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Entropy of Non-static spacetimes with horizons

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Abstract

We present the entropy of non-static spacetimes with horizons and associate temperature β^1 which satisfies $S=\frac{1}{2}\beta E$ where E as the source of gravitational acceleration a^i as a integral of $\left(T_{ab}-\frac{1}{2}Tg_{as}\right)u^a u^b$. Hence, one may define the free energy F with $\beta^F=\beta U-S.$

1. Introduction

In view of Birrell[1] and review by Padmanabhan[2], it is obvious that spacetimes with horizons bear a similarity with thermodynamic systems. Several class of static or stationary horizons presented in literature provides to a periodicity in the Euclidean time coordinate, which provides motivation to introduce a temperature with the horizon. The temperature T is connected to the magnitude of the surface gravity k of the horizon as $T=K/2\pi$ in certain situations. Bekenstein[4] associated entropy to the black hole horizons before temperature was associated. As horizons black information, it appears reasonable to associate an entropy with the horizons. Though in general horizons are observer dependents and entropy be an observer-independent, which creates problems. Let us attempt to associate thermodynamical variables such as entropy, free energy etc. with a static horizon. Padmanabhan[3] presented a scheme by giving a general ansatz for gravitational entropy to associate an entropy which is proportional to the horizon area for accelerated observer. The author, for any static spacetime with a horizon associated temperature β^{-1} with entropy S, presented a relation $S=\frac{1}{2}\beta E$ where E be the energy source for gravitational acceleration as the integral of $\left(T_{ab}-\frac{1}{2}g_{ab}T\right)u^a u^b$ and

$\beta F = \beta U - S$ where F as free energy and U as the integral of $T_{ab}u^a u^b$ and showed that $S\alpha E^2$ and/or $S\alpha U^2$, thereby generalised the results known for black holes.

In this paper we have attempted to associate an entropy to any horizon *i.e.* an entropy of non-static spacetimes by giving a suitable mathematical expression for entropy. Hence, we have associated other thermodynamical variables such as free energy F and temperature β^{-1}.

2. *The entropy of non-static Spacetimes :*

Let us consider metric of the form

$$ds^2 = -\left(1 - \frac{2m(v,r)}{r}\right)dv^2 + 2\in dvdr + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \qquad (1)$$

with $\in = \pm 1$, the energy momentum tensor satisfying the conditions $T_r^v = 0$ and $T_\theta^\theta = kT_r^r$ (k as a constant) and

$$m(v,r) = \begin{cases} M(v) + \frac{1}{6}\Lambda r^3 & if \quad C(v) = 0 \\ M(v) - \frac{8\pi C(v)}{2}\frac{1r^{2k+1}}{(2k+1)} + \frac{1}{6}\Lambda r^3 & if \quad C(v) \neq 0 \text{and}\, k = -1/2 \\ M(v) - \frac{8\pi C(v)}{2}\ell nr + \frac{1}{6}\Lambda r^3 & if \quad C(v) \neq 0 \text{ and}\, k = -1/2 \end{cases} \qquad (2)$$

$$T_b^a = \frac{C(v)}{r^{2(1-k)}} diag[1,1,k,k] - \frac{\Lambda}{8\pi} diag[1,1,1,1] \qquad (3)$$

and

$$T_v^r = \begin{cases} \frac{1}{8\pi r^2} 2\frac{\partial M}{\partial v} - \frac{1}{2k+1}\frac{\partial C}{dv} r^{2k-1} \; if \;\; k \neq -1/2 \\ \frac{1}{8\pi r^2} 2\frac{\partial M}{\partial v} - \frac{1}{r^2}\frac{\partial C}{\partial v}\ell nr \; if \;\; k = -1/2 \end{cases}. \qquad (4)$$

where $M(v)$, $C(v)$, are the constants, and Λ as cosmological constant. In view of York (1984), a null vector decomposition for the metric (1), assumes the form

$$g_{ab} = -n_a \ell_b - \ell_a n_b + \gamma_{ab} \qquad (5)$$

with

$$\begin{cases} n_a = \delta_a^v, \ell_a = \frac{1}{2}\left[1 - \frac{2m(v,r)}{r}\right]\delta_a^v + \delta_a^r \\ \gamma_{ab} = r^2 \delta_a^\theta \delta_b^\theta + r^2 \sin^2\theta \delta_a^\theta \delta_b^\theta \\ \ell_a \ell^a = n_a n^a = 0, \ell_a n^a = -1 \\ \ell^a \gamma_{ab} = 0, \gamma_{ab} n^b = 0. \end{cases} \qquad (6)$$

