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Products of Modified Multivariable H-Function and General class of Polynomial on a Boundary Value Problem and its Solution

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Abstract

In this paper, we construct a model of a boundary value problem and then obtain its solution involving Product General Class of Polynomials and Modified Multivariable H-function.

Introduction

The operational techniques[1-3] are important tools to solve various problems in many fields of sciences and engineering. These techniques are used in the works of Agrawal and Kumar[4], Chandel and Sengar[5], Chaurasia[6] and Kumar[7] to find out many results in several problems in different field of science. Here we construct a model problem in a rectangular plate for temperature distribution under predefine boundary conditions and then evaluate its solution involving Modified Multivariable H-function with product of general class of polynomials involving Modified Multivariable H-function[8-10].

The general class of Polynomials is defined by Srivastva and Panda as:

$$S_{n_1,\dots,n_r}^{m_1,\dots,m_r}(x_1,\dots,x_r)=\sum_{k_1=0}^{[n_1/m_1]}\dots\sum_{k_r=0}^{[n_r/m_r]}\frac{(-n_1)}{k_1!}m_1k_1\dots\frac{(-n_r)}{k_r!}m_rk_rF(n_1,k_1;\dots;n_r,k_r)x_1^{k_1}\dots x_r^{k_r} \tag{1.1}$$

where $m_1,\dots,m_r$ are arbitrary positive integers and the coefficients $F(n_1,k_1;\dots;n_r,k_r)$ are arbitrary constants real or complex. Here we derive some new particular cases and find their applications also.

The modified multi-variable H-function will be defined and represented as follows:

$$H_{p,q:|\ R:p_1,q_1;\dots;\ p_r,q_r}^{m,n:|R':m_1,n_1;\dots;m_r,n_r}\left[\begin{matrix}z_1\\ \vdots\\ z_r\end{matrix}\left|\begin{matrix}(a_j;\alpha'_j,\dots,\alpha_j^{(r)})_{1,p}:(e_j;u'_jg'_j,\dots,u_j^{(r)}g_j^{(r)})_{1,R'}:(c'_j,\gamma'_j)_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta'_j,\dots,\beta_j^{(r)})_{1,q}:(l_j;U'_jf'_j,\dots,U_j^{(r)}f_j^{(r)})_{1,R}:(d'_j,\delta'_j)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right.\right]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1} \cdots \int_{L_r} \Phi_1(\xi_1) \ldots \Phi_r(\xi_r)\psi(\xi_1,\ldots,\xi_r) z_1^{\xi_1} \ldots z_r^{\xi_r} d\xi_1 \ldots d\xi_r \qquad \ldots\ldots 1.2$$

where

$$\Phi_i(\xi_i) = \frac{\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)} - \delta_j^{(i)}\xi_i\right)\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}-\gamma_j^{(i)}\xi_i\right)}{\prod_{j=m_i+1}^{q_i}\Gamma\left(1-d_j^{(i)}+\delta_j^{(i)}\xi_i\right)\prod_{j=n_i+1}^{p_i}\Gamma\left(c_j^{(i)}-\gamma_j^{(i)}\xi_i\right)} \; (i = 1,2,\ldots,r) \qquad \ldots\ldots\ldots\ldots 1.3$$

$$\psi(\xi_i,\ldots,\xi_r) = \frac{\prod_{j=1}^{m_i}\Gamma\left(b_j - \sum_{i=1}^{r}\beta_j^{(i)}\xi_i\right)\prod_{j=1}^{n}\Gamma\left(1-a_j + \sum_{i=1}^{r}\alpha_j^{(i)}\xi_i\right)\prod_{j=1}^{|R'}\Gamma\left(e_j + \sum_{i=1}^{r}u_j^{(i)}g_j^{(i)}\xi_i\right)}{\prod_{j=m+1}^{p}\Gamma\left(a_j - \sum_{i=1}^{r}\alpha_j^{(i)}\xi_i\right)\prod_{j=n+1}^{q}\Gamma\left(1-b_j + \sum_{i=1}^{r}\beta_j^{(i)}\xi_i\right)\prod_{j=1}^{|R}\Gamma\left(l_j + \sum_{i=1}^{r}U_j^{(i)}f_j^{(i)}\xi_i\right)} \ldots 1.4$$

