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Some Results Based on Product of Homogeneous Generalized Hypergeometric Function and Modified Multivariable H-Function

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Abstract

The aim of this chapter is to obtain some relations between Modified Multivariable H-function and Homogenous generalized hyper-geometric function and various well-known polynomials. These relations are very general in nature and consequently contain a large number of few and known relation as special cases.

Introduction

The homogeneous generalized hyper-geometric function ${}_pB_q[\alpha_r\beta_t; z]$ was defined by Basister A.W.[1] in 1967 as

$$ {}_pB_q[\alpha_r;\beta_t;z] = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \dots (\alpha_p)_n}{(\beta_1)_n \dots (\beta_q)_n} \Omega(\alpha_{p+n}, \beta_{q+n}) \frac{z^n}{n!}, \tag{1.1} $$

where Ω is he generalized modified struve function (Basister[1], 1967, p. 96) defined as

$$ \Omega(a,c,z) = (2)^{-1-c}(\pi)^{-2}e^{-i\pi c}\Gamma(1-\mathrm{a})\Gamma(\mathrm{c})\Gamma(1+\mathrm{a}-\mathrm{c}) \times $$

$$ \mathrm{e}^{(1/2)_z}\left[\left(1-e^{2\pi i a}\right)\int_0^{(1+)} e^{(1/2)zu)}(1+u)^{\alpha-1}(1-u)^{c-a-1}du\right. $$

$$ +\{(1-e^{2\pi i a(c-a)}\}\int_0^{(-1+)} e^{(1/2)zu)}(1+u)^{\alpha-1}(1-u)^{c-a-1}du \tag{1.2} $$

If $(R(\beta_q) > R(\alpha_p)$, the series (1.1) converges for all z, if $p \leq q$, converges for $|z|<1$, if $p=q+1$, diverges for all non zero z, if $p>q+1$. We shall take $2 \leq p \leq q+1$.

The modified Multi-variable H-function is defined by Prasad and Singh[10] on the basis of Srivastava and Panda[11], Prasad and Murya[9] is as follows:

$$H^{m,n:|R':m_1,n_1;\ldots;m_r,n_r}_{p,q:|R:p_1,q_1;\ldots;p_r,q_r}\begin{bmatrix} z_1 \\ \vdots \\ z_r \end{bmatrix}\begin{matrix}(a_j;\alpha'_j,\ldots,\alpha_j^{(r)})_{1,p}:(e_j;u'_j g'_j,\ldots,u_j^{(r)}g_j^{(r)})_{1,R'}:(c'_j,\gamma'_j)_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r} \\ (b_j;\beta'_j,\ldots,\beta_j^{(r)})_{1,q}:(l_j;U'_j f'_j,\ldots,U_j^{(r)}f_j^{(r)})_{1,R}:(d'_j,\delta'_j)_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\Bigg]$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\Phi_1(\xi_1)\ldots\Phi_r(\xi_r)\psi(\xi_1,\ldots,\xi_r)z_1^{\xi_1}\ldots z_r^{\xi_r}d\xi_1\ldots d\xi_r \qquad (1.3)$$

where

$$\Phi_i(\xi_i) = \frac{\prod_{j=1}^{m_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}\xi_i\right)\prod_{j=1}^{n_i}\Gamma\left(1-c_j^{(i)}-\gamma_j^{(i)}\xi_i\right)}{\prod_{j=m_i+1}^{q_i}\Gamma\left(1-d_j^{(i)}+\delta_j^{(i)}\xi_i\right)\prod_{j=n_i+1}^{p_i}\Gamma\left(c_j^{(i)}-\gamma_j^{(i)}\xi_i\right)}(i = 1,2,\ldots,r) \qquad (1.4)$$

