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Magnetohydrodynamic Heat and Mass Transfer Flow Past an Accelerated Plate

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Abstract

An exact solution of magnetohydrodynamic convective flow with mass transfer past an accelerated vertical plate embedded in a porous medium is investigated. The governing equations are solved using the Laplace-transform technique. The influence of various parameters on the temperature, concentration and velocity as well as on the skin fraction, the rate of heat and mass transfer is studied graphically.

Key words: Convective flow with mass transfer, accelerated surface, Laplace-transform technique.

Nomenclature

B_0 : magnetic field component along y-axis,

C' : concentration in dimensional form,

C : concentration in non-dimensional form,

C'_w : concentration in the equilibrium state,

C'_∞ : initial concentration of the plate and the fluid,

D_T : chemical molecular diffusivity,

g : gravitational acceleration,

K' : permeability of the medium in dimensional form,

K : permeability of the medium in non-dimensional form,

K_T : thermal conductivity of the fluid,

L_R : reference length,

M : magnetic parameter,

N : ratio of the change in concentration and temperature,

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Pr : Prandtl number,

Sc : Schmidt number,

T' : dimensional temperature,

T : non- dimensional temperature,

T'_w : temperature in the equilibrium state,

T'_∞ : initial temperature of the plate and the fluid,

t' : dimensional time,

t : non-dimensional time,

t_R : reference time,

u' : dimensional velocity of the fluid,

u : non- dimensional velocity of the fluid,

U_R : reference velocity,

x', y' : dimensional coordinates,

x, y : non-dimensional coordinates,

Greek symbols :

β' : coefficient of volume expansion for mass transfer,

β : coefficient of volume expansion for heat transfer,

ρ : density of the fluid,

σ : electrical conductivity of the fluid,

μ : viscosity of the fluid,

υ : kinematic viscosity of the fluid,

ω : frequency of the oscillation,

ε : amplitude (constant),

$\omega' t'$: dimensional phase-angle,

ωt : non- dimensional phase-angle.

Introduction

Magnetohydrodynamic convective flow with mass transfer is of considerable interest due to its frequent occurrence in MHD power generator system and Industrial technology. Muthucumaraswamy *et al.*[1] discussed natural convection on a moving isothermal vertical plate with chemical reaction. Sharma *et al.*[2] investigated radiative and free convective effects on MHD flow through porous medium between infinite parallel plates with periodic cross-flow velocity. Singh *et al.*[3] discussed mixed convection flow past a porous vertical plate bounded by a porous medium in a rotating system in the presence of a magnetic field. Singh *et al.*[4] discussed effects of periodic permeability and suction velocity on three-dimensional free convection flow past a vertical porous plate embedded in highly porous medium. Recently, Singh *et al.*[5] investigated hydromagnetic convection flow in a porous medium bounded between vertical wavy wall and parallel flat wall. While Chaudhary and Jain[6] are studied an exact solution of magnetohydrodynamic convection flow past an accelerated surface embedded in a porous medium. In the present paper, it is proposed to study mass transfer effects on flow field which enhances the usefulness of the investigation of Chaudhary and Jain[6].

Formulation of the Problem :

We consider one dimensional unsteady flow of an incompressible, electrically conducting viscous fluid along an infinite non-conducting vertical plate embedded in a homogeneous porous medium. The x'-axis is considered on the plate and y'-axis normal to it. Initially, the plate and the fluid are at the same temperature T'_∞ and concentration C'_∞. At time $t' > 0$, the plate starts moving impulsively in its own plane with a velocity U_R, its temperature and concentration is raised to T'_w and C'_w respectively. Periodic temperature and concentration is assumed to be superimposed on this mean constant temperature of the fluid and concentration at the plate. A magnetic field of uniform strength is applied perpendicular to the plate. In the analysis magnetic Reynolds number is assumed to be small so that the induced magnetic field is neglected. Now, under the usual Boussinesq's approximation, the flow is governed by the following equations:

$$\frac{\partial u'}{\partial t'} = \upsilon \frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T' - T'_\infty\right) + g\beta'\left(C' - C'_\infty\right) - \frac{\sigma B_0^2}{\rho} u' - \frac{\upsilon}{K'} u' \qquad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{K_T}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} \qquad (2)$$

$$\frac{\partial C'}{\partial t'} = D_T \frac{\partial^2 C'}{\partial y'^2} \qquad (3)$$

