Ultra Scientist Vol. 27(2)B, 95-104 (2015).

On **gs-Closed Sets in Topological Spaces

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(Acceptance Date 22nd June, 2015)

Abstract

In this paper, we introduce and study a new class of sets namely **gs-closed sets which lie in between the class of *gs-closed sets[32] and the class of gs-closed sets[8]. we investigate their basic properties together with the relationship of these sets with some other recently introduced sets. As applications of **gs-closes sets, we introduce new separation properties, namely $\overset{*}{T}_s$ -spaces, T_s**-spaces and **T_s-spaces. Further we introduce and study new types of continuous functions called **gs-continuous functions and **gs-irresolute functions. Finally, we introduce and study **gs-compactness and **gs-connectedness.

Key words: **gs-closed sets; $\overset{*}{T}_s$ -spaces; T_s**-spaces; **T_s-spaces; **gs-continuity; **gs-compactness; **gs-connectedness.

1. Introduction

The study of g-closed sets in a topological space was initiated by Levine[3] in 1970. Earlier, Levine[1] also introduced the concept of semi-open sets and semi-continuity in a topological space in 1963. Bhattacharya and Lahiri[7] introduced sg-closed sets in 1987. Arya and Nour[8] defined gs-closed sets in 1990. Njastad[2] introduced the concepts of α-closed sets for topological spaces in 1965. Dontchev[16] (resp. Palaniappan and Rao[14], Gnanambal[18]) introduced gsp-closed (resp. rg-closed, gpr-closed) sets in 1995 (resp. 1993, 1997). Veera Kumar introduced $\hat{g}$-closed sets[22], ψ-closed sets[20], *g-closed sets[25], g*-closed sets[21], #g-closed sets[26] and #gs-closed sets[29]. Manoj *et al.* introduced the concepts of $\hat{\hat{g}}$-closed sets[24], **g-closed sets[30] and *gs-closed sets[32] in 2007.

In this paper we study relationships of **gs-closed sets with the above mentioned sets. Levine[3], Bhattacharya and Lehiri[7] and Devi *et al.*[12] introduced $T_{1/2}$-spaces, semi-$T_{1/2}$ spaces and T_b-spaces respectively. Veera Kumar introduced $\hat{T}_b$ -spaces[23], $T^*_{1/2}$-spaces[21] and semi $T_{1/3}$-spaces[20]. Recently, Manoj *et al.*[32] introduced T_s-spaces, T_s*-spaces and *T_s-spaces.

We introduce and study new classes of spaces, namely the class of $\overset{*}{T}_s$-spaces, the class of T_s^{**}-spaces and the class of $^{**}T_s$-spaces. We also characterize and study some relationships of these spaces with the above defined spaces.

We introduced **gs-continuous maps, **gs-irresolute maps and investigate some of their properties. Finally, we introduce and study the concept of **gs-compactness and **gs-connectedness.

Throughout this paper (X, τ), (Y, σ) and (Z, η) represent non-empty topological spaces on which no separation axioms are assumed unless or otherwise mentioned. For a subset A of a space (X, τ), cl(A), int(A), pcl(A) and A^c denote respectively the closure of A, the interior of A, pre-closure of A and the complement of A.

2. *Preliminaries :*

We begin by recalling the following definitions which are useful in our investigation.

Definition 2.01 : A subset A of a topological space (X, τ) is called semi-open[1] (resp. semi-closed[1], pre-open[5], pre-closed[5], α-open[2], semi-preopen[6], semi-pre closed[6], regular open[38], regular closed[38]) if $A \subseteq \mathrm{cl}(\mathrm{int}(A))$ (resp. $\mathrm{int}(\mathrm{cl}(A)) \subseteq A$, $A \subseteq \mathrm{int}(\mathrm{cl}(A))$, $\mathrm{cl}(\mathrm{int}(A)) \subseteq A$, $A \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$, $A \subseteq \mathrm{cl}(\mathrm{int}(\mathrm{cl}(A)))$, $\mathrm{int}(\mathrm{cl}(\mathrm{int}(A))) \subseteq A$, $A = \mathrm{int}(\mathrm{cl}(A))$, $A = \mathrm{cl}(\mathrm{int}(A))$).