The multiple integral (1.1) converges absolutely if

$$|arg z_i| < \frac{1}{2}U_i\pi, \text{ (i=1,2,…,r).}$$

where

$$U_i = \sum_{j=1}^{m}\beta_j^{(i)} - \sum_{j=m+1}^{q}\beta_j^{(i)} + \sum_{j=1}^{n}\alpha_j^{(i)} - \sum_{j=n+1}^{p}\alpha_j^{(i)}\sum_{j=1}^{m}\delta_j^{(i)} - \sum_{j=m_i+1}^{q_i}\delta_j^{(i)}\sum_{j=1}^{n_1}\gamma_j^{(i)} -$$

$$\sum_{j=n_i+1}^{p_i}\gamma_j^{(i)} + \sum_{j=1}^{R'}g_j^{(i)} - \sum_{j=1}^{R}f_j^{(i)} > 0 \text{(i=1,2…,r)}$$

2. A Boundary Value Problem:

We consider a rectangular plate such that

$$U = f(x) \qquad \left(\frac{a}{2},\frac{b}{2}\right)$$

where the boundary value conditions are:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = a, 0 < x < \frac{a}{2}, 0 < y < \frac{b}{2} \text{.......(2.1)}$$

$$\left.\frac{\partial U}{\partial x}\right|_{x=0} = \left.\frac{\partial U}{\partial x}\right|_{x=\frac{a}{2}} = 0, 0 < y < \frac{b}{2} \text{.......(2.2)}$$

$$U(x,0) = 0, 0 < x < \frac{a}{2} \text{.......(2.3)}$$

$$U\left(x, \frac{b}{2}\right) = f(x) = \left(\cos\frac{\pi x}{a}\right) S_{n_1,\dots,n_r}^{m_1,\dots,m_r}\left[y\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right]$$

$$H_{p,q:|\,R:p_1,q_1;\dots;\,p_r,q_r}^{m,n;|R':m_1,n_1;\dots;m_r,n_r}\left[\begin{matrix} z_1\cos\left(\frac{\pi x}{a}\right)^{2\sigma_1} \\ \vdots \\ z_r cos\left(\frac{\pi x}{a}\right)^{2\sigma_r}\end{matrix} \begin{matrix} (a_j;\alpha'_j,\dots,\alpha_j^{(r)})_{1,p}:(e_j;u'_j g'_j,\dots,u_j^{(r)} g_j^{(r)})_{1,R'}\ (c'_j,\gamma'_j)_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ (b_j;\beta'_j,\dots,\beta_j^{(r)})_{1,q}:(L_j;U'_j f'_j,\dots,U_j^{(r)} f_j^{(r)})_{1,R}\ (d'_j,\delta'_j)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right]$$

.....(2.4)

where, $0 < x < \dfrac{a}{2}$ provided the Re(η)>-1, (σ_1,......σ_r)>0

U(x, y) is the temperature distribution in the rectangular plate at point (x,y).

3. Main Integral:

In our investigations we use the modified formula of Kumar[6] as

$$\int_0^{a/2}\left(\cos\frac{\pi x}{a}\right)^{\eta}\cos\frac{2m\pi x}{a}\,dx = \frac{a\Gamma(\eta+1)}{2^{\eta+1}\left(\frac{\eta}{2}+m+1\right)\left(\frac{\eta}{2}-m+1\right)} \dots\dots (3.1)$$

where, m is positive integer and Re(η)>-1, then we evaluate an applicable integral.

$$\int_0^{a/2}\left(\cos\frac{\pi x}{a}\right)^{\eta}\cos\frac{2m\pi x}{a} S_{n_1,\dots,n_r}^{m_1,\dots,m_r}\left[y\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right]$$

$$H_{p,q:|\,R:p_1,q_1;\dots;\,p_r,q_r}^{m,n;|R':m_1,n_1;\dots;m_r,n_r}\left[\begin{matrix} z_1\cos\left(\frac{\pi x}{a}\right)^{2\sigma_1} \\ \vdots \\ z_r\cos\left(\frac{\pi x}{a}\right)^{2\sigma_r}\end{matrix} \begin{matrix} (a_j;\alpha'_j,\dots,\alpha_j^{(r)})_{1,p}:(e_j;u'_j g'_j,\dots,u_j^{(r)} g_j^{(r)})_{1,|R'}\ (c'_j,\gamma'_j)_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ (b_j;\beta'_j,\dots,\beta_j^{(r)})_{1,q}:(L_j;U'_j f'_j,\dots,U_j^{(r)} f_j^{(r)})_{1,|R}\ (d'_j,\delta'_j)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right]$$