The initial and boundary conditions relevant to the present problem are :

$$t' \le 0 : u' = 0, \; T' = T'_\infty, \; C' = C'_\infty \text{ for all } y',$$

$$t' > 0 : u' = U_R, \; T' = T'_w + \varepsilon\left(T'_w - T'_\infty\right)\cos\omega' t',$$

$$C' = C'_w + \varepsilon\left(C'_w - C'_\infty\right)\cos\omega' t' \text{ at } y' = 0,$$

$$u' \to 0, T' \to T'_\infty, C' \to C'_\infty \text{ as } y' \to \infty. \qquad (4)$$

The symbols are defined in the nomenclature.

We introduce the following non-dimensional variables and parameters:

$$u = \frac{u'}{U_R}, \quad y = \frac{y'}{L_R}, \quad t = \frac{t'}{t_R}, \quad \omega = \omega' t_R,$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \; C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \; \Delta T = T'_w - T'_\infty,$$

$$\Delta C = C'_w - C'_\infty, \; K = \frac{U_R^2 K'}{\upsilon^2}, \; M = \frac{\sigma B_0^2 \upsilon}{\rho U_R^2},$$

$$Pr = \frac{\mu C_p}{K_T}, \; Sc = \frac{\upsilon}{D_T}, \; U_R = \left(\upsilon g\beta\Delta T\right)^{1/3},$$

$$L_R = \left(\frac{g\beta\Delta T}{\upsilon^2}\right)^{-1/3}, t_R = \left(g\beta\Delta T\right)^{-2/3}\upsilon^{1/3},$$

$$N = \frac{\beta'\Delta C}{\beta\Delta T}.$$

Introducing above mentioned variables and parameters in equations (1)-(3), we obtain:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+T+NC-M_1 u \qquad (5)$$

$$\frac{\partial T}{\partial t}=\frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} \qquad (6)$$

$$\frac{\partial C}{\partial t}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \qquad (7)$$

where $M_1 = M + \frac{1}{K}$.

The boundary conditions (4) become:

$t \le 0$: $u=0$, $T=0$, $C=0$ for all y,

$t > 0$: $u=1$, $T=1+\varepsilon \cos \omega t$,

$C=1+\varepsilon \cos \omega t$ at y=0,

$u \to 0$, $T \to 0$, $C \to 0$ as $y \to \infty$. (8)

Method of Solution :

Laplace transform of equations (5)-(7) yield:

$$\frac{\partial^2 \bar{u}}{\partial y^2}-(p+M_1)\bar{u}=-\bar{T}-N\bar{C}, \qquad (9)$$

$$\frac{\partial^2 \bar{T}}{\partial y^2}=p\,Pr\,\bar{T}, \qquad (10)$$

$$\frac{\partial^2 \bar{C}}{\partial y^2}=p\,Sc\,\bar{C}. \qquad (11)$$

Where $\bar{u}=\int_0^\infty u\,e^{-pt}\,dt$, $\bar{T}=\int_0^\infty T\,e^{-pt}\,dt$,

$\bar{C}=\int_0^\infty C\,e^{-pt}\,dt$.