Definition 2.02 : A subset A of a topological space (X, τ) is called g-closed[3] (resp. sg-closed[7], gs-closed[8], $^{\#}$g-closed[26], **g-closed[30], rg-closed[14], gpr-closed[18], gsp-closed[16], $\hat{g}$-closed[22], ψ-closed[20],*g-closed[25], $\hat{\hat{g}}$-closed[24], g*-closed[21], *gs-closed[32], $^{\#}$gs-closed[29]) if $\mathrm{cl}(A) \subseteq U$ (resp. $\mathrm{scl}(A) \subseteq U$, $\mathrm{scl}(A) \subseteq U$, $\mathrm{cl}(A) \subseteq U$, $\mathrm{cl}(A) \subseteq U$, $\mathrm{cl}(A) \subseteq U$, $\mathrm{pcl}(A) \subseteq U$, $\mathrm{spcl}(A) \subseteq U$, $\mathrm{cl}(A) \subseteq U$, $\mathrm{scl}(A) \subseteq U$, $\mathrm{cl}(A) \subseteq U$, $\mathrm{cl}(A) \subseteq U$, $\mathrm{cl}(A) \subseteq U$, $\mathrm{scl}(A) \subseteq U$ and $\mathrm{scl}(A) \subseteq U$) whenever $A \subseteq U$ and U is open (resp. semi-open, open, *g-open, $\hat{\hat{g}}$-open, regular-open, regular open, open, semi-open, sg-open, $\hat{g}$-open, sg-open, g-open, -open and *g-open) in (X, τ).

Definition 2.03 : A function $f : (X, \tau) \rightarrow (Y, \sigma)$ is called continuous[10] (resp. sg-continuous[11], gs-continuous[17], α-continuous[37], ψ-continuous[20], *gs-continuous[32], gsp-continuous[16], *g-continuous[25], g*-continuous[21], **g-continuous[30], $^{\#}$g-continuous[26], $^{\#}$gs-continuous[29] semi-continuous[1], irresolute[4], gc-irresolute[10], sg-irresolute[11], $\hat{g}$-irresolute[23], $\hat{\hat{g}}$-irresolute[24], *g-irresolute[25], *gs-irresolute[32], ψ-irresolute[20], **g-irresolute[30]) if $f^{-1}(V)$ is closed (resp. sg-closed, gs-closed, α-closed, ψ-closed, *gs-closed, gsp-closed, *g-closed, g*-closed, **g-closed, $^{\#}$g-closed, $^{\#}$gs-closed, semi-closed, semi-closed, gc-closed, sg-closed, $\hat{g}$-closed, $\hat{\hat{g}}$-closed, *g-closed, *gs-closed, ψ-closed, **g-closed) set in (X, τ) for every closed (resp. closed, closed, closed, closed, closed, closed, closed, closed, closed, closed,

closed, closed, semi-closed, gc-closed, sg-closed, $\hat{g}$-closed, $\hat{\hat{g}}$-closed, *g-closed, *gs-closed, ψ-closed, **g-closed) set in (Y, σ).

Definition 2.04 : A space (X, τ) is called $T_{1/2}$[3] (resp. T_b[12]], T_s[32], T_s[32], $*T_s$[32], semi-$T_{1/2}$[7], semi-$T_{1/3}$[35], $\hat{T}_b$[23], $T*_{1/2}$[21], $*T_{1/2}$[21], T_c[21], T_u[30]) space if every g-closed (resp. gs-closed, *gs-closed, *gs-closed, gs-closed, sg-closed, ψ-closed, gs-closed, g*-closed, g-closed, gs-closed, **g-closed) set is a closed (resp. closed, closed, semi-closed, *gs-closed, semi-closed, semi-closed, $\hat{g}$-closed, closed, g*-closed, g*-closed, closed) set.

