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Hydrodynamic Unsteady flow and Heat Transfer in an Elastico-Viscous Liquid over an Oscillating Porous Plate in a Rotating Frame

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Abstract

This paper deals with Hydrodynamic unsteady flow and heat transfer in an elastic-viscous liquid over an oscillating porous plate in a rotating frame. The constitutive equation of motion and energy have been developed and solved with the approved boundary conditions. Profiles of velocity of flow and temperature have been drawn varying the fluid parameters involved. The values of skin-friction and the rate of heat transfer are entered in the tables. It is observed that the non-Newtonian parameter, thermal Grashof number, porosity parameter and rotation parameter affect the flow appreciably. Likewise, Prandtl number, porosity parameter and angular velocity influence the temperature field and the rate of heat transfer to a great extent.

Key words: Hydrodynamic unsteady flow, heat transfer, elastic-viscous liquid, porous plate, rotating frame.

1. Introduction

The study of rotational flow of visco-elastic fluids has attracted the attention of many researchers because of its applications in technology. The unsteady flow of a visco-elastic fluid of $Walters'\ B'$ model[1] over a plate has been studied by Gulati[2] Soundalgekar and Puri[3]. Puri[4] alone has analysed the flow induced by oscillations of a plate in a rotating frame of reference. The rotational flow of an elastico-viscous liquid due to the time-dependent rotation of a circular cylinder has been studied by Mukherjee and Mukherjee[5]. Datta and Jana[6] have analysed the flow and heat transfer in an elastico-visous liquid over an oscillating plate in a rotating frame. In all these problems, the effects of porousity have not been accounted for, though the unsteady flow can be changed by the influence of the permeability of porous mediums. Recently Biswal, Roy and Mishra[7] have studied theoretically the magnetohydrodynamic flow of a rotating visco-elastic fluid past an isothermal vertical porous plate. Biswal[8] has analysed the unsteady free convection flow and heat transfer of a visco-elastic fluid past an impulsively started porous wall. Biswal, Dash and Nayak[9] have studied magnetohydrodynamic flow of a viscous conducting fluid past a stretched vertical permeable surface with heat source/sink and chemical reaction. Heat and mass transfer effects on the marginal stability of rotating dusty ferro fluid flowing through a porous medium has been investigated by Patra, Biswal and Mishra[10]. Sahoo, Biswal and Jena[11] have studied the heat and mass transfer effects on magnetohydrodynamic free convection flow of a viscous conducting fluid in a porous vertical channel with Hall current. Mishra, Roy and Biswal[12] have analysed the heat transfer in the rotational flow of a visco-elastic liquid due to rotation of an infinite porous disk in the presence of heat sources. Heat and mass transfer in the unsteady Couette flow of Oldroyd liquid between two horizontal parallel porous plates with heat sources, chemical reaction and soret effect when the lower plate moves with time varying velocity has been investigated by Paikaray and Dash[13], Free convective flow of a visco-elastic fluid inside a porous vertical channel with constant suction and heat sources has been investigated by Dash and Biswal[14]. Unsteady free convection flow of an elasto-viscous fluid past an infinite plate with constant suction and heat sources has been studied by the above authors[15]. Taneja and Jain[16] have analysed the non-linear analysis of free convection flow in a rotating wavy channel with temperature dependent heat source. Molla, Hessain and Yao[17] have investigated the problem of natural convection flow along a vertical wavy surface with uniform surface temperature in the presence of heat generation/absorption.

In the present study, our aim is to analyse the hydrodynamic unsteady flow and heat transfer in an elastico-visous fluid over an oscillating porous plate in a rotating frame of reference.