Laplace transform of boundary conditions (8) yield:

$\bar{u}=0$, $\bar{T}=0$, $\bar{C}=0$ for all y,

$\bar{u}=\frac{1}{p}$, $\bar{T}=\frac{1}{p}+\varepsilon\frac{p}{p^2+\omega^2}$,

$\bar{C}=\frac{1}{p}+\varepsilon\frac{p}{p^2+\omega^2}$ at $y=0$,

$\bar{u}\to 0$, $\bar{T}\to 0$, $\bar{C}\to 0$ as $y\to\infty$. (12)

Solutions of the equations (9)-(11) satisfying the boundary conditions (12) are given by:

$$\bar{u}=\left[\frac{1}{p}+\frac{K_1}{p(p-\alpha_1)}+\frac{NK_2}{p(p-\beta_1)}+\frac{\varepsilon}{2}\left\{\frac{2pK_1}{(p-\alpha_1)(p^2+\omega^2)}+\frac{2NK_2 p}{(p-\beta_1)(p^2+\omega^2)}\right\}\right]e^{-\sqrt{p+M_1}\,y}$$

$$-\frac{K_1}{p(p-\alpha_1)}e^{-\sqrt{p\,Pr}\,y}-\frac{NK_2}{p(p-\beta_1)}e^{-\sqrt{p\,Sc}\,y}$$

$$-\frac{\varepsilon}{2}\left[\frac{K_1}{(p-\alpha_1)(p-i\omega)}e^{-\sqrt{p\,Pr}\,y}+\frac{K_1}{(p+i\omega)(p-\alpha_1)}e^{-\sqrt{p\,Pr}\,y}\right.$$

$$\left.+\frac{NK_2}{(p-\beta_1)(p-i\omega)}e^{-\sqrt{p\,Sc}\,y}+\frac{NK_2}{(p-\beta_1)(p+i\omega)}e^{-\sqrt{p\,Sc}\,y}\right], \qquad (13)$$

$$\bar{T}=\frac{1}{p}e^{-\sqrt{p\,Pr}\,y}+\varepsilon\frac{p}{(p^2+\omega^2)}e^{-\sqrt{p\,Pr}\,y}, \qquad (14)$$

$$\bar{C}=\frac{1}{p}e^{-\sqrt{p\,Sc}\,y}+\varepsilon\frac{p}{(p^2+\omega^2)}e^{-\sqrt{p\,Sc}\,y}. \qquad (15)$$

Where $\alpha_1 = \frac{M_1}{Pr-1}$, $\beta_1 = \frac{M_1}{Sc-1}$, $K_1 = \frac{1}{Pr-1}$,

$K_2 = \frac{1}{Sc-1}$.

Inverse Laplace transform of the equations (13)-(15) satisfying the boundary conditions (12) is given by :

$$u = H_1(y,1,M_1,t) + \frac{K_1}{\alpha_1}\Big[e^{\alpha_1 t}H_2(y,1,\alpha_1+M_1,t) - H_1(y,1,M_1,t)\Big] + \frac{NK_2}{\beta_1}\Big[e^{\beta_1 t}H_3(y,1,\beta_1+M_1,t) - H_1(y,1,M_1,t)\Big] + \frac{\varepsilon}{2}\Bigg[\frac{2\alpha_1 K_1}{\alpha_1^2+\omega^2}e^{\alpha_1 t}H_2(y,1,\alpha_1+M_1,t) - \frac{K_1}{\alpha_1+i\omega}e^{-i\omega t}H_4(y,1,M_1-i\omega,t) - \frac{K_1}{\alpha_1-i\omega}e^{i\omega t} H_5(y,1,M_1+i\omega,t) + \frac{2NK_2}{\beta_1^2+\omega^2}e^{\beta_1 t}H_6(y,1,M_1+\beta_1,t) - \frac{NK_2}{\beta_1+i\omega}e^{i\omega t}H_4(y,1,M_1-i\omega,t) - \frac{NK_2}{\beta_1-i\omega}e^{-i\omega t}H_5(y,1,M_1+i\omega,t)\Bigg]$$

