}{\rho(1+m^2)}(u+mw)$$
$$+\ g\beta\,(T-T_\infty)+g\beta^*\,(C-C_\infty) \qquad (4)$$

$$\frac{\partial w}{\partial t}+v\frac{\partial w}{\partial y}=v\frac{\partial^2 w}{\partial y^2}-\frac{\sigma\mu_e^2B_0^2}{\rho(1+m^2)}(w-mu) \qquad (5)$$

$$\frac{\partial T}{\partial t}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+S(T-T_\infty) \qquad (6)$$

$$\frac{\partial C}{\partial t}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-K_c(C-C_\infty) \qquad (7)$$

where v is the y-component of the velocity vector.

The initial and the boundary conditions are:

$u\,(y, t) = w\,(y, t) = 0,\ T\,(y, t) = T_\infty,\ C$

$u(y, t)=w(y, t)=0,\ T(y, t)=T_\infty\ C(y,t)=C_\infty$ every where for $t \le 0$

$$\left.\begin{array}{l} u(0, t)=0,\ \ w(0, t)=0,\ T(0, t)=T_w, \\ \quad C(0, t) = C_w \\ u(\infty, t) = w(\infty, t)=0,\ T(\infty, t)=T_\infty, \\ \quad C(\infty, t) = C_\infty \end{array}\right\} \text{for } t > 0 \tag{8}$$

Let us introduce the following dimensionless quantities

$$u^* = \frac{u}{U_0}, v^* = \frac{v}{U_0}, w^* = \frac{w}{U_0}, t^* = \frac{U_0^2 t}{\nu},$$

$$a^* = \frac{a\nu}{U_0^2},$$

$$y^* = \frac{U_0}{\nu} y, T^* = \frac{T - T_\infty}{T_w - T_\infty}, C^* = \frac{C - C_\infty}{C_w - C_\infty},$$

$$, P_r = \frac{\mu c_p}{k}, S_c = \frac{\nu}{D},$$

$$G_r = \frac{g\beta\nu(T_w - T_\infty)}{U_0^3},\ G_c = \frac{g\beta^* \nu(C_w - C_\infty)}{U_0^3},$$

$$M = \frac{\sigma\mu_e^2 B_0^2 \nu}{\rho U_0^2},$$

$$S^* = \frac{S\nu t^*}{U_0^2},\ K_c^* = \frac{K_c V t^*}{U_0^2}$$

where U_0 is some reference velocity, G_r the Grashoff number for heat transfer, G_c the Grashoff number for mass transfer, P_r the Prandtl number, S^* the source/sink parameter and K_c^* the chemical reaction parameter. Now introducing these non-dimensional quantities in equations (3)-(7) and dropping the asterisks for convenience we get the following non-dimensional equations:

$$\frac{\partial v}{\partial y} = 0 \tag{9}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \frac{M}{1+m^2}(u + mw) + G_r T + G_c C \tag{10}$$

$$\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} - \frac{M}{1+m^2}(w - mu) \tag{11}$$

$$\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} + \frac{ST}{t} \tag{12}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} - \frac{K_c C}{t} \tag{13}$$

The corresponding initial and boundary conditions are now given by:

$$u(y,t) = 0,\ w(y,t) = 0,\ T(y,t) = 0,\ C(y,t) = 0 \quad \forall\ y \text{ for } t \le 0$$

$$\left.\begin{array}{l} u(0,t)=0,\ w(0,t)=0,\ T(0,t) = 1, \\ \quad C(0.t) = 1 \\ u(\infty,t) = 0,\ w(\infty,t)=0,\ T(\infty,t)=0, \\ \quad C(\infty,t) = 0 \end{array}\right\} \text{for } t > 0 \tag{14}$$

From the equation (9) it follows that the function v is either constant or a function of time t. Similarity solutions of the equations (10) – (13) with the boundary conditions (14) exist only if we take

$$v = -\alpha t^{-1/2} \tag{15}$$

where α is a non-dimensional transpiration parameter and for suction $\alpha > 0$ and $\alpha < 0$ for injection or blowing. This assumption is valid only for small values of *t*. Now solving equations (10) and (11), assuming the factor *Mt* to be small, by expanding the variables *u* and *w* in powers of *Mt* as :

$$u(\eta)= t\sum_{i=0}^{\infty}(Mt)^i f_i (\eta),\; w(\eta)= t\sum_{i=0}^{\infty}(Mt)^i g_i (\eta) \qquad (16)$$

$$\text{where } \eta = \frac{y}{2\sqrt{t}} \qquad (17)$$

is the similarity variable.