2. *Formulation of the Problem :*

The mathematical model for $Walters'\ B'$ liquid is given by

$$P_{ik} = -Pg_{ik} + P'_{ik}, \tag{2.1}$$

and

$$P'_{ik} = 2\int_{-\infty}^{t} \psi(t-t') \frac{\partial x^i}{\partial x'^m} \cdot \frac{\partial x^k}{\partial x'^r} e^{(1)mr} dt', \tag{2.2}$$

where P_{ik} is the stress tensor,
P an arbitrary isotropic pressure,
g_{ik} the metric tensor,
$e^{(1)ik}$ the rate of strain tensor and

$$\psi(t-t') = \int_0^\infty \frac{N(\tau)}{\tau} e^{-(t-t')/\tau} d\tau, \tag{2.3}$$

Where $N(\tau)$ is the distribution function of relaxation times. For liquids with short memories (i.e. short relaxation times), the above equations give

$$P^{ik} = 2\eta e^{(1)ik} - 2K_0 \frac{\delta}{\delta t} e^{(1)ik}, \tag{2.4}$$

where $\eta = \int_0^\infty N(\tau) d\tau$ is the limiting viscosity at small rates of shear.

$K_0 = \int_0^\infty \tau N(\tau) d\tau$ and $\frac{\delta}{\delta t}$ denotes the convected differentiation of a tensor.

In a rotating frame of reference, the equation of continuity and the equation of motion are

$$\frac{\partial u_i}{\partial x^i} = 0, \qquad (2.5)$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x^j} + 2 \in_{ijk} V_j \Omega_k^*,$$

$$= -\frac{1}{P}\frac{\partial P^*}{\partial x^i} + \frac{1}{P}\frac{\partial P'_{ik}}{\partial x^k} - \frac{\nu}{K'} u_i, \qquad (2.6)$$

Where u_i is the velocity vector, Ω_k^* the angular velocity vector and P^* is the modified fluid pressure which includes the centrifugal force.

Let us consider the flow of an viscoelastic liquid occupying the space z>0 induced by simple harmonic oscillations U_0 Cos w^*t in the x-direction of an infinite plate z = 0 in a rotating frame of reference. The plate is rotating in unison with an angular velocity Ω^* about the z-axis. Since the plate is infinite, all physical quantities will be functions of z and t only. The equation of continuity together with the no-slip condition at the plate then shows that the z-component of the velocity vanishes everywhere.

As the plate temperature oscillates with the same frequency as that of the oscillating plate, the energy equation becomes

$$\frac{\partial T^*}{\partial t} = \alpha^* \frac{\partial^2 T^*}{\partial z^2} + \frac{\nu}{C_p}\left[\left(\frac{\partial u_x}{\partial z}\right)^2 + \left(\frac{\partial u_y}{\partial z}\right)^2\right] \qquad (2.7)$$

Where α^* is the thermal diffusivity, ν the kinetic co-efficient of viscosity, C_p the specific heat at constant pressure and T^* the temperature of the fluid. In writing equation (2.7), we have neglected elastic dissipation of the fluid.

Introducing the non-dimensional parameters

$$\xi = \frac{U_0 Z}{\nu}, T = \frac{U_0^2 t}{\nu}, u_1 = \frac{u_x}{U_0},$$

$$u_2 = \frac{u_y}{U_0}, \Omega = \frac{\Omega^* \nu}{U_0^2}, R_c = \frac{K^* U_0^2}{\nu^2}, \qquad (2.8)$$

$$\theta = \frac{T^* - T_\infty}{T_w - T_\infty}, P_r = \frac{\alpha^*}{\nu}, E = \frac{U_0^2}{C_p (T_w - T_\infty)},$$

where $\nu = \eta_0 / \rho$ and $K^* = K_0 / \rho$,

u_x, u_y denote the velocity components along x and y-directions, the momentum equation (2.6) with the help of equations (2.1) and (2.4) takes the form

$$\frac{\partial^2 u_1}{\partial \xi^2} - R_c \frac{\partial^3}{\partial T \partial \xi^2} - \frac{\partial u_1}{\partial T} + 2\Omega u_2 + \frac{1}{K^*} u_2 = 0, \qquad (2.9)$$

$$\frac{\partial^2 u_2}{\partial \xi^3} - R_c \frac{\partial^3 u_2}{\partial T \partial \xi^2} - \frac{\partial u_2}{\partial T} + 2\Omega u_1 + \frac{1}{K^*} u_1 = 0 \qquad (2.10)$$

and the energy equation (2.7) becomes

$$P_r \frac{\partial \theta}{\partial T} = \frac{\partial^2 \theta}{\partial \xi^2} + P_r E\left[\left(\frac{\partial u_1}{\partial \xi}\right)^2 + \left(\frac{\partial u_2}{\partial \xi}\right)^2\right] \qquad (2.11)$$