Again Raychaudhuri equation reads

$$\frac{d\theta}{dv} = K\theta - R_{ab}\ell^a \ell^b - \frac{1}{2}\theta^2 - \sigma_{ab}\sigma^{ab} + \omega_{ab}\omega^{ab} \qquad (7)$$

where θ as expansion, ω twist, σ shear and K as surface gravity. The expansion of null rays read

$$\theta = \nabla_a \ell^a - K, \qquad (8)$$

and surface gravity K as

$$K = -n^a \ell^a \nabla_b \ell_a. \qquad (9)$$

Now apparent horizons are as surfaces such that $\theta \simeq 0$ and event horizons are surfaces such that $\frac{d\theta}{dv} \simeq 0$. Hence for $k \neq -1/2$, one obtains

$$\theta = \frac{1}{r}\left[1 - \frac{2M(v)}{r} + Q^2(v) r^{2k-1} - \chi^2 r^2\right] \qquad (10)$$

where

$$Q^2(v) = \frac{8\pi C(v)}{2k+1}, \chi^2 = \frac{\Lambda}{3}. \qquad (11)$$

Therefore apparent horizons satisfy $(\theta \simeq 0)$

$$\chi^2 r^3 - Q^2 r^{2k+1} - r + 2M(v) = 0, \qquad (12)$$

which has two positive solutions.

For $\chi^2 = Q^2 = 0$, one obtains Schwarzschild horizon r = 2M and for $Q^2 = M = 0$, we obtain de Sitter horizon $r = \frac{1}{\chi}$. For general k, eq. (12) does not admit simple closed form solutions. However, for

$$Q^2 = Q_c^2 = \frac{-1}{2k+1}\left(\frac{2M(2k+1)}{2k-1}\right)^{-2k} \qquad (13)$$

with $\chi^2 = 0$, two roots coincide and there is only one horizon

$$r = \left(\frac{2M(2k+1)}{2k-1}\right) \qquad (14)$$

for $Q^2 \lesssim Q_c^2$, one obtains two horizons, namely a cosmological horizon and black hole horizon, and for $Q^2 > Q_c^2$, no horizons are formed.

Hence, one may use, the definition of entropy as

$$S = \frac{1}{8\pi G}\int \sqrt{g} d^4 x \nabla_i \left(a^i + K u^i\right) \quad (15)$$

If one assumes

$$\beta E = 2\int d^4 x \sqrt{-g}\left[\left(T_{ab} - \tfrac{1}{2} T g_{ab}\right) u^a u^b + \left({}^3R - 16\pi G T_{ab} u^a u^b\right)\right] \quad (16)$$

we obtain

$$S = \frac{1}{2}\beta E \quad (17)$$

such that

$$U = \frac{1}{8\pi G}\int d^4 x \sqrt{-g}\left({}^3R - 8\pi G T_{ab} u^a u^b\right) \quad (18)$$

$$= \frac{1}{16\pi G}\int d^4 x \sqrt{-g}\left({}^3R + K_{ab} K^{ab} - K^2\right).$$

Hence, one may define

$$\beta F = \beta U - S.$$

Under these conditions we get

$$S \alpha E^2$$

and/or

$$S \alpha U^2,$$

which generalises the known results of black hole.

3. Concluding Remarks

We have presented that in non-static spacetime with horizon with temperature β^{-1}, the entropy reads $U = \frac{1}{2}\beta E$ where E as the source for gravitational acceleration a^i obtained as the integral of $\left(T_{ab} - \frac{1}{2} T g_{ab}\right) u^a u^b$ and free energy as $\beta F = \beta U - S$ where U as the integral of $T_{ab} u^a u^b$. Hence, one obtains $S \alpha E^2$ and/or $S \alpha U^2$.

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