$$\psi(\xi_i,\ldots,\xi_r)=$$

$$\frac{\prod_{j=1}^{m_i}\Gamma\left(b_j-\sum_{i=1}^{r}\beta_j^{(i)}\xi_i\right)\prod_{j=1}^{n}\Gamma\left(1-a_j+\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i\right)\prod_{j=1}^{|R'}\Gamma\left(e_j+\sum_{i=1}^{r}u_j^{(i)}g_j^{(i)}\xi_i\right)}{\prod_{j=m+1}^{p}\Gamma\left(a_j-\sum_{i=1}^{r}\alpha_j^{(i)}\xi_i\right)\prod_{j=n+1}^{q}\Gamma\left(1-b_j+\sum_{i=1}^{r}\beta_j^{(i)}\xi_i\right)\prod_{j=1}^{|R}\Gamma\left(l_j+\sum_{i=1}^{r}U_j^{(i)}f_j^{(i)}\xi_i\right)} \qquad (1.5)$$

The multiple integral (1.5) converges absolutely if

$$|arg z_i| < \frac{1}{2}U_i\pi, (i = 1,2,\ldots,r)$$

Where $U_i = \sum_{j=1}^{m}\beta_j^{(i)} - \sum_{j=m+1}^{q}\beta_j^{(i)} + \sum_{j=1}^{n}\alpha_j^{(i)} - \sum_{j=n+1}^{p}\alpha_j^{(i)}\sum_{j=1}^{m}\delta_j^{(i)} -$

$$\sum_{j=m_i+1}^{q_i}\delta_j^{(i)}\sum_{j=1}^{n_i}\gamma_j^{(i)} - \sum_{j=n_i+1}^{p_i}\gamma_j^{(i)} + \sum_{j=1}^{R'}g_j^{(i)} - \sum_{j=1}^{R}f_j^{(i)} > 0 \quad (i=1,2\ldots,r) \qquad (1.4)$$

2. Required Results

(i)$\int_0^1 C_n^{\lambda}(1-2y^2)(1-y^2)^{\lambda-1/2}y^{2\lambda+2r+2u}dy$

$$=\frac{\sqrt{\pi}}{2^{2\lambda}\,\Gamma(\mathrm{d})}\frac{\Gamma(n+2\lambda)(-1)^n}{n!}\frac{\Gamma(u+r+1)\Gamma\left(u+r+\lambda+\frac{1}{2}\right)}{\Gamma(u+r-n+1)\Gamma(U+r+n+2\lambda+1)} \quad (2.1)$$

where λ + r + u >-1/2 and $C_n^{\lambda}(x)$ is an Ultraspherical polynomial defined in[5] as $C_n^{\lambda}(x)=\frac{(2\upsilon)_n p_n^{\left(\upsilon-\frac{1}{2},\upsilon-\frac{1}{2}\right)}(x)}{(\upsilon+1/2\,)_n}$, where $p_n^{(\alpha,\alpha)}$ is the well known Jacobi polynomial also discussed in[5]

(ii) The following recurrence relations will be used in our investigations given in[5]

$$=(n+1)C_{n+1}^{\lambda}(x)=2(n+\lambda)xC_n^{\lambda}(x)-(n+2\lambda-1)C_{n-1}^{\lambda}(x) \quad (2.2)$$

$$=\ 2\lambda(1-x^2)C_{n-1}^{\lambda+1}(x)=(n+2\lambda-1)C_{n-1}^{\lambda}(x)-nxC_n^{\lambda}(x) \quad (2.3)$$

(iii) The following recurrence relations will be also used in our investigation given in[6-8].

$$\frac{1}{2}(2+\alpha+\beta+2n)(x+1)P_n^{(\alpha,\beta+1)}(x)$$

$$=(n+1)P_{n+1}^{(\alpha,\beta)}(x)+(1+\beta+n)P_n^{(\alpha,\beta)}(x) \quad (2.4)$$

$$(\alpha+\beta+2n)P_n^{(\alpha,\beta-1)}(x)=(\alpha+\beta+n)P_n^{(\alpha,\beta)}(x)+(\alpha+n)P_{n-1}^{(\alpha,\beta)}(x), \quad (2.5)$$

$$(x+1)P_n^{(\alpha,\beta+1)}(x)+(1-x)P_n^{(\alpha+1,\beta)}(x)=2P_n^{(\alpha,\beta)}(x), \quad (2.6)$$