*3. Relationships of **gs-Closed sets with some other sets :*

In this section we study the relationship of **gs-closed sets with other sets.

Definition 3.01 : A subset A of topological (X, τ) is called **gs-closed set if scl(A) $\subseteq$ U whenever A $\subseteq$ U and U is $\hat{\hat{g}}$-open set in (X, τ).

The complement of **gs-closed set is called **gs-open set.

Theorem 3.01 : (i) Every closed, semi-closed, α-closed, *g-closed, ψ-closed, g*-closed, *gs-closed, **g-closed, #g-closed and #gs-closed set is **gs-closed set.

(ii) Every **gs-closed set is gsp-closed set.

Next examples show that converse of the above theorem is not true in general.

Example 3.01 : Let X = {a, b, c}, τ = {ϕ, {a}, {c}, {a, c}, X}. Consider A = {c} then A is not a closed set, g*-closed set, *g-closed set, $\hat{\hat{g}}$-closed set, **g-closed set. However A is a **gs-closed set.

Example 3.02 : Let X = {a, b, c}, τ = {ϕ, {a}, {b, c}, X}. Consider B = {a, b} then B is not a *gs-closed set. However B is a **gs-closed set. Consider C = {b, c} then C is not a ψ-closed. However C is **gs-closed.

Example 3.03 : Let X = {a, b, c}, τ = {ϕ, {a}, X}. Consider D = {a, b} then D is not a #g-closed and #gs-closed. However D is **gs-closed set. Consider E = {b} then E is not a #g-closed set. However E is **gs-closed set. If F = {a, b} then F is not a semi-closed. However F is a **gs-closed.

Example 3.04 : Let X = {a, b, c}, τ = {ϕ, {c}, {a, c}, X}. Consider G = {b, c} then G is not a α-closed set. However G is a **gs-closed set.

Example 3.05 : Let X = {a, b, c}, τ = {ϕ, {a, b}, X}. Consider H = {a} then H is a gsp-closed set. However H is not a **gs-closed set.

Remark 3.01 : **gs-closed set is independent from rg-closed set and gpr-closed set.

Example 3.06 : In example (3.01), consider A = {a} then A is **gs-closed set. However A is not a rg-closed set or gpr-closed set.

Example 3.07 : In example (3.03), consider B = {a} then B is rg-closed set and gpr-closed set. However B is not a **gs-closed set.

Remark 3.02 : **gs-closed set is not necessarily g-closed set and sg-closed set as it can be seen from the following examples.

Example 3.08 : In example (3.01), consider A = {c} then A is **gs-closed set. However A is not g-closed set.

Example 3.09 : In example (3.04), consider B = {b, c} then B is **gs-closed set. However B is not a sg-closed set.

Theorem 3.02 : Every **gs-closed set is gs-closed.

Therefore the class of **gs-closed sets is properly contained in the class of gs-closed sets and gsp-closed sets. Also this new class is properly contains the class of closed sets, the class of semi-closed sets, the class of α-closed sets, the class of *g-closed sets, the class of ψ-closed sets, the class of g*-closed sets, the class of *gs-closed sets, the class of **g-closed sets, the class of $^{\#}$g-closed sets and the class of $^{\#}$gs-closed sets.

The following diagram shows the relationships of **gs-closed set with other sets.