With the boundary conditions

$$\left.\begin{array}{lll} u_1 = e^{iwt}, & u_2 = 0 \text{ at } \xi = 0 & \\ u_1 \to 0, & u_2 \to 0 \text{ at } \xi \to \infty & \\ T^* - T_\infty = (T_w - T_\infty)\,\mathrm{Cos}\, w^* t \text{ at } z = 0 & & \\ T^* \to T_\infty \text{ as } & z \to \infty & \\ \text{Or,} \quad \theta = \mathrm{Cos}\, wt & \text{at } \xi = 0 & \\ \text{And} \quad \theta = 0 & \text{as } \xi \to \infty & \end{array}\right\} \quad (2.12)$$

3. *Solution of the Equations:*

Assuming

$u_1 = f_1 e^{iwt}$ and $u_2 = f_2 e^{iwt}$, (3.1)

equations (2.9) and (2.10) become

$$(1 - iR_c w)\frac{d^2 f_1}{d\xi^2} - iwf_1 + \left(2\Omega + \frac{1}{K^*}\right)f_2 = 0 \quad (3.2)$$

$$\text{and } (1 - iR_c w)\frac{d^2 f_2}{d\xi^2} - iwf_2 - \left(2\Omega - \frac{1}{K^*}\right)f_1 = 0 \quad (3.3)$$

The modified boundary conditions for velocity become

$$\left.\begin{array}{l} f_1 = 1 \text{ and } f_2 = 0 \text{ at } \xi = 0 \\ \text{and} \quad f_1 = 0 \text{ and } f_2 = 0 \text{ at } \xi \to \infty \end{array}\right\} \quad (3.4)$$

Eliminating f_2 from equations (3.2) and (3.3), we have

$$(1 - iR_c w)^2 \frac{d^4 f_1}{d\xi^4} - 2iw(1 - iR_c w)\frac{d^2 f_1}{d\xi^2} - \left(w^2 - 4\Omega^2 - \left(\frac{1}{K^*}\right)^2\right) f_1 = 0 \quad (3.5)$$

Solving equation (3.5), we obtain f_1. Having found f_1, we can find f_2 from equn. (3.2). Obtaining f_1 and f_2, we can find expressions for u_1 and u_2. These expressions are

$$u_1 = \frac{1}{2}\left[e^{\{-A\xi + i(wT - B)\}} + e^{\{-A_1\xi + i(wT - B_1\xi)\}}\right] \text{ for } w > 2\Omega \quad (3.6)$$

$$u_2 = -\frac{i}{2}\left[e^{\{-A\xi + i(wT - B\xi)\}} - e^{\{-A_1\xi + i(wT - B_1\xi)\}}\right] \quad (3.7)$$

and

$$u_1 = \frac{1}{2}\left[e^{\{-A\xi + i(wT - B)\}} + e^{\{-A_2\xi + i(wT + B_1\xi)\}}\right] \text{ for } w < 2\Omega \quad (3.8)$$

$$u_2 = -\frac{i}{2}\left[e^{\{-A\xi + i(wT - B)\}} - e^{\{-A_2\xi + i(wT - B_2\xi)\}}\right] \quad (3.9)$$

where

$$A = \frac{1}{\sqrt{2}}\left(\frac{w + 2\Omega + \frac{1}{K^*}}{1 + R_c^2 w^2}\right)(\alpha - \beta)$$

$$B = \frac{1}{\sqrt{2}}\left(\frac{w + 2\Omega + \frac{1}{K^*}}{1 + R_c^2 w^2}\right)^{1/2}(\alpha + \beta)$$