(iv) $$\int_0^1 P_n^{\alpha_1\beta}(2y^2-1)(1-y^2)^{\alpha}y^{2\sigma+1}dy=\frac{\frac{1}{2}\Gamma(\sigma+1)\Gamma(\alpha+n+1)\Gamma(\alpha-\beta+1)}{\frac{1}{2}\Gamma(\sigma-\beta-n+1)\Gamma(\alpha+\sigma+n+2)} \quad (2.7)$$

Where $P_n^{\alpha,\beta}(x)$ is Jacobi polynomial and {(2σ +1)>-1)}.

Proof: On putting $x=1-2y^2$ in the result of Gradshteyn and Ryzhik ([12], 4, p. 834) we get the result (2.1) after little simplification.

Also we put $2y^2 - 1 = x$ in the result of Gradshteyn and Ryzhik ([12], 3,p.847) we get the result (2.7) after little simplification.

3. Main Results

First Integral

$$\int_0^1 C_n^{\lambda}(1-2y^2)\,(1-y^2)^{\lambda-1/2}y^{2\lambda+2r}\,{}_pB_q[\alpha_s;\beta_t;(zy)^2] \times H^{m,n:|R':m_1,n_1;\ldots;m_r,n_r}_{p,q:|\,R:p_1,q_1;\ldots;\,p_r,q_r}$$

$$\left[\begin{matrix}(xy^2)^{\sigma_1}\\ \vdots \\ (xy^2)^{\sigma_r}\end{matrix}\left|\begin{matrix}(a_j;\alpha'_j,\ldots,\alpha_j^{(r)})_{1,p}:(e_j;u'_jg'_j,\ldots,u_j^{(r)}g_j^{(r)})_{1,R'}:(c'_j,\gamma'_j)_{1,p_1};\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta'_j,\ldots,\beta_j^{(r)})_{1,q}:(l_j;U'_jf'_j,\ldots,U_j^{(r)}f_j^{(r)})_{1,R}:(d'_j,\delta'_j)_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right.\right]$$

dy

$$= \frac{\Gamma(n+2\lambda)\sqrt{\pi}(-1)^n}{4^{\lambda}n!\,\Gamma(\lambda)}\sum_{k=0}^{n}\frac{(\alpha_1)_k\ldots(\alpha_p)_k}{(\beta_1)_k\ldots(\beta_q)_k}\Omega(\alpha_{p+k},\beta_{q+k},0)\frac{z^{2k}}{k!}\times$$

$$H^{m,n+2:|R':m_1,n_1;\ldots;m_r,n_r}_{P+2,Q+2:|\,R:p_1,q_1;\ldots;\,p_r,q_r}\left[\begin{matrix}(x)^{\sigma_1}\\ \vdots \\ (x)^{\sigma_r}\end{matrix}\left|\begin{matrix}(-k-r,\sigma_1\ldots\sigma_r,1),(\frac{1}{2}-k-r-\lambda,\sigma_1\ldots\sigma_r,1),\\ (b_j;\beta'_j,\ldots,\beta_j^{(r)})_{1,q}:\quad(l_j;U'_jf'_j,\ldots,U_j^{(r)}f_j^{(r)})_{1,R}:\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j;\alpha'_j,\ldots,\alpha_j^{(r)})_{1,p}:(e_j;u'_jg'_j,\ldots,u_j^{(r)}g_j^{(r)})_{1,R'}:(c'_j,\gamma'_j)_{1,p_1};\ldots\ldots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (d'_j,\delta'_j)_{1,q_1};\ldots;(d_j^{(r)},\delta_j^{(r)}:)_{1,q_r}:(-k-r+n,\sigma_1\ldots\sigma_r;1),(-k-r-n-2\lambda,\sigma_1\ldots\sigma_r,1)\end{matrix}\right] \quad (3.1)$$