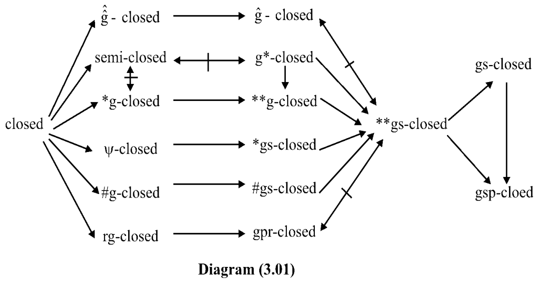

Diagram (3.01)

Where $A \longrightarrow B$ (resp. $A \nleftrightarrow B$) represents A implies B but converse is not necessary true (resp. A and B are independent).

4. Basic Properties of **gs-Closed sets :

In this section we introduce the following properties.

Theorem 4.01 : Union (intersection) of any **gs-closed sets is not necessarily **gs-closed set. The following example supports the above theorem.

Example 4.01 : In example (3.01), consider A = {a} and B = {c} then A and B are **gs-closed sets. However their union {a, c} is not **gs-closed set.

Example 4.02 : In example (3.03), consider A = {a, b} and B = {a, c} then A and B are **gs-closed sets. However their intersection {a} is not **gs-closed set.

Theorem 4.02 : If A be a **gs-closed set in a space (X, τ) and $A \subseteq B \subseteq \mathrm{scl}(A)$ then B is also a **gs-closed set.

Theorem 4.03 : A is **gs-closed set of (X, τ) iff scl(A) - A does not contain any non-empty $\hat{\hat{g}}$ -closed set.

Definition 4.01 : Let (X, τ) be a topological space and $E \subseteq X$. We define the **gs-closure of A (briefly **gs-cl(A)) to be the intersection of all **gs-closed sets containing A. In symbols, **gs-cl(A) = $\cap$ {B : $A \subseteq B$ and $B \in$ **gs-closed set}.

Theorem 4.04 : Let (X, τ) be a topological space and $A \subseteq X$. The following properties are hold.

(i) **gs-cl(A) is the smallest **gs-closed set containing A and

(ii) A is **gs-closed iff **gs-cl(A) = A.

Theorem 4.05 : For any two subsets A and B of (X, τ)

(i) $A \subseteq B$ then **gs-cl(A) $\subseteq$ **gs-cl(B).

(ii) **gs-cl($A \cap B$) $\subseteq$ **gs-cl(A) $\cap$ **gs-cl(B).

Theorem 4.06 : If $B \subseteq A \subseteq X$, B is a **gs-closed set relative to A and A is open and **gs-closed in (X, τ). Then B is **gs-closed in (X, τ).

Corollary 4.01 : If A is a **gs-closed set and F is a closed set, then $A \cap F$ is a **gs-closed set.

Definition 4.03 : Let (X, τ) be a topological space and $A \subseteq X$. We define the **gs-interior of A (briefly **gs-int(A)) to be the union of all **gs-open sets contained in A.

Theorem 4.07 : For any $A \subseteq X$, int(A) $\subseteq$ **gs-int(A) $\subseteq$ A.

Proof : Since every open set is **gs-open, the proof follows immediately.

*5. Applications of **gs-Closed sets :*

In this section we introduce the following definitions.

Definition 5.01 : A topological space (X, τ) is called $\overset{*}{T}_s$-space if every **gs-closed set is clsoed.

Definition 5.02 : A topological space (X, τ) is called T_s**-space if every **gs-closed set is semi-closed.

Definition 5.03 : A topological space (X, τ) is called **T_s-space if every gs-closed set is **gs-closed.

Theorem 5.01 : Every $\overset{*}{T}_s$-space is semi-$T_{1/3}$-space, $T^*_{1/2}$-space, T_u-space, T_s-space, T_s*-space, T_s**-space.

The converse of the above theorem is not true as it can be seen from the following examples.

Example 5.01 : In example (3.03), (X, τ) is semi-$T_{1/3}$-space but not a $\overset{*}{T}_s$-space.

Example 5.02 : In example (3.01), (X, τ) is T_u-space, $T^*_{1/2}$-space and T_s**-space. However it is not a $\overset{*}{T}_s$-space.