$$-\frac{K_1}{\alpha_1}\Big[e^{\alpha_1 t}H_7(y,Pr,\alpha_1,t) - H_8(y,Pr,0,t)\Big] - \frac{NK_2}{\beta_1}\Big[e^{\beta_1 t}H_9(y,Sc,\beta_1,t) - H_{10}(y,Sc,0,t)\Big] - \frac{\varepsilon}{2}\Bigg\{\frac{K_1}{\alpha_1-i\omega}\Big[e^{\alpha_1 t}H_7(y,Pr,\alpha_1,t) - e^{i\omega t}H_{11}(y,Pr,i\omega,t)\Big] + \frac{K_1}{\alpha_1+i\omega}\Big[e^{\alpha_1 t}H_7(y,Pr,\alpha_1,t) - e^{-i\omega t}H_{12}(y,Pr,-i\omega,t)\Big] + \frac{NK_2}{\beta_1-i\omega}\Big[e^{\beta_1 t}H_9(y,Sc,\beta_1,t) - e^{i\omega t}H_{13}(y,Sc,i\omega,t)\Big] + \frac{NK_2}{\beta_1+i\omega}\Big[e^{\beta_1 t}H_9(y,Sc,\beta_1,t) - e^{-i\omega t}H_{14}(y,Sc,-i\omega,t)\Big]\Bigg\}, \quad (16)$$

$$T = H_8(y,Pr,0,t) + \frac{\varepsilon}{2}\Big[e^{i\omega t}H_{11}(y,Pr,i\omega,t) + e^{-i\omega t}H_{12}(y,Pr,-i\omega,t)\Big], \quad (17)$$

$$C = H_{10}(y,Sc,0,t) + \frac{\varepsilon}{2}\Big[e^{i\omega t}H_{13}(y,Sc,i\omega,t) + e^{-i\omega t}H_{14}(y,Sc,-i\omega,t)\Big]. \quad (18)$$

Where

$$F(Z_1, Z_2, Z_3, Z_4) = \frac{1}{2}\Bigg[\exp\left(-Z_1\sqrt{Z_2 Z_3}\right)erfc\left(\frac{Z_1\sqrt{Z_2}}{2\sqrt{Z_4}} - \sqrt{Z_3 Z_4}\right) + \exp\left(Z_1\sqrt{Z_2 Z_3}\right)erfc\left(\frac{Z_1\sqrt{Z_2}}{2\sqrt{Z_4}} + \sqrt{Z_3 Z_4}\right)\Bigg].$$

Nusselt number, Sherwood number and Shear-stress:

The rate of heat and mass transfer at the plate $y = 0$ in terms of Nusselt number (Nu) is obtained as follows:

$$(Nu)_{y=0} = -\left(\frac{\partial T}{\partial y}\right)_{y=0}$$

$$= f_1(0,Pr,0,t) + \frac{\varepsilon}{2}\sqrt{Pr}\left(\sqrt{i\omega}\,e^{i\omega t} + \sqrt{-i\omega}\,e^{-i\omega t}\right), \quad (19)$$

The rate of mass transfer at the plate $y = 0$ in terms of Sherwood number (Sh) is obtained as follows:

$$(Sh)_{y=0} = -\left(\frac{\partial C}{\partial y}\right)_{y=0}$$

$$= f_2(0, Sc, 0, t) + \frac{\varepsilon}{2}\sqrt{Sc}\left(\sqrt{i\omega}\, e^{i\omega t} + \sqrt{-i\omega}\, e^{-i\omega t}\right), \quad (20)$$

The Shear-stress at the plate $y = 0$ denoted by $(\tau)_{y=0}$ is obtained in the following form:

$$(\tau)_{y=0} = -\left(\frac{\partial u}{\partial y}\right)_{y=0}$$

$$= \sqrt{M_1} + \frac{K_1}{\alpha_1}\left[\sqrt{\alpha_1 + M_1}\, e^{\alpha_1 t} - \sqrt{M_1}\right]$$

$$+ \frac{NK_2}{\beta_1}\left[\sqrt{\beta_1 + M_1}\, e^{\beta_1 t} - \sqrt{M_1}\right]$$

$$+ \frac{\varepsilon}{2}\left\{\frac{2K_1\alpha_1}{\left(\alpha_1^2 + \omega^2\right)}\sqrt{\alpha_1 + M_1}\, e^{\alpha_1 t}\right.$$