Where the following conditions are satisfied[13]

(i) Modified Multivariable h-function must satisfy the conditions of convergence given by (1.6)

(ii) $R(\beta_q) > R(\alpha_p) for 0 \le p \le q+1,$

(iii) $(\lambda + r) > -1/2.$

Second Integral:

$$\int_0^1 {}_pB_q[\alpha_s;\beta_t;(zy)^2](1-y^2)^{\lambda-\frac{1}{2}}y^{2r} \times H^{m,n:|R':m_1,n_1;\ldots;m_r,n_r}_{p,q:|\,R:p_1,q_1;\ldots;\,p_r,q_r}$$

$$\left[\begin{matrix}(xy^2)^{\sigma_1}\\ \vdots\\ (xy^2)^{\sigma_r}\end{matrix}\left|\begin{matrix}(a_j;\alpha'_j,\dots,\alpha_j^{(r)})_{1,p}:(e_j;u'_jg'_j,\dots,u_j^{(r)}g_j^{(r)})_{1,R'}:(c'_j,\gamma'_j)_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta'_j,\dots,\beta_j^{(r)})_{1,q}:(l_j;U'_jf'_j,\dots,U_j^{(r)}f_j^{(r)})_{1,R}:(d'_j,\delta'_j)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right.\right]dy$$

$$=\frac{\sqrt{\pi}}{\Gamma(\lambda)}\sum_{n=0}^{\infty}\frac{\Gamma(n+2\lambda)(-1)^n}{n!}\sum_{k=0}^{\infty}\frac{(\alpha_1)_k\dots(\alpha_p)_k}{(\beta_1)_k\dots(\beta_q)_k}\Omega(\alpha_{p+k},\beta_{q+k},0)\frac{z^{2k}}{k!}$$

$$H_{P+2,Q+2:|\,R:p_1,q_1;\dots;\,p_r,q_r}^{m,n+2:|R':m_1,n_1;\dots;m_r,n_r}\left[\begin{matrix}(x)^{\sigma_1}\\ \vdots\\ (x)^{\sigma_r}\end{matrix}\left|\begin{matrix}(-k-r,\sigma_1\dots\sigma_r,1),\left(\frac{1}{2}-k-r-\lambda,\sigma_1\dots\sigma_r,1\right),\\ (b_j;\beta'_j,\dots,\beta_j^{(r)})_{1,q}:\ (l_j;U'_jf'_j,\dots,U_j^{(r)}f_j^{(r)})_{1,R}:\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(a_j;\alpha'_j,\dots,\alpha_j^{(r)})_{1,p}:(e_j;u'_jg'_j,\dots,u_j^{(r)}g_j^{(r)})_{1,R'}:(c'_j,\gamma'_j)_{1,p_1};\dots\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (d'_j,\delta'_j)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}:(-k-r+n,\sigma_1\dots\sigma_r;1),(-k-r-n-2\lambda,\sigma_1\dots\sigma_r,1)\end{matrix}\right] \quad (3.2)$$

The conditions of convergence are as below

(i) Modified Multivariable h-function must satisfy the conditions of convergence given by (1.6)

(ii) $R(\beta_q)>R(\alpha_p) for 0\le p\le q+1,$

(iii) $r>-1/2.$

Proof: The integral (3.1) can be established by expressing the modified Multivariable H-function as given in (1.3) and the homogeneous generalized function ${}_pB_q[\alpha_s;\beta_t;(zy)^2]$ as defined by (1.1) and changing the order of integration we get

$$\int_0^1 C_n^{\lambda}(1-2y^2)\,(1-y^2)^{\lambda-\frac{1}{2}}y^{2\lambda+2r}\,{}_pB_q[\alpha_r;\beta_t;(zy)^2]$$