Example 5.03 : In example (3.02), (X, τ) is T_s-space and T_s*-space. However it is not a $\overset{*}{T}_s$-space.

Theorem 5.02 : Every T_b-space is T_s**-space and **T_s-space.

The converse of the above theorem is not true as it can be seen from the following examples.

Example 5.04 : In example (3.01), (X, τ) is T_s**-space but not a T_b-space.

Example 5.05 : In example (3.05), (X, τ) is **T_s-space but not a T_b-space.

Theorem 5.03 : Every T_c-space and $\hat{T}_b$-space is **T_s-space.

The converse of the above theorem is not true as it can be seen from the following examples.

Example 5.06 : Let X = {a, b, c, d}, τ = {ϕ, {a}, {b}, {a, b}, {a, d}, {a, b, c}, {a, b, d}, X}. Then (X, τ) is not a $\hat{T}_b$-space. However it is $^{**}T_s$-space.

Example 5.07 : Let X = {a, b, c, d}, τ = {ϕ, {a}, {a, b, c}, X}. Then (X, τ) is not a T_c-space. However it is $^{**}T_s$-space.

Theorem 5.04 : Every T_s^{**}-space is T_s^{*}-space.

The converse of the above theorem is not true as seen from the following example.

Example 5.08 : In example (3.02), (X, τ) is T_s^{*}-space but not a T_s^{**}-space.

Remark 5.01 : $^{**}T_s$-space is not necessarily T_s^{**}-space as it can be seen from the following example.

Example 5.09 : In example (3.03), (X, τ) is $^{**}T_s$-space but not a T_s^{**}-space.

Remark 5.02 : $T_{1/2}$-space (resp. $^{**}T_s$-space) is not necessarily $\overset{*}{T}_s$-space (resp. T_b-space and $^{*}T_s$-space) as seen from the following examples.

Example 5.10 : In example (3.01), (X, τ) is $T_{1/2}$-space and **Ts-space but not a $\overset{*}{T}_s$-space and T_b-space.

Example 5.11 : In example (3.02), (X, τ) is $^{**}T_s$-space but not $^{*}T_s$-space.

Theorem 5.05 : If a topological space (X, τ) is T_s^{**}-space and $^{**}T_s$-space then it is $T_{1/2}$-space.

Theorem 5.06 : Every $\overset{*}{T}_s$-space is T_u-space.

The converse of the above theorem is not true as seen from the following example.

Example 5.12 : In example (3.01), (X, τ) is T_u-space but not a $\overset{*}{T}_s$-space.

The following diagram shows the relationships between the separation axioms discussed in this section.

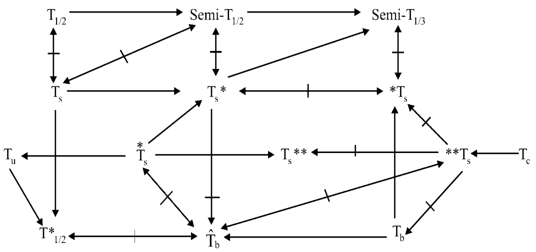

Diagram (5.01)

6. **gs-Continuous and **gs-Irresolute Maps :

In this section we introduce the following definitions.

Definition 6.01 : A map f : (X, τ) $\rightarrow$ (Y, σ) is said to be **gs-continuous if the inverse image of every σ-closed set in Y is **gs-closed in X.

Example 6.01 : Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{b\}, \{c\}, \{b, c\}, \{a, c\}, X\}$, $\sigma = \{\phi, \{a\}, X\}$. Define $f : (X, \tau) \to (X, \sigma)$ by $f(a) = c$, $f(b) = b$ and $f(c) = a$ then f is a **gs-continuous map.

Definition 6.02 : A map $f : (X, \tau) \to (Y, \sigma)$ is said to be **gs-irresolute if the inverse image of every **gs-closed set in Y is **gs-closed set in X.

