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Some results on Electrical networks in graph theory

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Abstract

The behavior or dynamics of a circuit is described by three systems of equations determined by Ohm's law, Kirchhoff's voltage law, and Kirchhoff's current law, Ohm's law specifies the relationship between the voltage and current variables associated with a circuits element. This relationship could be a linear or non linear. If the relationship is linear, then the circuit element is called a linear element, other wise, it is a non linear element. In this paper we shall develop most of those results that form the foundation of graph theoretic study of electrical circuits.

Key words : dynamics, electrical circuits, graphical study, inductance.

1. Introduction

An electrical network is an interconnection of electrical network elements such as resistances, capacitances, inductances, voltage and current sources, etc. Each network element is associated with two variables, the voltage variable, $v(t)$ and the current variable $i(t)$. We e also assign reference direction to the network element s (see fig. 1) so that $i(t)$ is positive whenever the current is in the direction of the arrow, and $v(t)$ is positive whenever the voltage drop in the network element is in the direction of the arrow,

Kirchoff's voltage law (KVL) : The algebraic sum of the voltge around any circuit is equal to zero.

Kirchof's current law (KCL) : The algebraic sum of the currents flowing out of a node is equal to zero.

2. Buliding a network:

As an example, the KVL equation for the circuit 1,3,5 and the KCL equation for the vertex b in the graph of fig 2 and

Circuit 1,3, $v_1 + v_3 + v_5 = 0$

Vertex b $-i_1 + i_2 + i_3 = 0$

It can be easily seen that KVL and KCL equations for an electrical network N can be written as $A_c I_e = 0$ and $B_c V_e = 0$.

Where A_c and B_c are, respectively, the indecence and circuit matrix of the directed graph representing N: I_e and V_e are respectively, the column vectors in N(node). Because each in the matrix Q_c can be expressed as a linear combination of the rows of the matrix, in the above we can replace A_c by Q_c; thus we have

KCL : $Q_c I_e = 0$

KVL : $B_c V_e = 0$

Thus, KCL can also be stated as : The algebraic sum of the currents in any cut of N is equal to zero.

If a network N has n- vertices and m-elements and its graph is connected ,then there are only (n-1) linearly independent cuts and only (m-n+1) linearly independent circuits. Thus, in writing KVL and KCL

Equations we need to use only B_f, a fundamental circuit matrix, and Q_f,a fundamental circuit matrix respectively, thus, we have

KCL : $Q_f I_e = 0$

KVL : $B_f V_e = 0$

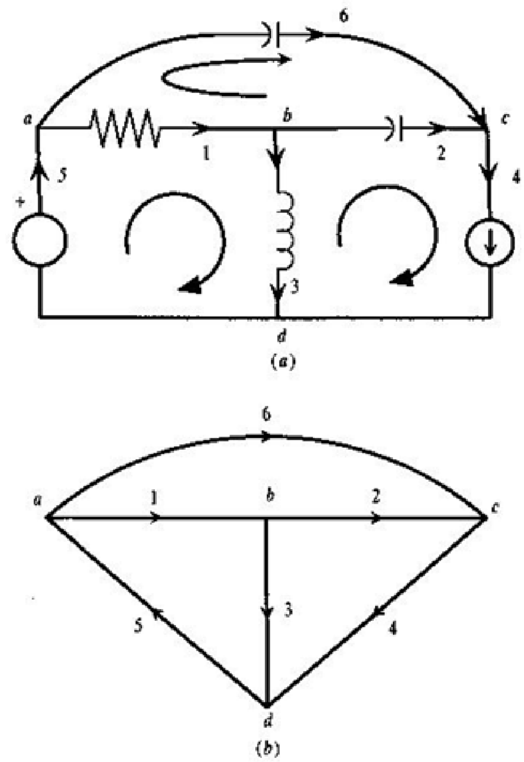

Fig (1) and Fig(2)

We note that the KCL and KVL equations depend only on the way network elements are interconnected and not on the nature of the network elements, Thus, several results in electrical network elements. Thus ,several results in electrical network theory are essentially graph theoretic in nature. Some results of interest in electrical network analysis are presented in the reminder of this paper.