$$\times H_{p,q:|\,R:p_1,q_1;\dots;\,p_r,q_r}^{m,n:|R':m_1,n_1;\dots;m_r,n_r}$$

$$\left[\begin{matrix}(xy^2)^{\sigma_1}\\ \vdots\\ (xy^2)^{\sigma_r}\end{matrix}\left|\begin{matrix}(a_j;\alpha'_j,\dots,\alpha_j^{(r)})_{1,p}:(e_j;u'_jg'_j,\dots,u_j^{(r)}g_j^{(r)})_{1,R'}:(c'_j,\gamma'_j)_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta'_j,\dots,\beta_j^{(r)})_{1,q}:(l_j;U'_jf'_j,\dots,U_j^{(r)}f_j^{(r)})_{1,R}:(d'_j,\delta'_j)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right.\right]$$

dy

$$= \sum_{k=0}^{\infty} \frac{(\alpha_1)_k \dots (\alpha_p)_k}{(\beta_1)_k \dots (\beta_p)_k} \Omega(\alpha_{p+k}, \beta_{p+k}, 0) \frac{(z)^{2k}}{k!}$$

$$\times \frac{1}{(2\pi\omega)^r} \int_{L_1} \cdots \int_{L_r} \Phi_1(\xi_1) \dots \Phi_r(\xi_r) \psi(\xi_1, \dots, \xi_r)$$

$$\left\{ \int_0^1 C_n^{\lambda}(1-2y^2)\,(1 - y^2)^{\lambda-1/2} y^{2\lambda+2r+2k+2(\sigma_1\xi_1,\dots\dots\dots,\sigma_r\xi_r)} dy \right\} d\xi_1 \dots d\xi_r$$

Now we use the result (2.1) and get the result (3.1) after a little simplification.
To calculate the integral (3.2) we use the following relation

$$(1-2xh+h^2)^{-\lambda} = \sum_{n=0}^{\infty} C_n^{\lambda}(x) h^n$$

put $x = 1 - 2y^2$ in the above relation and then multiplying by

$${}_pB_q[\alpha_s; \beta_t; (zy)^2] y^{2\lambda+2r} (1-y^2)^{\lambda-1/2} H^{m,n:|R':m_1,n_1;\dots;m_r,n_r}_{p,q:|\,R:p_1,q_1;\dots;\,p_r,q_r}$$

$$\left[\begin{matrix}(xy^2)^{\sigma_1}\\ \vdots \\ (xy^2)^{\sigma_r}\end{matrix}\left|\begin{matrix}(a_j;\alpha'_j,\dots,\alpha_j^{(r)})_{1,p}:(e_j;u'_j g'_j,\dots,u_j^{(r)} g_j^{(r)})_{1,R'}:(c'_j,\gamma'_j)_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta'_j,\dots,\beta_j^{(r)})_{1,q}:(l_j;U'_j f'_j,\dots,U_j^{(r)} f_j^{(r)})_{1,R}:(d'_j,\delta'_j)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right.\right] dy$$

on both sides and then integrating with respect to y between the limits 0 to 1 and using the results (2.1) we get the result (3.2).

Special Cases:

(i) If we put $m = |R' = |R = n = p = q = 0$ in (3.1) the Modified Multivariable H-function degenerates into fox's H-function we get the result.

$$\int_0^1 C_n^{\lambda}(1-2y^2)(1-y^2)^{\lambda-1/2} y^{2\lambda+2r}\,{}_pB_q[\alpha_s; \beta_t; (zy)^2]\, H^{m,n:|R':m_1,n_1;\dots;m_r,n_r}_{p,q:|\,R:p_1,q_1;\dots;\,p_r,q_r}$$

$$\left[\begin{matrix}(xy^2)^{\sigma_1}\\ \vdots\\ (xy^2)^{\sigma_r}\end{matrix}\left|\begin{matrix}(a_j;\alpha'_j,\dots,\alpha_j^{(r)})_{1,p}:(e_j;u'_jg'_j,\dots,u_j^{(r)}g_j^{(r)})_{1,R'}(c'_j,\gamma'_j)_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}:\\ (b_j;\beta'_j,\dots,\beta_j^{(r)})_{1,q}:\ (l_j;U'_jf'_j,\dots,U_j^{(r)}f_j^{(r)})_{1,R}:(d'_j,\delta'_j)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right.\right]$$