Now substituting equations (16) and (17) in equations (10) to (13) and then equating the coefficients of equal powers of *Mt* we get

$$f_0'' + 2(\eta+\alpha) f_0' - 4 f_0 = -4G_r T - 4G_c C, \qquad (18)$$

$$f_1'' + 2(\eta+\alpha) f_1' - 8 f_1 = \frac{4}{1+m^2}(f_0 + mg_0), \qquad (19)$$

$$f_2'' + 2(\eta+\alpha) f_2' - 12 f_2 = \frac{4}{1+m^2}(f_1 + mg_1), \qquad (20)$$

$$g_0'' + 2(\eta+\alpha) g_0' - 4 g_0 = 0, \qquad (21)$$

$$g_1'' + 2(\eta+\alpha) g_1' - 8 g_1 = \frac{4}{1+m^2}(g_0 - mf_0), \qquad (22)$$

$$g_2'' + 2(\eta+\alpha) g_2' - 12 g_2 = \frac{4}{1+m^2}(g_1 - mf_1) \qquad (23)$$

$$T'' + 2P_r(\eta+\alpha)T' + 4P_r ST = 0 \qquad (24)$$

and

$$C'' + 2S_c(\eta+\alpha)C' - 4S_c K_c T = 0 \qquad (25)$$

where primes denote differentiation with respect to η. The corresponding boundary conditions are as follows:

$$f_0(0) = 0,\; f_i(0) = 0,\; g_{i-1}(0) = 0,\; f_{i-1}(\infty) = 0,\; g_{i-1}(\infty) = 0,\; (i = 1, 2,3\ldots.) \qquad (26)$$

Let us consider the function

$$Hh_n(x)= \begin{cases} \displaystyle\int_x^{\infty} \frac{(u-x)^n}{n!} e^{-u^2/2} du & n=0,1,2\ldots \\ (-1)^{n-1}\left(\frac{d}{dx}\right)^{-n-1} e^{-x^2/2}, & n=0,1,2\ldots \end{cases} \qquad (27)$$

which satisfies the equation

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} - ny = 0 \qquad (28)$$

where $n = \ldots\ldots -2, -1, 0, 1, 2 \ldots.$

Solution for heat absorption with chemical reaction (S = –1, K_c = 1), $P_r \neq 1$, $S_c \neq 1$:

Solving equations (18)- (25) for $S = -1, K_c=1$ subject to the boundary conditions (14) and (26), we get

$$T(\eta)= \frac{Hh_2\left(\sqrt{2P_r}(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2P_r}\,\alpha\right)} \qquad (29)$$

$$C(\eta)= \frac{Hh_2\left(\sqrt{2S_c}(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2S_c}\,\alpha\right)} \qquad (30)$$

$$f_0(\eta)=\frac{Hh_2\left(\sqrt{2}(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2}\alpha\right)}\left[\frac{2G_r}{(P_r-1)}\frac{Hh_2\left(\sqrt{2P_r}\alpha\right)}{Hh_0\left(\sqrt{2P_r}\alpha\right)}+\frac{2G_c}{(S_c-1)}\frac{Hh_2\left(\sqrt{2S_c}\alpha\right)}{Hh_0\left(\sqrt{2S_c}\alpha\right)}\right]$$

$$-\frac{2G_r}{(P_r-1)}\frac{\left[Hh_2\left(\sqrt{2P_r}(\eta+\alpha)\right)\right]^2}{Hh_2\left(\sqrt{2P_r}\alpha\right).Hh_0\left(\sqrt{2P_r}(\eta+\alpha)\right)}-\frac{2G_c}{(S_c-1)}\frac{\left[Hh_2\left(\sqrt{2S_c}(\eta+\alpha)\right)\right]^2}{Hh_2\left(\sqrt{2S_c}\alpha\right)Hh_0\left(\sqrt{2S_c}(\eta+\alpha)\right)} \tag{31}$$

$$g_0(\eta)=0 \tag{32}$$

$$f_1(\eta)=\frac{4G_r}{(1+m^2)(P_r-1)^2}\left[\frac{Hh_2\left(\sqrt{2}(\eta+\alpha)\right)Hh_2\left(\sqrt{2P_r}\alpha\right)Hh_4\left(\sqrt{2P_r}(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2}\alpha\right)Hh_0\left(\sqrt{2P_r}\alpha\right)Hh_2\left(\sqrt{2P_r}(\eta+\alpha)\right)}\right.$$

$$\left.-\frac{Hh_4\left(\sqrt{2P_r}(\eta+\alpha)\right)Hh_2\left(\sqrt{2P_r}(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2P_r}\alpha\right).Hh_0\left(\sqrt{2P_r}(\eta+\alpha)\right)}\right]$$