Example 6.02 : Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{a, b\}, X\}$, $\sigma = \{\phi, \{a, b\}, X\}$. Define $f : (X, \tau) \to (X, \sigma)$ by identity mapping then f is a **gs-irresolute map.

Theorem 6.01 : (i) Every **gs-irresolute map is **gs-continuous map.

(ii) Every continuous map, semi-continuous map, α-continuous map, *g-continuous map, ψ-continuous map, g*-continuous map, *gs-continuous map, **g-continuous map, $^{\#}$g-continuous map and $^{\#}$gs-continuous map is **gs-continuous map.

(iii) Every **gs-continuous map is gs-continuous map and gsp-continuous map.

The converse of the above theorem is not true as seen from the following examples.

Example 6.03 : The function f in example (6.01) is **gs-continuous but not **gs-irresolute.

Example 6.04 : Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, X\}$ and $\sigma = \{\phi, \{b\}, X\}$. Define $f : (X, \tau) \to (X, \sigma)$ by $f(a) = a$, $f(b) = c$ and $f(c) = b$ then f is not continuous, semi-continuous, α-continuous, ψ-continuous, g*-continuous, $^{\#}$g-continuous, $^{\#}$gs-continuous. However f is **gs-continuous map.

Example 6.05 : Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, X\}$. Define $f : (X, \tau) \to (X, \sigma)$ by identity mapping then f is not *g-continuous and *gs-continuous. However f is **gs-continuous.

Example 6.06 : Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{c\}, \{a, c\}, X\}$ and $\sigma = \{\phi, \{a\}, \{a, b\}, X\}$. Define $f : (X, \tau) \to (X, \sigma)$ by identity mapping then f is not **g-continuous. However f is **gs-continuous map.

Example 6.07 : Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a, b\}, X\}$ and $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. Define $f : (X, \tau) \to (X, \sigma)$ by identity mapping then f is gsp-continuous. However f is not **gs-continuous.

Remark 6.01 : The composition of two **gs-continuous functions need not be **gs-continuous for consider the following example.

Example 6.08 : Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{b\}, \{a, b\}, X\}$, $\sigma = \{\phi, \{a\}, \{a, c\}, X\}$ and $\eta = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$. Define $f : (X, \tau) \to (X, \sigma)$ by identity mapping and $g : (X, \eta) \to (X, \tau)$ by $g(a) = b$, $g(b) = a$ and $g(c) = c$. Here f and g are **gs-continuous but $fog : (X, \eta) \to (X, \sigma)$ is not **gs-continuous.

Theorem 6.02 : The composition of two **gs-irresolute function is again **gs-irresolute.

Theorem 6.03 : Let (X, τ), (Y, σ) and (Z, η) be any three topological spaces. Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \eta)$ be any two functions then $gof : (X, \tau) \to (Z, \eta)$ is

(i) **gs-continuous if g is continuous and f is

**gs-continuous.

(ii) **gs-irresolute if g is **gs-irresolute and f is **gs-irresolute.

(iii) **gs-continuous if g is **gs-continuous and f is **gs-irresolute.

Therefore the class of **gs-continuous maps properly contains the class of continuous maps, the class of semi-continuous maps, the class of α-continuous maps, the class of *g-continuous maps, the class of ψ-continuous maps, the class of g*-continuous maps, the class of *gs-continuous maps, the class of **g-continuous maps, the class of $^{\#}$g-continuous maps and the class of $^{\#}$gs-continuous maps. Also this class properly contained in the class of gs-continuous maps and the class of gsp-continuous maps.

Theorem 6.04 : Let f : $(X, \tau) \to (Y, \sigma)$ be a **gs-continuous map. If (X, τ) is $\overset{*}{T_s}$-space, then f is continuous map.

Theorem 6.05 : Let f : $(X, \tau) \to (Y, \sigma)$ be a **gs-continuous map. If (X, τ) is T_s**-space, then f is semi-continuous map.