$$= \frac{a}{2^{\eta+1}}\sum_{k_1=0}^{[n_1/m_1]}\dots\dots\sum_{k_r=0}^{[n_r/m_r]}\frac{(-n)_{m_1k_1}}{k_1!}\dots\frac{(-n)_{m_rk_r}}{k_r!}F(n_1,k_1;\dots;n_r,k_r)H(k)\left(\frac{y}{4\rho}\right)^{k_1}\dots\left(\frac{y}{4\rho}\right)^{k_r}$$

..... (3.2)

where

$$H(k) = H^{m,n+1:|R':m_1,n_1;...;m_r,n_r}_{p+1,q+1:|R:p_1,q_1;...;p_r,q_r}\left[\begin{array}{c}\frac{Z_1}{4^{\sigma_1}}\\ \vdots \\ \frac{Z_r}{4^{\sigma_r}}\end{array}\middle| \begin{array}{l}(-\eta-2\rho k_1 \ldots -2\rho k_r\,;2\sigma_1 \ldots 2\sigma_r;1)(a_j;\alpha'_j,\ldots,\alpha_j^{(r)})_{1,p}:\\ (b_j;\beta'_j,\ldots,\beta_j^{(r)})_{1,q}:(l_j;U'_jf'_j,\ldots,U_j^{(r)}f_j^{(r)})_{1,|R}\end{array}\right.$$

$$\left.\begin{array}{l}(e_j;u'_jg'_j,\ldots,u_j^{(r)}g_j^{(r)})_{1,|R'}(c'_j,\gamma'_j)_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (d'_j,\delta'_j)_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\left(\frac{-\eta}{2}-m-\rho k_1,\ldots-\rho k_r\,;\sigma_1,\ldots,\sigma_r\,;1\right),\left(\frac{-\eta}{2}+m-\rho k_1,\ldots,-\rho k_r\,;\ \sigma_1,\ldots,\sigma_r\,;1\right)\end{array}\right] \quad \text{.... (3.3)}$$

provided that F(n_1,k_1;....;n_r, k_r] are arbitrary functions of n_1,k_1;....;n_r, k_r real or complex independent of x, y, ρ. the conditions of 2.4 and (3.1) are satisfied and

$$R_e\left(\eta+\sigma_i\frac{b_j}{\beta_j}\right) > -1, |arg z| \frac{\le 1}{2} U_i \Pi \ (i = 1, 2, \ldots r)$$

4. Solution of Boundary Value Problem:

In this section we obtain the Solution of the Boundary Value Problem define in the Section 2 as using eq^n. (2.1), (2.2) and (2.3) with the help of the techniques referred to Zill[13] as:

$$U(x,y) = A_0 y + \sum_{p=1}^{\infty} A_p sinh\frac{2p\pi y}{a}\ cos\ \frac{2p\pi x}{a},$$

$$0 < x < \frac{a}{2}, 0 < y < \frac{b}{2} \ldots (4.1)$$

for $y = \frac{b}{2}$, we find that

$$U\left(x,\frac{b}{2}\right) = f(x) = \frac{A_0 b}{2} + \sum_{p=1}^{\infty} A_p sinh\frac{p\pi b}{a}\ cos\ \frac{2p\pi x}{a},$$

$$0 < x < \frac{a}{2} \ldots . (4.2)$$

Now we use eq^n (2.4) and (4.2) and then integrating both sides with respect to x from 0 to $\frac{a}{2}$, we get

$$A_0 = \frac{2}{b\sqrt{\pi}} \sum_{k_1=0}^{[n_1|m_1]} \ldots . \sum_{k_r=0}^{[n_r|m_r]} (-n_1)_{m_1 k_1} \ldots (-n_r)_{m_r k_r} F(n_1, k_1; \ldots, n_r, k_r) H_1(k) \frac{y^{k_1}}{k_1!} \cdots \frac{y^{k_r}}{k_r!}$$