$$+\frac{4G_c}{(1+m^2)(S_c-1)^2}\left[\frac{Hh_2\left(\sqrt{2}(\eta+\alpha)\right)Hh_2\left(\sqrt{2S_c}\alpha\right)Hh_4\left(\sqrt{2S_c}(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2}\alpha\right)Hh_0\left(\sqrt{2S_c}\alpha\right)Hh_2\left(\sqrt{2S_c}(\eta+\alpha)\right)}\right.$$

$$\left.-\frac{Hh_4\left(\sqrt{2S_c}(\eta+\alpha)\right)Hh_2\left(\sqrt{2S_c}(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2S_c}\alpha\right)Hh_0\left(\sqrt{2S_c}(\eta+\alpha)\right).}\right] \tag{33}$$

$$g_1(\eta)=-mf_1(\eta) \tag{34}$$

$$f_2(\eta)=\frac{8(1-m^2)G_r}{(1+m^2)^2(P_r-1)^3}\left[\frac{Hh_2\left(\sqrt{2}(\eta+\alpha)\right)Hh_2\left(\sqrt{2P_r}\alpha\right)Hh_6\left(\sqrt{2P_r}(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2}\alpha\right)Hh_0\left(\sqrt{2P_r}\alpha\right)Hh_2\left(\sqrt{2P_r}(\eta+\alpha)\right)}\right.$$

$$\left.-\frac{Hh_2\left(\sqrt{2P_r}(\eta+\alpha)\right)Hh_6\left(\sqrt{2P_r}(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2P_r}\alpha\right).Hh_0\left(\sqrt{2P_r}(\eta+\alpha)\right)}\right]$$

$$+\frac{8(1-m^2)G_c}{(1+m^2)^2(S_c-1)^3}\left[\frac{Hh_2\left(\sqrt{2}(\eta+\alpha)\right)Hh_2\left(\sqrt{2S_c}\alpha\right)Hh_6\left(\sqrt{2S_c}(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2}\alpha\right)Hh_0\left(\sqrt{2S_c}\alpha\right)Hh_2\left(\sqrt{2S_c}(\eta+\alpha)\right)}\right.$$

$$-\frac{Hh_6\left(\sqrt{2S_c}\,(\eta+\alpha)\right)Hh_2\left(\sqrt{2S_c}\,(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2S_c}\,\alpha\right)Hh_0\left(\sqrt{2S_c}\,(\eta+\alpha)\right).}\Bigg] \tag{35}$$

$$g_2\,(\eta) = -\frac{2m}{(1-m^2)}\,f_2(\eta) \tag{36}$$

Solution for heat generation with chemical reaction (S=1, K_c=1), $P_r \neq 1$, $S_c \neq 1$:

Now solving equations (18) – (25) for $S = 1, K_c = 1$ subject to the boundary conditions (14) and (26), we get

$$T(\eta) = \frac{Hh_{-2}\left(\sqrt{2P_r}\,(\eta+\alpha)\right)}{Hh_{-2}\left(\sqrt{2P_r}\,\alpha\right)} \tag{37}$$

The value of $C(\eta)$ is same as in equation (30)

$$f_0(\eta) = \frac{Hh_2\left(\sqrt{2}(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2}\alpha\right)}\left[\frac{2G_r}{(P_r-1)}\frac{Hh_2\left(\sqrt{2P_r}\,\alpha\right)}{Hh_0\left(\sqrt{2P_r}\,\alpha\right)}+\frac{2G_c}{(S_c-1)}\frac{Hh_2\left(\sqrt{2S_c}\,\alpha\right)}{Hh_0\left(\sqrt{2S_c}\,\alpha\right)}\right]$$

$$-\frac{2G_r}{(Pr-1)}\frac{Hh_{-2}\left(\sqrt{2P_r}\,(\eta+\alpha)\right)Hh_2\left(\sqrt{2P_r}\,(\eta+\alpha)\right)}{Hh_{-2}\left(\sqrt{2P_r}\,\alpha\right).Hh_0\left(\sqrt{2P_r}\,(\eta+\alpha)\right)}$$

$$-\frac{2G_c}{(S_c-1)}\frac{Hh_{-2}\left(\sqrt{2S_c}\,(\eta+\alpha)\right)Hh_2\left(\sqrt{2S_c}\,(\eta+\alpha)\right)}{Hh_{-2}\left(\sqrt{2S_c}\,\alpha\right)Hh_0\left(\sqrt{2S_c}\,(\eta+\alpha)\right)} \tag{38}$$

$$g_0\,(\eta) = 0 \tag{39}$$

$$f_1(\eta) = \frac{4G_r}{(1+m^2)(P_r-1)^2}\left[\frac{Hh_2\left(\sqrt{2}(\eta+\alpha)\right)Hh_2\left(\sqrt{2P_r}\,\alpha\right)Hh_4\left(\sqrt{2P_r}\,(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2}\alpha\right)Hh_0\left(\sqrt{2P_r}\,\alpha\right)Hh_2\left(\sqrt{2P_r}\,(\eta+\alpha)\right)}\right.$$