$$A_1 = \frac{1}{\sqrt{2}}\left(\frac{w - 2\Omega - \frac{1}{K^*}}{1 + R_c^2 w^2}\right)^{1/2}(\alpha - \beta)$$

$$B_1 = \frac{1}{\sqrt{2}}\left(\frac{w - 2\Omega - \frac{1}{K^*}}{1 + R_c^2 w^2}\right)^{1/2}(\alpha + \beta)$$

$$\alpha = \frac{1}{\sqrt{2}}\left\{\left(1 + R_c^2 w^2\right)^{1/2} + 1\right\}^{1/2}$$

$$\beta = \frac{1}{\sqrt{2}}\left\{\left(1 + R_c^2 w^2\right)^{1/2} - 1\right\}^{1/2}$$

$$A_2 = \frac{1}{\sqrt{2}}\left(\frac{2\Omega + \frac{1}{K^*} - w}{1 + R_c^2 w^2}\right)^{1/2}\left\{\left(1 + R_c^2 w^2\right)^{1/2} + R_c w\right\}^{1/2}$$

$$B_2 = \frac{1}{\sqrt{2}}\left(\frac{2\Omega + \frac{1}{K^*} - w}{1+R_c^2 w^2}\right)^{1/2}\left\{\left(1+R_c^2 w^2\right)^{1/2} - R_c w\right\}^{1/2}$$

In the absence of rotation ($\Omega = 0$), $u_2 = 0$ and u_1 is given by

$$u_1 = e^{\left\{-A^*\xi + i\left(wT - B^*\xi\right)\right\}}, \qquad (3.10)$$

where A^* and B^* are obtained from the above constants by putting $\Omega = 0$. It is observed from (3.10) that the two layers merge into one, which oscillates with amplitude $U_0 e^{-A^*\xi}$ and a phase lag $B^*\xi$.

Components of shear stress at the plate ($\xi = 0$):

$$\tau_{zx} = \left[\left(1 - R_c \frac{\partial}{\partial T}\right)\frac{\partial u_1}{\partial \xi}\right]_{\xi=0}, \qquad (3.11)$$

and

$$\tau_{zy} = \left[\left(1 - R_c \frac{\partial}{\partial T}\right)\frac{\partial u_2}{\partial \xi}\right]_{\xi=0}, \qquad (3.12)$$

Using equations (3.6)–(3.9) in equations (3.11) and (3.12), we have

$$\tau_{zx} = -\frac{1}{2} R_1\, e^{i\left(wT + \theta_1\right)}, \qquad (3.13)$$

and

$$\tau_{zy} = -\frac{1}{2} R_2\, e^{i\left(wT - \theta_2\right)}, \qquad (3.14)$$

where

$$\left.\begin{aligned}
R_1 &= \sqrt{2}\left[\left(1+R_c^2 w^2\right)^{1/2}\left\{w + \left(w^2 - 4\Omega^2 - \left(\frac{1}{K^*}\right)^2\right)^{1/2}\right\}\right]^{1/2}, \\
R_2 &= \sqrt{2}\left[\left(1+R_c^2 w^2\right)^{1/2}\left\{w - \left(w^2 - 4\Omega^2 - \left(\frac{1}{K^*}\right)^2\right)^{1/2}\right\}\right]^{1/2}, \\
\operatorname{Tan}\theta_1 &= \frac{(\alpha+\beta) - R_c w(\alpha-\beta)}{(\alpha-\beta) + R_c w(\alpha+\beta)}, \\
\operatorname{Tan}\theta_2 &= \frac{(\alpha-\beta) + R_c w(\alpha+\beta)}{(\alpha+\beta) - R_c w(\alpha-\beta)},
\end{aligned}\right\} \quad w > 2$$