$$- \frac{K_1}{(\alpha_1 + i\omega)}\sqrt{M_1 - i\omega}\, e^{-i\omega t}$$

$$- \frac{K_1}{(\alpha_1 - i\omega)}\sqrt{M_1 + i\omega}\, e^{i\omega t}$$

$$+ \frac{2NK_2\beta_1}{\left(\beta^2 + \omega^2\right)}\sqrt{M_1 + \beta_1}\, e^{\beta_1 t}$$

$$- \frac{NK_2}{(\beta_1 + i\omega)}\sqrt{M_1 - i\omega}\, e^{-i\omega t}$$

$$\left.- \frac{NK_2}{(\beta_1 - i\omega)}\sqrt{M_1 + i\omega}\, e^{i\omega t}\right\}$$

$$- \frac{K_1}{\alpha_1}\left\{\sqrt{\alpha_1 Pr}\, e^{\alpha_1 t} - f_1(0, Pr, 0, t)\right\}$$

$$- \frac{NK_2}{\beta_1}\left\{\sqrt{\beta_1 Sc}\, e^{\beta_1 t} - f_2(0, \mathrm{Sc}, 0, t)\right\}$$

$$- \frac{\varepsilon}{2}\left\{\frac{K_1}{(\alpha_1 - i\omega)}\left[\sqrt{\alpha_1 Pr}\, e^{\alpha_1 t} - \sqrt{i\omega Pr}\, e^{i\omega t}\right]\right.$$

$$+ \frac{K_1}{(\alpha_1 + i\omega)}\left[\sqrt{\alpha_1 Pr}\, e^{\alpha_1 t} - \sqrt{-i\omega Pr}\, e^{-i\omega t}\right]$$

$$+ \frac{NK_2}{(\beta_1 - i\omega)}\left[\sqrt{\beta_1 Sc}\, e^{\beta_1 t} - \sqrt{i\omega Sc}\, e^{i\omega t}\right]$$

$$\left.+ \frac{NK_2}{(\beta_1 + i\omega)}\left[\sqrt{\beta_1 Sc}\, e^{\beta_1 t} - \sqrt{-i\omega Sc}\, e^{-i\omega t}\right]\right\}, \quad (21)$$

Verification of the Problem :

In absence of the concentration equation (3), the corresponding term in momentum equation (1) disappears. Hence equations governing the flow in non-dimensional form reduces to:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + T - \left(M + \frac{1}{K}\right)u, \quad (22)$$

$$\frac{\partial T}{\partial t} = \frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} \quad (23)$$

Non-dimensional boundary conditions reduced to:

$t \leq 0: u = 0, T = 0$ for all $y = 0,$

$t > 0: u = 1, T = 1 + \varepsilon \cos \omega t$ at $y = 0,$

$u \to 0, T \to 0$ as $y \to \infty.$ (24)

This problem is studied by Chaudhary and Jain[6].

Results and Discussion

In order to gain physical insight into the problem, the value of ε is chosen 0.01.

Fig.1 reveals the effect of M, K, Pr on the transient velocity profiles. The velocity decreases with increasing magnetic parameter M because the application of transverse magnetic field will result a resistive type force (Lorentz force) similar to drag force which tends to resist the flow field. The presence of the porous medium resists to the flow field resulting in decrease in the velocity. The behaviour is depicted by the decrease in the velocity as K decreases and when $K \to \infty$ (i.e. the porous medium effect is vanished), the velocity is highest in the flow field. Besides, an increase in Prandtl number increases the velocity.

Fig. 2 depicts the transient temperature profiles against ωt. It is observed that the temperature falls owing to an increase in the value of ωt for both air $(Pr = 0.71)$ and water $(Pr = 7)$. Besides, the magnitude of temperature for air is greater than water. This is due to the fact that thermal conductivity of fluid decreases with increases Pr, resulting a decrease in thermal boundary layer thickness.