$$\left.-\frac{Hh_{-2}\left(\sqrt{2P_r}\,(\eta+\alpha)\right)Hh_4\left(\sqrt{2P_r}\,(\eta+\alpha)\right)}{Hh_{-2}\left(\sqrt{2P_r}\,\alpha\right).Hh_0\left(\sqrt{2P_r}\,(\eta+\alpha)\right)}\right]$$

$$+\frac{4G_c}{(1+m^2)(S_c-1)^2}\left[\frac{Hh_2\left(\sqrt{2}(\eta+\alpha)\right)Hh_2\left(\sqrt{2S_c}\,\alpha\right)Hh_4\left(\sqrt{2S_c}\,(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2}\alpha\right)Hh_0\left(\sqrt{2S_c}\,\alpha\right)Hh_2\left(\sqrt{2S_c}\,(\eta+\alpha)\right)}\right.$$

$$-\frac{Hh_{-2}\left(\sqrt{2S_c}(\eta+\alpha)\right)Hh_4\left(\sqrt{2S_c}(\eta+\alpha)\right)}{Hh_{-2}\left(\sqrt{2S_c}\,\alpha\right)Hh_0\left(\sqrt{2S_c}(\eta+\alpha)\right).}\Bigg] \tag{40}$$

$$g_1(\eta) = -mf_1(\eta) \tag{41}$$

$$f_2(\eta)=\frac{8(1-m^2)}{\left(1+m^2\right)^2}\Bigg[\frac{G_r}{(P_r-1)^3}\Bigg\{\frac{Hh_2\left(\sqrt{2}(\eta+\alpha)\right)Hh_2\left(\sqrt{2P_r}(\eta+\alpha)\right)Hh_6\left(\sqrt{2P_r}(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2}\alpha\right)Hh_0\left(\sqrt{2P_r}\,\alpha\right)Hh_2\left(\sqrt{2P_r}(\eta+\alpha)\right)}$$

$$-\frac{Hh_{-2}\left(\sqrt{2P_r}(\eta+\alpha)\right)Hh_6\left(\sqrt{2P_r}(\eta+\alpha)\right)}{Hh_{-2}\left(\sqrt{2P_r}\,\alpha\right).Hh_0\left(\sqrt{2P_r}(\eta+\alpha)\right)}\Bigg\}$$

$$+\frac{G_c}{(S_c-1)^3}\Bigg\{\frac{Hh_2\left(\sqrt{2}(\eta+\alpha)\right)Hh_2\left(\sqrt{2S_c}(\eta+\alpha)\right)Hh_6\left(\sqrt{2S_c}(\eta+\alpha)\right)}{Hh_2\left(\sqrt{2}\alpha\right)Hh_0\left(\sqrt{2S_c}\,\alpha\right)Hh_2\left(\sqrt{2S_c}(\eta+\alpha)\right)}$$

$$-\frac{Hh_{-2}\left(\sqrt{2S_c}(\eta+\alpha)\right)Hh_6\left(\sqrt{2S_c}(\eta+\alpha)\right)}{Hh_{-2}\left(\sqrt{2S_c}\,\alpha\right)Hh_0\left(\sqrt{2S_c}(\eta+\alpha)\right).}\Bigg\}\Bigg] \tag{42}$$

$$g_2(\eta) = -\frac{2m}{(1-m^2)}f_2(\eta) \tag{43}$$

Equation (16) along with equations (31) – (36) give the required expressions for the primary and secondary velocity components u and w in case of heat absorption respectively. Similarly equation (16) along with equations (38) – (43) give the expressions for the primary and secondary components of velocity in case of heat generation.

The x-component skin-friction at the surface of the plate for heat absorption (τ_{ax}) and heat generation (τ_{gx}) is given by

$$\tau_{ax} or\ \tau_{gx} = t\left[f_0'(0)+(Mt)f_1'(0)+(Mt)^2 f_2'(0)\right] \tag{44}$$

and the z - component is given by

$$\tau_{az} \text{ or } \tau_{gz} = Mt^2\left[g_1'(0)+(Mt)g_2'(0)\right] \tag{45}$$

The expressions for $f_0'(0)$, $f_1'(0)$, $f_2'(0)$, $g_1'(0)$ and $g_2'(0)$ can be obtained from the expressions (31)- (36) for heat absorption and (38) – (43) for heat generation separately.