And

$$\left.\begin{aligned}
R_1 &= \left(1+R_c^2w^2\right)^{1/2}\left[\left(A+A_2\right)^2+\left(B-B_2\right)^2\right]^{1/2}\\
R_2 &= \left(1+R_c^2w^2\right)^{1/2}\left[\left(A-A_2\right)^2+\left(B+B_2\right)^2\right]^{1/2}\\
\operatorname{Tan}\theta_1 &= \frac{\left(B-B_2\right)-R_cw\left(A+A_2\right)}{\left(A+A_2\right)+R_cw\left(B-B_2\right)}\\
\operatorname{Tan}\theta_2 &= \frac{\left(A-A_2\right)+R_cw\left(B+B_2\right)}{\left(B+B_2\right)-R_cw\left(A-A_2\right)}
\end{aligned}\right\}\qquad w<2$$

Heat transfer:

Assuming

$$\theta\ (\xi,\ T)=\theta_1\ (\xi)\ e^{iwt}+\theta_2\ (\xi)\ e^{2iwt}, \tag{3.15}$$

and using the equations (3.6) – (3.9), equation (2.11) has been integrated under the boundary conditions (2.12) to give

$$\theta = \exp\left\{-(1+i)\left(P_r\, {w}/{2}\right)^{1/2}\xi\right\}e^{iwt} + P_rE(m_1-in_1)$$

$$\left[\exp\left\{-(1+i)\left(P_rw\right)^{1/2}\xi\right\}-\exp\left\{-\left[\left(A+A_1\right)+i(B+B_1)\right]\xi\right\}\right]$$

$$e^{2iwt} \text{ for } w>2\ \Omega, \tag{3.16}$$

and

$$\theta = \exp\left\{-(1+i)\left(P_r\, {w}/{2}\right)^{1/2}\xi\right\}e^{iwt} + PE(m_4+in_4)$$

$$\left[\exp\left\{-(1+i)\left(P_rw\right)^{1/2}\xi\right\}-\exp\left\{-\left[\left(A+A_1\right)+i(B-B_2)\right]\xi\right\}\right]$$

$$e^{2iwt} \text{ for } w>2\ \Omega, \tag{3.17}$$

where

$$m_1 = \left(w^2-4\Omega^2-4\left({1}/{K^*}\right)^2\right)^{1/2}\left\{w(1-P_r)+\left(w^2-4\Omega^2\right)^{1/2}\right\}/\,2R,$$

$$n_1 = \left(w^2-4\Omega^2-4\left({1}/{K^*}\right)^2\right)^{1/2}P_rR_cw^2/2R,$$

$$R = \left\{(1-P_r)w + \left(w^2 - 4\Omega^2 - 4\left(1/K^*\right)^2\right)^{1/2}\right\}^2 + P_r^2 R_c^2 w^4$$

$m_2 = AA_2 - BB_2$, $n_2 = AB_2 - BA_2$,

$m_3 = m_2 (1 + R_c^2 w^2) - R_c w^2$,

$n_3 = (n_2 + P_r w)(1 + R_c^2 w^2) - w$,

$m_4 = (m_2 m_3 + n_2 n_3)/2 (m_3^2 + n_3^2)$,

$n_4 = (m_2 n_3 - n_2 m_3)/2 (m_3^2 + n_3^2)$,

The rate of heat transfer at the plate ($\xi = 0$) is

$$Nu_1 = \left(\frac{d\theta}{d\xi}\right)_{\xi=0} = -\left[(P_r w/2)^{1/2} (\text{Cos } wT - \text{Sin } wT)\right.$$
$$+ P_r E \{(m_1 m_5 + n_1 n_5) \text{Cos } 2wT,$$
$$\left.- (m_1 n_5 - n_1 m_5) \text{Sin } 2wT\}\right]; \tag{3.18}$$
for $w > 2\Omega$

and

$$Nu_2 = \left(\frac{d\theta}{d\xi}\right)_{\xi=0} = -\left[(P_r w/2)^{1/2} (\text{Cos } wT - \text{Sin } wT)\right.$$
$$+ P_r E \{(m_4 m_6 - n_4 n_6) \text{Cos } 2wT,$$
$$\left.- (m_4 n_6 + n_4 m_6) \text{Sin } 2wT\}\right]; \tag{3.19}$$
for $w < 2\Omega$