3. Loop and Cut set Transformations :

As we observed that, the problem of network analysis is to determine the voltages and currents associated with the element of an electrical network. These voltages and currents can be determined from kirchoff's equations and the element voltage –current (in short, v-i) relations given by Ohm's law. However, these equations involve a large number of variables. As can be seen from the loop and cut set transformations, not all these variables are independent furthermore, in place of KCL equations we can use the loop transformation which involves only chord currents as variables. Similarly, KVL equations can be replaced by the cutset transformation which involves only these transformations to establish different system of network equations known a the loop and cutset system. In deriving the loop system we use the loop transformation in place of KCL, and in this case the loop variables will serve as independent variables.

In deriving the cutset transformation in place of KVL, and the cutset variables (tree branch voltages) will serve as the independent variable in this case.

Consider a connected electrical network N. We assume that N consists of only resistance (R) capacitances (C), inductance (L), and independent voltage and current Sources, we also assume that all initial inductor currents and initial capacitor voltages have been replaced by appropriate sources. Further, the voltage and current variables are all of frequencies s, In N there can be no circuit consisting of only independent voltage sources, for, if such a circuit of sources were present, then KVL there would be a linear relationship among the corresponding voltages, violating independence of the voltage sources. For the same reason, in N there can be no cutset consisting of only independent current sources. So there exists in N a spanning tree containing all the voltage sources but not current sources. Such a tree is the starting point for the development of both the loop and cutset systems of equations.

Let T be a spanning tree of the given network such that T contains all the voltage sources but no current sources, Let us partition the element voltage V_e and the element current vector I_e as follows :

$$V_e = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} \text{ and } I_e = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix}$$

Where the subscripts 1,2 and 3 refer to the vectors corresponding to, the current sources, RCL elements and voltage sources, respectively. Let B_f be the fundamental circuit matrix of N, and Q_f the fundamental cutset matrix of N with respect to T. Then the KVL and KCL equations can be written as

$$\text{KVL : } B_f V_e = \begin{bmatrix} U & B_{12} & B_{13} \\ 0 & B_{22} & B_{23} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = 0$$

$$\text{KCL: } Q_f I_e = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & U \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = 0$$

4. Cut set Method Network Analysis :

Step 1: Solve the following for the vector V_b .

$$Y_b V_b = -Q_{11} I_1 - Q_{12} Y_2 Q_{22} Y_b \qquad (1)$$

Where Y_2 is the admittance matrix of RLC elements and $Y_b = Q_{12} Y_2 Q_{12}$. Equation (1) Is called cutset system of equations.

Step 2 : Calculating V_2 using :

$$V_2 = Q_{12} V_b + Q_{22} V_3 \qquad (3)$$

Then , $I_2 = Y_2 Y_2$ (4)

Step 3: Determine and using the following :

$$V_1 = Q_{12}V_b + Q_{21}V_3 \quad (5)$$

$$I_3 = -Q_{21}I_1 - Q_{22}I_2 \quad (6)$$

Note that I_1 and V_3 have specified values, since they correspond to current and voltage sources.

5. Loop Method of Network Analysis:

Step 1 : Solve the following for vector I_1

$$Z_1I_1 = -B_{23}V_3 - B_{22}Z_2B_{12}I_1 \quad (7)$$

Equation (7) Is called the loop system of equation.

Step 2 : Calculating I_2 using :

$$I_2 = B_{12}I_1 + B_{22}I_1 \quad (8)$$

Then , $V_2 = Z_2I_2$ (9)

Step 3: Determine V_1 and I_3 using the following:

$$V_1 = -B_{12}V_2 - B_{13}V_3 \quad (10)$$

$$I_3 = B_{13}I_1 + B_3I_1 \quad (11)$$

Note that I_1 and V_3 have specified values, since they correspond to current and voltage sources, respectively.

Concussion

The system of equations determined by the applications of Kirchoff's voltage and current laws depend on the structure of graph of the circuit. In other words ,they depend only on the way the circuit elements are interconnected. Thus ,the graph of a circuit play a fundamental role in the study of circuits. We have discussed the several interesting properties of circuits depend only on the structure of the circuits. Thus we conclude that the theory of graphs has played a important role in discovering structural properties of electrical circuits.

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