Results and Discussion

The governing equations characterizing the flow, heat and mass transfer phenomena with the given boundary conditions admit similarity solution with the assumption $v = -\alpha t^{-\frac{1}{2}}$, where α is the transpiration parameter representing both suction and blowing. This assumption is valid for small values of t and hence a perturbation method is suggested by expanding the velocity component u and w in powers of Mt. Thus the governing equations are reduced to a set of ordinary differential equation whose solutions are obtained in terms of hyper geometric function $Hh_n(x)$. The results are discussed with the effect of the parameters m the Hall current, Mt the hydromagnetic parameter, G_r the Grashoff number for heat transfer, G_c the Grashoff number for mass transfer, S the internal heat absorption/ generation parameter, α the transpiration parameter, S_c the Schmidt parameter and K_c the chemical reaction parameter for Prandtl number $P_r = 0.71$ which represents air. While computing the velocity function, values of prandtl number P_r and time tare taken for convenience as $P_r = 0.71$ corresponds to air at 20°C and $t=0.2$. For concentration distribution, the values of Schmidt number are chosen in such a way that those represent diffusing species of common interest in air due to the presence of H_2,CO_2,H_2O and NH_3 with S_c=0.22,0.30,0.60 and 0.78 respectively.

Fig. 1 and Fig. 2 depict the primary velocity distribution with and without chemical reaction respectively. In case of heat absorption, the effect of suction ($(\alpha > 0)$ and injection $(\alpha > 0)$, curves(curve I, $\alpha = -0.5$ and curve II, $\alpha = 0.5$) have been examined and observed that the velocity increases in case of suction. Comparing the curve II (G_r=1.0) with curve III (G_r=0.0) it is interesting to observe that in the absence of thermal buoyancy force , the velocity decreases in the presence of destructive chemical reaction($K_c > 0$). Further, it is to note from the curves I,IV,X and XI that appearance of backflow is due to injection of the fluid ($\alpha = -0.5$), absence of mass buoyancy (G_c=0.0) and with higher values of S_c (S_c= 0.60 and 0.78). Therefore, injection gives rise to backflow while suction prevents it, which is an established result for controlling the flow reversal. Further, the injection of heavier species is not desirable as it gives rise to backflow. This result is in good agreement with the earlier work, reported in literature.

Now, the effect of modified Grashoff number G_c is also to increase the primary velocity at all points (curve II,IV,V). Thus it is concluded that the presence of foreign species is to enhance the fluid motion. Another important physical parameter is the Hall current (m). It also increases the primary velocity at all points (curve II,VI,VII) but the reverse effect is observed in case of magnetohydro-dynamic parameter Mt and Schmidt number S_c.

Hence, the Lorentz force due to transverse magnetic field decelerates the fluid particles as the ponder motive force acts in the opposite direction of the flow, which is also an established fact. Moreover, it is concluded that heavier species decreases the primary velocity.

Now, we shall discuss the effect of heat generation on primary velocity with and without chemical reaction. It is seen that the effect of all the parameters discussed above are same for both the cases *i.e.* with and without chemical reaction.

Now, Fig. 3 shows the secondary velocity distribution in the presence of chemical reaction . It is remarkable to note that variation of velocity is confined within a few layers of fluid resulting in a thin boundary layer in case of heat absorption but thickness of the boundary layer increases substantially in case of heat generation. Due to diffusion of more thermal energy into the fluid layer velocity increases as a result of which attainment of free stream velocity is delayed span wise. One most striking feature of the velocity profile is that the presence of injection ($\alpha<0$) gives rise to backflow with maximum negative value which asserts that the suction prevents it. The back flow is also experienced due to the heavier species and in the absence of mass bouncy.

Now Fig. 4 shows the case of without chemical reaction, comparing with Fig 5. we arrive at the following results. Presence of heavier species causes no back flow. Effect of injection causes back flow irrespective of the presence/absence of chemical reaction. So is the case of absence of mass bouncy. Hence it may be concluded that concentration difference gives rise to a bouncy force which prevents back flow.

The temperature distribution in the presence of a suction ($\alpha > 0$) and injection ($\alpha < 0$) are shown in fig 5. In case of heat absorption (continuous line), smooth variation in temperature distribution is observed which leads to thinning of thermal boundary layer, where as high fluctuation is marked with negative values in case of blowing ($\alpha=-0.5$) for heat generation (dotted line). Hence it may concluded that heat generation combined with blowing leads to sharp fall in temperature.

The concentration variation with and without chemical reaction are shown in Figs. 6 and 7 for different diffusing species such as H_2, CO_2, H_2O *and* NH_3 having Schmidt number 0.22,0.30,0.60 and 0.78 respectively. Comparing the Fig. 6 $(K_C \neq 0)$ with Fig. 7 $(K_C = 0)$ it is seen that concentration variation attains a constant value earlier in the presence of chemical reaction.