Theorem 6.06 : Let f : $(X, \tau) \to (Y, \sigma)$ be a gs-continuous map. If (X, τ) is **T_s-space, then f is **gs-continuous map.

Theorem 6.07 : If f : $(X, \tau) \to (Y, \sigma)$ be a bijective, **gs-irresolute map. If (X, τ) is T_s**-space, then f is an irresolute map.

Theorem 6.08 : Let f : $(X, \tau) \to (Y, \sigma)$ be a surjective, $\hat{\hat{g}}$-irresolute and pre-semi-closed map then for every **gs-closed set A of (X, τ), f(A) is a **gs-closed set of (Y, σ).

Proof : Let A be a **gs-closed set of (X, τ). Let U be a $\hat{\hat{g}}$-open set of (Y, σ) such that $f(A) \subseteq U$. Since f is surjective, -irresolute map, $f^{-1}(U)$ is a $\hat{\hat{g}}$-open set of (X, τ). Then $scl(A) \subseteq f^{-1}(U)$ since A is **gs-closed set and $A \subseteq f^{-1}(A)$. This implies $f(scl(A)) \subseteq U$. Since f is pre-semi-closed so $f(scl(A)) \subseteq scl(f(scl(A)))$. Now $scl(f(A)) \subseteq scl(f(scl(A))) = f(scl(A)) \subseteq U$. Therefore f(A) is **gs-closed set of (Y, σ).

Theorem 6.09 : Let f : $(X, \tau) \to (Y, \sigma)$ be a surjective, **gs-irresolute and pre-semi-closed map. If (X, τ) is a T_s**-space then (Y, σ) is also a T_s**-space.

Theorem 6.10 : If f : $(X, \tau) \to (Y, \sigma)$ is **gs-continuous and Y is -space then f is **gs-irresolute map.

*7. **gs-Compactness and **gs-Connectedness :*

In this section we introduce the following definitions.

Definition 7.1 : A collection $\{U_i : i \in \Lambda\}$ of **gs-open sets in a topological space X is said to be **gs -open cover of a subset V of X if $V \subseteq \cup \{U_i : i \in \Lambda\}$ holds.

Definition 7.2 : A topological space X is **gs -compact if every **gs -open cover of X has a finite subcover.

Definition 7.3 : A subset V of a topological space X is said to be **gs -compact relative to X if for every collection $\{U_i : i \in \Lambda\}$of **gs -open subset of X such that $V \subseteq \cup \{U_i : i \in \Lambda\}$ there exists a finite subset Λ_0 of Λ such $V \subseteq \cup \{U_i : i \in \Lambda_0\}$.

Definition 7.4 : A subset U of a topological space X is said to be **gs-compact if U is **gs-compact as a subspace of X.

Theorem 7.5 : Every **gs -closed subset of a **gs-compact space X is **gs-compact relative to X.

Theorem 7.6 : The **gs-continuous image of a **gs-compact space is compact.

Theorem 7.7 : If f : X → Y is a **gs-irresolute and a subset V of X is **gs-compact relative to X then f(V) is **gs-compact relative to Y.

Definition 7.8 : A topological space X is said to be **gs-connected if X can not be written as a disjoint union of two non-empty **gs-open sets.

A subset of X is **gs-connected if it is **gs-connected as a subspace.

Remark 7.9 : Every **gs-connected space is connected.

Theorem 7.10 : For a topological space X the following are equivalent :

(i) X is **gs-connected.

(ii) The only subset of X which are both **gs-open and **gs-closed are ϕ and X.

(iii) Every **gs-continuous map of X into a discrete space Y with at least two points is a constant map.

Theorem 7.11 : Let f : X → Y is **gs-continuous, surjection and X is **gs-connected then Y is connected.

Theorem 7.12 : If f : X → Y is **gs-irresolute, surjection and X is **gs-connected then Y is **gs-connected.

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