where

$m_5 = (P_r w)^{1/2} - (A + A_1)$, $n_5 = (P_r w)^{1/2} - (B + B_1)$

$n_6 = (P_r w)^{1/2} - (A + A_1)$, $n_6 = (P_r w)^{1/2} - (B - B_1)$

4. **Results and Discussions**

The effects of the fluid parameter like the elastic or non-Newtonian parameter R_c, rotation parameter Ω, porosity parameter K^*, Eckert number E and Prandtl number P_r on the velocity, temperature, skin-friction and the rate of heat transfer can be explained from the expressions of u_1, u_2, θ, τ_{zx}, τ_{zy}, Nu_1 and Nu_2 respectively.

Velocity of the fluid:

It is observed that the velocity consists of damped harmonic oscillations with amplitudes $U_0 e^{-A\xi}$ and $U_0 e^{-A_1\xi}$ for $w > 2\Omega$ and having phase lags $B\xi$ and $B_1\xi$ respectively relative to the wall. The depths of penetration or wave lengths of the two layers are respectively $2\pi\nu/U_0 B$ and $2\pi\nu/U_0 B_1$. Because of the presence of elasticity of the fluid, A and B decrease while A and A_1 increase for $w < R_c w \leq 0.58$. This implies that the depths of penetration $2\pi\nu/U_0 B$ and $2\pi\nu/U_0 B_1$ decrease for $0 < R_c w \leq 0.58$ and increase for $0.59 \leq R_c w$.

For $w < 2\Omega$, the velocity consists of two damped oscillations of which one is the same as that for the case $w > 2\Omega$ while the second one has an amplitude $U_0 e^{-A_2\xi}$ and has a phase advance $B_2\xi$ with respect to the plate. The depth of penetration of the second layer is $2\pi\nu/U_0|B_2|$ which decreases with $R_c w$.

For $w = 2\Omega$, we have the phenomenon of resonance similar to the result obtained by Thornley[18] in her study of non-torsional oscillations of an infinite non-porous plate rotating in accord with a viscous fluid. This resonance implies that the whole liquid is affected by the motion of the plate and the oscillation is not confined to a well-defined Ekman layer near the plate.

Fig. 1 illustrates the effects of porosity parameter on f_1, the amplitude of fluid velocity u_1 for a non-Newtonian fluid. It is observed that the decrease in the value of permeability parameter (k^*) reduces the amplitude of the fluid velocity u_1.

The effects of porosity parameter on the amplitude of the fluid velocity u_2 have been exhibited by the curves of Fig. 2. It is noticed that the decrease in the value of the permeability parameter (k^*) reduces the value of f_2. But, f_2 rises with ξ for each value of k^*.

Temperature of the fluid :

Fig. 3 shows the effects of Prandtl number P_r and rotational parameter w on the temperature of the fluid. It is observed that the increase in P_r reduces θ. As the rotation parameter increases, the temperature of the fluid first decreases and then rises beyond $\xi=0.2$ for water ($P_r=7.0$) and beyond $\xi=0.7$ for liquid sodium ($P_r = 1.0$).

Amplitude of the Shear stress:

The amplitudes of shear stresses τ_{zx} and τ_{zy} are given by the values of R_1 and R_2. The phases are represented by $\tan\theta_1$ and $\tan\theta_2$ as mentioned earlier. For $w>2\Omega$, both R_1 and R_2 increase with increase in R_c and w and are fixed while for fixed R_c and Ω, R_1 increase and R_2 decreases with increase in w. For $w<2\Omega$, the values of R_1 and R_2 are given in Tables 1 and 2 respectively.