From application point of view the effect of physical parameters (G_r=0.0) on skin friction is very useful. Therefore *x*-component and *z*-component of skin friction for heat absorption/ generation on the surfaces are calculated and presented in tables 1 and 2.

Table 1 presents the skin friction τ_{ax} *and* τ_{az} for heat absorption. In case of suction ($\alpha > 0$), *x* and *z*-component of skin friction τ_{ax} *and* τ_{az} decrease with an increase in G_r,*m* *and* S_c but reverse effect is observed in case of *Mt* and G_c. The reduction of skin friction is desirable in view of application. In case of injection ($\alpha < 0$) τ_{ax} *and* τ_{az} decrease as G_r, G_c, S_c and *m* increases but reverse effect is observed in case of *Mt*.

Table 2 presents the skin friction τ_{gs} *and* τ_{gz} for heat generation. In case of injection

Table 1. Values of skin friction τ_{ax} and τ_{az} are entered in the upper and lower lines respectively for different values of G_r, G_c, m, Mt and S_c when P_r =0.71, t=0.1, K_c=1.0 in case of heat absorption

G_r	G_c	m	Mt	S_c	$\alpha = -0.5$		$\alpha = 0.5$	
					τ_{ax}	τ_{az}	τ_{ax}	τ_{az}
0	2	0.2	0.2	0.22	-0.1237	-0.0023	0.0498	0.0009
1	2	0.2	0.2	0.22	-0.7286	-0.0187	-0.3243	-0.0037
1	0	0.2	0.2	-	-0.6849	-0.0164	-0.4394	-0.0046
1	3	0.2	0.2	0.22	-0.7505	-0.0199	-0.2667	-0.0033
1	2	0.4	0.2	0.22	-0.7339	-0.0377	-0.3258	-0.0071
1	2	0.2	0.5	0.22	-0.7028	0.0374	-0.3065	-0.0018
1	2	0.2	0.2	0.30	-0.8218	-0.0226	-0.4041	-0.0063
1	2	0.2	0.2	0.60	-1.2914	-0.0604	-0.7355	-0.0324
1	2	0.2	0.2	0.78	-1.7991	-0.1172	-0.9527	-0.1241

Table 2. Values of skin friction τ_{ax} and τ_{az} are entered in the upper and lower lines respectively for different values of G_r, $G_{c,}$,m, Mt and S_c when P_r =0.71, t=0.1, K_c=1.0 in case of heat generation.

G_r	G_c	m	Mt	S_c	$\alpha = -0.5$		$\alpha = 0.5$	
					τ_{ax}	τ_{az}	τ_{ax}	τ_{az}
0	2	0.2	0.2	0.22	0.8103	-0.0541	-0.0112	0.1292
1	2	0.2	0.2	0.22	1.1556	-0.0651	0.7976	0.1492
1	0	0.2	0.2	-	0.3559	-0.0110	0.8054	0.0200
1	3	0.2	0.2	0.22	1.5555	-0.0921	0.7936	0.2138
1	2	0.4	0.2	0.22	1.1251	-0.1196	0.8723	0.2698
1	2	0.2	0.5	0.22	1.5623	-0.1029	-0.2578	0.3263
1	2	0.2	0.2	0.30	1.1656	-0.0532	0.8138	0.1560
1	2	0.2	0.2	0.60	1.3591	0.0156	0.5414	0.2880
1	2	0.2	0.2	0.78	1.1582	0.2891	-0.9029	0.7167

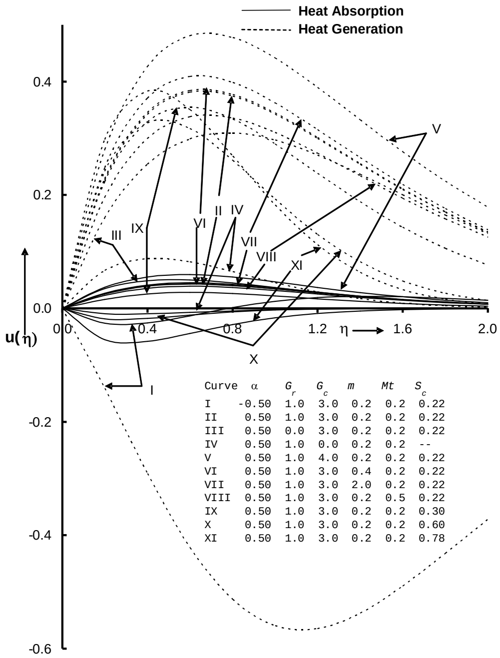

Fig. 1. Primary velocity component *u* for different values of α, G_r, G_c, *m*, *Mt* and S_c when *Pr* = 0.71, *t* = 0.2 and *Kc* = 1.0