..... (4.3)

where

$$H_1(k) = H^{m,n+1:|R':m_1,n_1;\ldots;m_r,n_r}_{p+1,q+1:|R:p_1,q_1;\ldots;p_r,q_r} \left[\begin{matrix} Z_1 \\ \vdots \\ Z_r \end{matrix} \middle| \begin{matrix} \left(-\frac{1}{2}-\frac{\eta}{2}-\rho_1 k_1, \ldots, \rho_r k_r ; \sigma_1 \ldots \sigma_r : 1\right) : (a_j; \alpha'_j, \ldots, \alpha_j^{(r)})_{1,p} \\ (b_j; \beta'_j, \ldots, \beta_j^{(r)})_{1,q} : (l_j; U'_j f'_j, \ldots, U_j^{(r)} f_j^{(r)})_{1,R} \end{matrix}\right.$$

$$\left.\begin{matrix} (e_j; u'_j g'_j, \ldots, u_j^{(r)} g_j^{(r)})_{1,R'} (c'_j, \gamma'_j)_{1,p_1}; \ldots; (c_j^{(r)}, \gamma_j^{(r)})_{1,p_r} \\ (d'_j, \delta'_j)_{1,q_1}; \ldots; (d_j^{(r)}, \delta_j^{(r)})_{1,q_r} \left(-\frac{\eta}{2}-\rho_1 k_1, \ldots, \rho_r k_r ; \sigma_1 \ldots \sigma_r : 1\right) \end{matrix}\right] \ldots . (4.4)$$

where all condition (2.4), (3.1) and (3.3) are satisfied

Again we use (2.4) and (4.2) and then multiplying by $\cos\frac{2m\pi x}{a}$ both sides and then integrating the result with respect to x from 0 to a/2, we get

$$A_m = \frac{1}{2^{\eta-1} \sinh\frac{p\pi b}{a}} \sum_{k_1=0}^{[n_1|m_1]} \ldots . \sum_{k_r=0}^{[n_r|m_r]} \frac{(-n_1)_{m_1 k_1}}{k_1!} \ldots \ldots \frac{(-n_r)_{m_r k_r}}{k_r!} F(n_1, k_1; \ldots; n_r, k_r) H(k) \left(\frac{y}{4\rho}\right)^{k_1} \ldots \ldots \left(\frac{y}{4\rho}\right)^{k_r}$$

..... (4.5)

provided that all conditions of (2.4), (3.1) and (3.3) are satisfied.

Finally, we use result (4.1), (4.3) and (4.5), derive the required solution of the boundary volume problem,

$$U(x,y) = \frac{2y}{b\sqrt{\pi}} \sum_{k_1=0}^{[n_1|m_1]} \ldots . \sum_{k_r=0}^{[n_r|m_r]} \left[\prod_{j=1}^{r} \left((-n_j)_{m_j k_j} \frac{y^{k_j}}{k_j!}\right)\right] F(n_1, k_1; \ldots, n_r, k_r) H_1(k) +$$

$$\sum_{m=1}^{\infty} \frac{Sinh\, \frac{2m\pi y}{a}\, Cos\, \frac{2m\pi x}{a}}{2^{\eta-1}\, Sinh \frac{m\pi b}{a}} \sum_{k_1=0}^{[n_1|m_1]} \ldots . \sum_{k_r=0}^{[n_r|m_r]} \left[\prod_{j=1}^{r} \left\{(-n_j)_{m_j k_j} \left(\frac{y}{4\rho}\right)^{k_j} \frac{1}{k_j!}\right\}\right] F(n_1 k_1; \ldots ; n_r k_r) H(k)$$

..... (4.6)

where all condition of (2.4), (3.1) and (3.3) are satisfied.

5. Expansion Formula:

We calculate the expansion formula with the help of (2.4) and (4.6) and then put $y = \frac{b}{2}$

$$\left(\cos\frac{\pi x}{a}\right)^{\eta} S^{m_1,\ldots,m_r}_{n_1,\ldots,n_r}\left[y\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right] H^{m,n:|R':m_1,n_1;\ldots;m_r,n_r}_{p,q\,|\,R:p_1,q_1;\ldots;p_r,q_r}$$

$$\left[\begin{matrix} z_1\left(\cos\frac{\pi x}{a}\right)^{2\sigma_1} \\ \vdots \\ z_r\left(\cos\frac{\pi x}{a}\right)^{2\sigma_r}\end{matrix}\left|\begin{matrix}(a_j;\alpha'_j,\ldots,\alpha_j^{(r)})_{1,p}:(e_j;u'_j g'_j,\ldots,u_j^{(r)}g_j^{(r)})_{1,|R'}\ (c'_j,\gamma'_j)_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ (b_j;\beta'_j,\ldots,\beta_j^{(r)})_{1,q}:(l_j;U'_j f'_j,\ldots,U_j^{(r)}f_j^{(r)})_{1,|R}\ (d'_j,\delta'_j)_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right.\right]$$