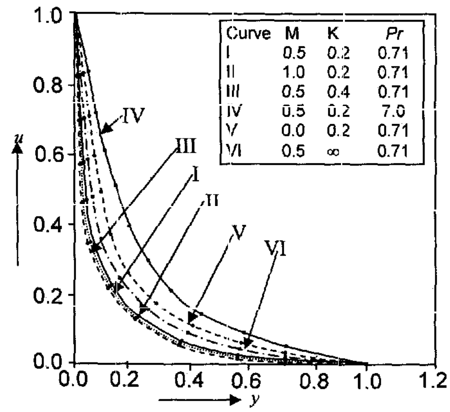

Fig. 1. The velocity distribution versus y for different values of M, K and *Pr* (N=1, ε =0.01, t=1.0, Sc=60 and ωt = π/2)

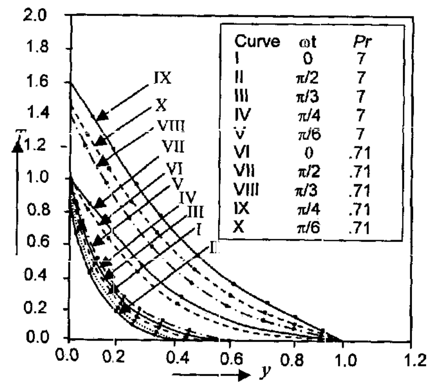

Fig. 2. The temperature distribution versus *y* for different values of ωt and *Pr* (N=1, ε =0.01, t=1.0, Sc= 60, M=0.5 and K= 0.2).

Fig. 3 represents variation in the concentration profiles against y across the boundary layer for different values of Schmidt number (Sc). It is observed that increasing values of Schmidt number lead to decreased concentration profiles. This is due to the fact that the increased values of Schmidt number are equivalent to the increasing chemical molecular diffusivity, which leads to decreased concentration profiles.

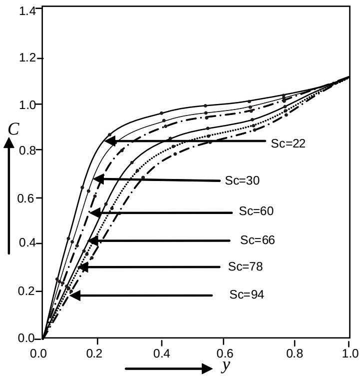

Fig. 3. The concentration distribution versus y for different values of Sc (N=1, ε =0.01, t =1.0, M=0.5, *Pr* =7, K= 0.2 and ωt = π/2).

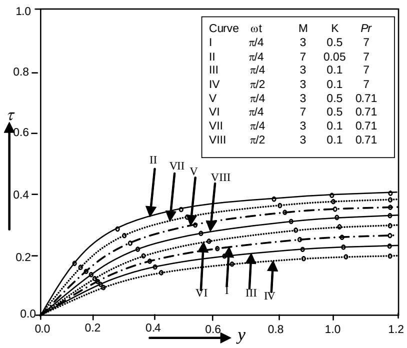

Fig. 4. Skin fraction τ at the plate y=0 for different values of M, K, *Pr* and ωt (N=1, ε =0.01, t=1.0, Sc= 60).

Fig. 4 depicts the skin-friction (τ) at the plate $(y=0)$ for different numerical values ωt , M , K and Pr . It is observed that the skin friction at the plate $(y=0)$ decreases due to increase in the permeability parameter K and for both air and water. We observed that the increase in magnetic parameter *M* increases the skin friction. Besides, an increase in Prandtl number *Pr* decreases the skin friction.

Conclusions

The conclusions of the study are as follows:

(i) The velocity decreases with increase in magnetic parameter *M*.

(ii) The velocity decreases as porous medium decreases, when the porous medium effect is vanished then the velocity is highest.

(iii) An increase in Prandtl number increases the velocity.

(iv) The temperature falls owing to an increase in the value of ωt for both air (*Pr=0.71*) and water (*Pr=7*).

(v) An increase in Schmidt number leads to decreased concentration profiles.

(vi) An increase in Prandtl number decreases the skin friction.

(vii) The skin friction decreases due to increase in the permeability parameter *K* and ωt for both air and water.

(viii) An increase in magnetic parameter *M* increases the skin friction.

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