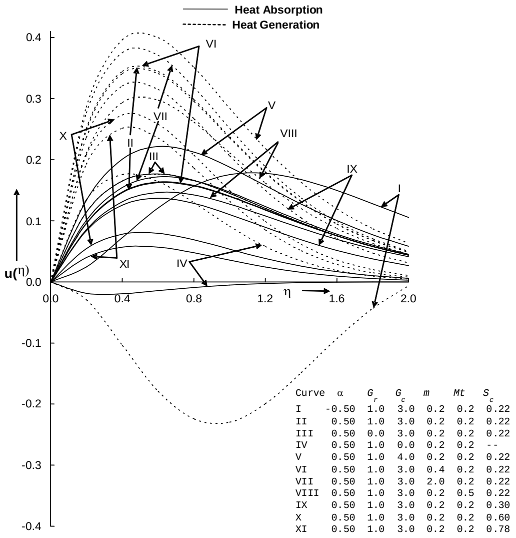

Fig. 2. Primary velocity component *u* for different values of α, G_r, G_c, *m*, *Mt* and *Sc* when *Pr*=0.71, *t*=0.2 and K_c=0.0

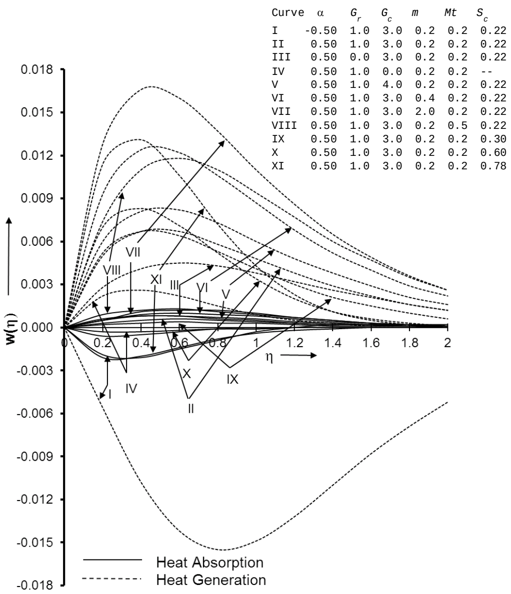

Fig. 3. Secondary velocity component *w* for different values of α, *Gr*, *Gc*, *m*, *Mt* and S_c when *Pr*=0.71, *t*=0.2 and *Kc*=1.0

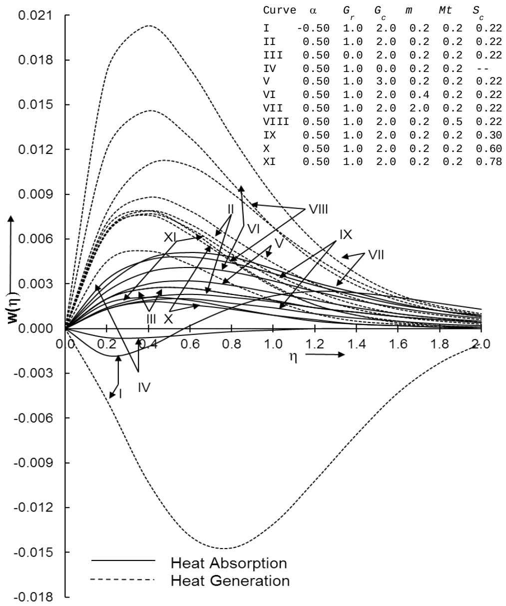

Fig. 4. Secondary velocity component *w* for different values of α, *Gr*, G_c, *m*, *Mt* and S_c when P_r=0.71, *t*=0.2 and K_c=0.0