$$= \frac{1}{\sqrt{\pi}}\sum_{k_1=0}^{[n_1|m_1]}\cdots\sum_{k_r=0}^{[n_r|m_r]}\left[\prod_{j=1}^{r}\left((-n_j)_{m_j k_j}\frac{y^{k_j}}{k_j!}\right)\right]F(n_1,k_1;\ldots,n_r,k_r)\ H(k) +$$

$$\sum_{m=1}^{\infty}\frac{\cos\frac{2m\pi x}{a}}{2^{\eta-1}}\sum_{k_1=0}^{[n_1|m_1]}\cdots\sum_{k_r=0}^{[n_r|m_r]}\left[\prod_{j=1}^{r}\left\{(-n_j)_{m_j k_j}\left(\frac{y}{4^{\rho}}\right)^{k_j}\frac{1}{k_j!}\right\}\right]F(n_1k_1;\ldots;n_rk_r)H(k) \quad \ldots.. (5.1)$$

where $0 < x < \frac{a}{2}$ provided that all condition of (2.4), (3.1) and (3.3) are satisfied.

6. Particular Cases and Applications:

In this section we use different parameters of our results and then derive some particular cases as taking $m_1 = \cdots m_r = \gamma$ and

$$F(n_1k_1;\ldots;n_rk_r) = \left(\frac{h}{(v)^{\gamma}}\right)^{k_1+k_2\ldots+k_r}\frac{1}{(1+p-n_1,-\cdots-n_r)_{\gamma(k_1+\cdots+k_r)}}$$

in (1.1) we get

$$S^{\gamma,\ldots,\gamma}_{n_1,\ldots,n_r}[x_1,\ldots,x_r] = \frac{(-v)^{-\eta_1,\ldots,-\eta_r}}{(-p)_{\eta_1+\cdots+\eta_r}}(x_1)^{n_r/\gamma},\ldots(x_r)^{n_r/\gamma}H^{(h,\gamma,v,p)}_{n_1,\ldots,n_r}\left[(x_1)^{-\frac{1}{\gamma}},\ldots,(x_r)^{-\frac{1}{\gamma}}\right] \quad \ldots. (6.1)$$

And thus, we obtain an integral for product of a class of polynomials of several variables and cosine functions as

$$\int_0^{a/2}\left(\frac{\cos\pi x}{a}\right)^{\eta}\cos\frac{2m\pi x}{a}\frac{(-v)^{-n_1,\dots,-n_r}}{(-p)_{n_1+\cdots+n_r}}\left[y\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right]^{n_1/\gamma}\dots.\left[y\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right]^{n_1/\gamma} H^{(h,\gamma,v,p)}_{n_1,\dots,n_r}\left[y\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right]^{n_1/\gamma}$$

$$H^{m,n:|R':m_1,n_1;\dots;m_r,n_r}_{p,q\,|\,R:p_1,q_1;\dots;p_r,q_r}\left[\begin{matrix} z_1\left(\cos\frac{\pi x}{a}\right)^{2\sigma_1} \\ \vdots \\ z_r\left(\cos\frac{\pi x}{a}\right)^{2\sigma_r}\end{matrix}\middle| \begin{matrix}(a_j;\alpha'_j,\dots,\alpha_j^{(r)})_{1,p}:(e_j;u'_j g'_j,\dots,u_j^{(r)}g_j^{(r)})_{1,|R'} \\ (b_j;\beta'_j,\dots,\beta_j^{(r)})_{1,q}:(l_j;U'_j f'_j,\dots,U_j^{(r)}f_j^{(r)})_{1,|R}\end{matrix}\right.$$

$$\left.\begin{matrix}(c'_j,\gamma'_j)_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ (d'_j,\delta'_j)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right]dx$$

$$=\frac{a}{2^{\eta+1}}\sum_{k_1=0}^{[n_1/\gamma]}\dots\sum_{k_r=0}^{[n_r/\gamma]}\frac{(-n)_{\gamma k_1}}{k_1!}\dots\frac{(-n)_{\gamma k_r}}{k_r!}\left[\frac{h}{(-v)^{\gamma}}\right]^{k_1+\cdots+k_r}\frac{1}{(1+\rho-n_1,-\cdots-n_r)_{\gamma(k_1+\cdots+k_r)}}H(k)\left(\frac{y}{4\rho}\right)^{k_1}\dots\left(\frac{y}{4\rho}\right)^{k_r} \quad \dots\dots (6.2)$$

Provided that all conditions of (2.4), (3.1) and (3.2) are satisfied.