$$=\frac{\Gamma(n+2\lambda)\sqrt{\pi}(-1)^n}{4^\lambda n!\,\Gamma(\lambda)}\sum_{k=0}^{\infty}\frac{(\alpha_1)_k\dots\dots(\alpha_p)_k}{(\beta_1)_k\dots(\beta_q)_k}\Omega\big(\alpha_{p+k},\beta_{q+k},+0\big)\frac{z^{2k}}{k!}H^{0,2:m_1,n_1;\dots;m_r,n_r}_{2,2:p_1,q_1;\dots;\,p_r,q_r}$$

$$\left[\begin{matrix}(xy^2)^{\sigma_1}\\ \vdots\\ (xy^2)^{\sigma_r}\end{matrix}\left|\begin{matrix}(-k-r,\sigma_1,\dots\sigma_r),\left(\frac{1}{2}-k-r-\lambda,\sigma_1,\dots\sigma_r\right),(c'_j,\gamma'_j)_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (d'_j,\delta'_j)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}(-k-r-n,\sigma_1,\dots\sigma_r),(-k-r-n,\sigma_1,\dots\sigma_r)\end{matrix}\right.\right]$$

(i) If we put $m=|R'=|R=n=p=q=0$ in (3.2) the Modified Multivariable H-function reduces to well-known Fox's H-function we get the result

$$\int_0^1 {}_p^B{}_q[\alpha_s;\beta_t;(zy)^2](1-y^2)^{\lambda-\frac{1}{2}}y^{2r}\times H^{m,n:|R':m_1,n_1;\dots;m_r,n_r}_{p,q:|\,R:p_1,q_1;\dots;\,p_r,q_r}$$

$$\left[\begin{matrix}(xy^2)^{\sigma_1}\\ \vdots\\ (xy^2)^{\sigma_r}\end{matrix}\left|\begin{matrix}(a_j;\alpha'_j,\dots,\alpha_j^{(r)})_{1,p}:(e_j;u'_jg'_j,\dots,u_j^{(r)}g_j^{(r)})_{1,R'}:(c'_j,\gamma'_j)_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (b_j;\beta'_j,\dots,\beta_j^{(r)})_{1,q}:\ (l_j;U'_jf'_j,\dots,U_j^{(r)}f_j^{(r)})_{1,R}:(d'_j,\delta'_j)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}\end{matrix}\right.\right]dy$$

$$=\frac{\sqrt{\pi}}{\Gamma(\lambda)}\sum_{n=0}^{\infty}\frac{\Gamma(n+2\lambda)(-1)^n}{n!}\sum_{k=0}^{\infty}\frac{(\alpha_1)_k\dots(\alpha_p)_k}{(\beta_1)_k\dots(\beta_q)_k}\Omega\big(\alpha_{p+k},\beta_{q+k},0\big)\frac{z^{2k}}{k!}$$

$$H^{0,2:m_1,n_1;\dots;m_r,n_r}_{2,2:p_1,q_1;\dots;\,p_r,q_r}\left[\begin{matrix}(x)^{\sigma_1}\\ \vdots\\ (x)^{\sigma_r}\end{matrix}\left|\begin{matrix}(-k-r,\sigma_1\dots\sigma_r;1),(\frac{1}{2}-k-r-\lambda,\sigma_1\dots\sigma_r;1)\\ (d'_j,\delta'_j)_{1,q_1};\dots;(d_j^{(r)},\delta_j^{(r)})_{1,q_r}(-k-r+n,\sigma_1,\dots\sigma_r;1)\end{matrix}\right.\right.$$

$$\left.\begin{matrix}(c'_j,\gamma'_j)_{1,p_1};\dots;(c_j^{(r)},\gamma_j^{(r)})_{1,p_r}\\ (-k-r-n-2\lambda,\sigma_1,\dots\sigma_r;1)\end{matrix}\right]$$

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