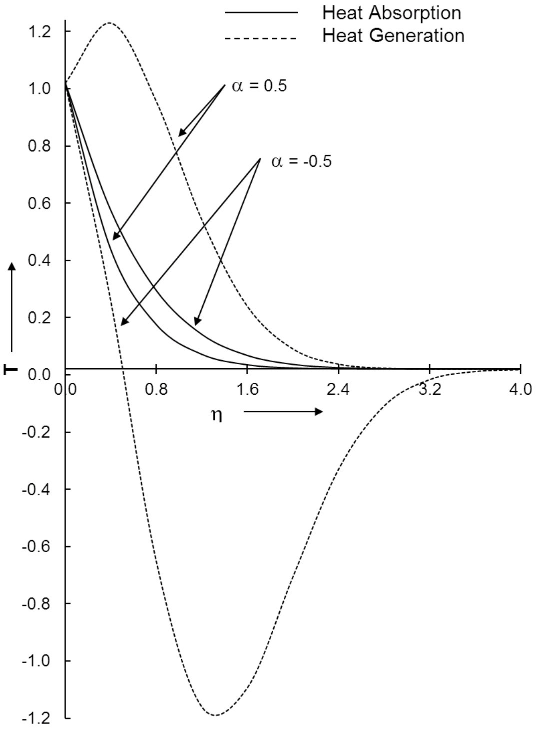

Fig. 5. Temperature profiles against η, $P_r = 0.71$

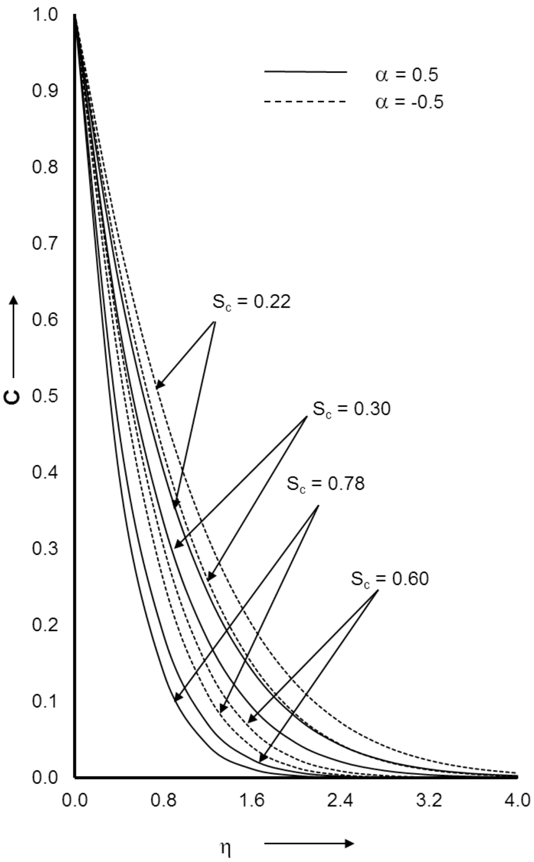

Fig. 6. Concentration profiles against η (with Chemical reaction)

($\alpha < 0$) x-component of skin friction (τ_{gx}) increases with an increase in the values of G_r, G_c, Mt and S_c but reverse effect is observed in case of m. Similarly, z-component of skin friction (τ_{gz}) decreases with an increase in G_r, G_c, m and Mt but (τ_{gz}) increases when S_c increases. In case of suction ($\alpha > 0$) τ_{gx} increases when G_r and m increases but reverse effect is observed incase of G_c, Mt and S_c. The z-component of skin friction increases with the increase of G_r, G_c, m, Mt and S_c.

4. Conclusion

From the present study the following conclusions are made.

(i) Presence of suction in a fluid with high diffusivity prevents back flow.
(ii) Presence of chemical reaction coupled with heat generation enhances/reduces secondary velocity by 40%.
(iii) Presence of injection gives rise to back flow with maximum magnitude. So injection is not suitable for the present physical situation.

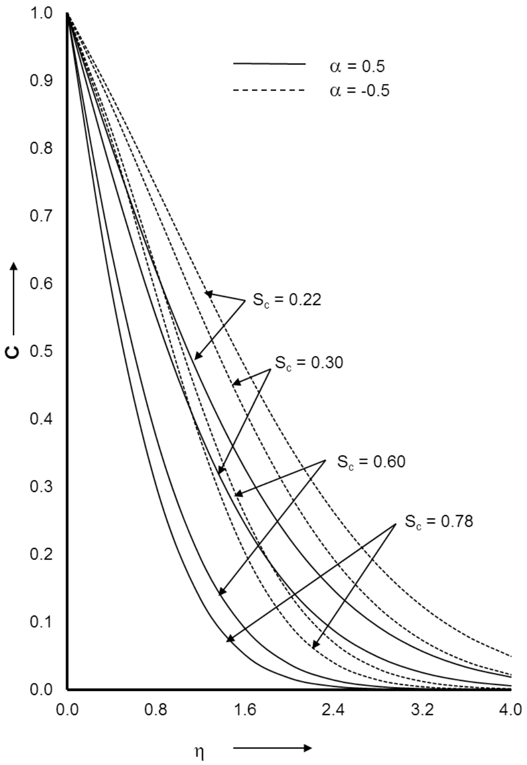

Fig. 7. Concentration profiles against η (without Chemical reaction)

(iv) Presence of heavier species prevents backflow.

(v) Effect of injection causes backflow irrespective of the presence/absence of chemical reaction.

(vi) Heat generation combined with blowing leads to a sharp fall of temperature.

(vii) Flow stability is increased in present study with suction.

(viii) Heat absorption reduces the fluctuation of velocity profiles.

(ix) On set of Hall current accelerates the primary velocity.

(x) Chemical reaction coupled with thermal buoyancy force decelerated the primary velocity.

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