The solution of the given problem is

$$U(x,y)=\frac{2y}{b\sqrt{\pi}}\sum_{k_1=0}^{[n_1/\gamma]}\dots\sum_{k_r=0}^{[n_r/\gamma]}\left[\prod_{j=1}^{r}\left[(-n_j)_{\gamma k_j}\left(\frac{hy}{(-v)^{\gamma}}\right)^{k_j}\frac{1}{k_j!}\right]\right]\frac{1}{(1+\rho-n_1,-\cdots-n_r)_{\gamma(k_1+\cdots+k_r)}}H_1(k)$$

$$+\sum_{m=1}^{\infty}\frac{\sinh\frac{2m\pi y}{a}\cos\frac{2m\pi x}{a}}{2^{\eta-1}\sinh\frac{m\pi b}{a}}\sum_{k_1=0}^{[n_1/\gamma]}\dots\sum_{k_r=0}^{\left[\frac{n_r}{\gamma}\right]}\left[\prod_{j=1}^{r}\left[(n_j)_{\gamma k_j}\left(\frac{hy}{(-v)^{\gamma}4^{\rho}}\right)^{k_j}\frac{1}{k_j!}\right]\frac{1}{(1+p-n_1,-\cdots-n_r)_{\gamma(k_1+\cdots+k_r)}}H(k)\right.$$

..... (6.3)

when $0<x<\frac{a}{2}, 0<y<\frac{b}{2}$, provided that all conditions of (2.4), (3.1) and (3.3) are satisfied.

The expansion formula is

$$\left(\cos\frac{\pi x}{a}\right)^{\eta}\frac{(-v)^{-n_1,\dots,-n_r}}{(-p)_{n_1+\cdots+n_r}}\left[y\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right]^{n_1/\gamma}\dots\dots\left[y\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right]^{n_1/\gamma}$$

$$H^{(h,\gamma,v,p)}_{n_1,\dots,n_r}\left[y\left(\cos\frac{\pi x}{a}\right)^{2\rho}\right]^{-r/\gamma} H^{m,n:|R':m_1,n_1;\dots;m_r,n_r}_{p,q\,|\,R:p_1,q_1;\dots;p_r,q_r}\left[\begin{matrix} z_1\left(\cos\frac{\pi x}{a}\right)^{2\sigma_1} \\ \vdots \\ z_r\left(\cos\frac{\pi x}{a}\right)^{2\sigma_r}\end{matrix}\right]$$

$$= \frac{1}{\sqrt{\pi}} \sum_{k_1=0}^{[n_1|\gamma]} \ldots\ldots \sum_{k_r=0}^{[n_r|\gamma]} \left[\prod_{j=1}^{r} (-n_j)_{\gamma k_j} \left(\frac{hy}{(-v)^{\gamma}} \right)^{k_j} \frac{1}{k_j!} \right] \frac{1}{(1+p-n_1, -\cdots - n_r)_{\gamma(k_1+\cdots.+k_r)}} H_1(k)$$

$$+ \sum_{m=1}^{\infty} \frac{cos\, \frac{2m\pi x}{a}}{2^{\eta-1}} \sum_{k_1=0}^{[n_1/\gamma]} \ldots \sum_{k_r=0}^{\left[\frac{n_r}{\gamma}\right]} \left[\prod_{j=1}^{r} \left[(-n_j)_{\gamma k_j} \left(\frac{hy}{(-v)^{\gamma} 4^{\rho}} \right)^{k_j} \frac{1}{k_j!} \right] \frac{1}{(1+p-n_1, -\cdots - n_r)_{\gamma(k_1+\cdots.+k_r)}} H(k) \quad \text{..... (6.4)}$$

when $0 < x < \frac{a}{2}$, provided that all conditions of (2.4), (3.1) and (3.3) are satisfied.

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