Table 1

Values of R_1 for $\Omega = 5.0$, $1/K^* = 0.1$

w/R_c	0.0	0.05	0.10
1.0	4.4720350	4.4739285	4.4813534
2.0	4.4720350	4.4828172	4.5052012
3.0	4.4720350	4.4961705	4.5484215
4.0	4.4720350	4.5051123	4.6223899

Table 2

Values of R_2 for $\Omega = 5.0$, $1/K^* = 0.1$

w/R_c	0.0	0.05	0.10
1.0	4.4721357	4.4638161	4.4743623
2.0	4.4721357	4.4721618	4.5051228
3.0	4.4721357	4.4861703	4.5584109
4.0	4.4721357	4.4952919	4.5522815

From the numerical values of R_1 and R_2 entered in the above tables, it is noticed that both R_1 and R_2 increase with increase in R_c (non–Newtonian flow). When $R_c=0$ (Newnotian flow), the frequency parameter w does not produce any effect on the amplitude of the shear stresses τ_{zx} and τ_{zy} under the action of high permeability factor.

The phases of the skin-friction tan θ_1 and tan θ_2 for both $w > 2\Omega$ and $w < 2\Omega$ have been plotted against R_c taking different values of w and Ω (Fig. 4). It is observed that keeping w fixed, tan θ_1 decreases with increase in R_c, whereas tan θ_2 increases with increase in either R_c or w for both cases $w>2\Omega$ and $w \angle 2\Omega$.

For the Newtonian fluid ($R_c=0$), both tan θ_1 and tan θ_2 are independent of frequency w when $w > 2\Omega$. In the presence of elasticity of the fluid ($R_c\neq0$), the frequency w influences the values of tan θ_1 and tan θ_2. However, for large rotation ($w<2\Omega$), the effect of w on tan θ_1 and tan θ_2 is prominent even in the case of Newtonian fluid ($R_c=0$) and for strong permeability.

Rate of heat transfer:

The values of the rate of heat transfer for $wT=\pi/2$, $E=0.02$ and $P_r=0.71$, $\Omega=2$, 5 and for different values of w and R_c are entered in Table 3 below.

Table 3

Rate of heat transfer for $wT = \pi/2$, $E = 0.02$ and $P_r = 0.71$ and $1/K^* = 0.1$

w	Ω/R_c	0.0	0.05	0.10
1.0	5.0	0.528717	0.526528	0.525215
2.0		0.793105	0.792834	0.790254
3.0		0.986762	0.984624	0.982561
5.0	2.0	1.287056	1.243435	1.222374
10.0		1.856986	1.8266724	1.809346
15.0		2.321056	2.205641	2.198762

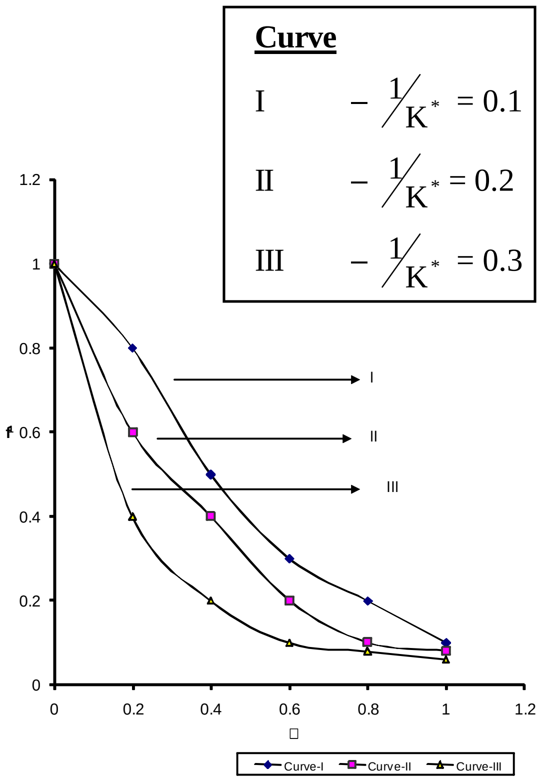

Fig. 1. Effects of porosity parameter on f_1 when $R_c = 0.05$, $E = 0.02$, $\Omega = 5.0$

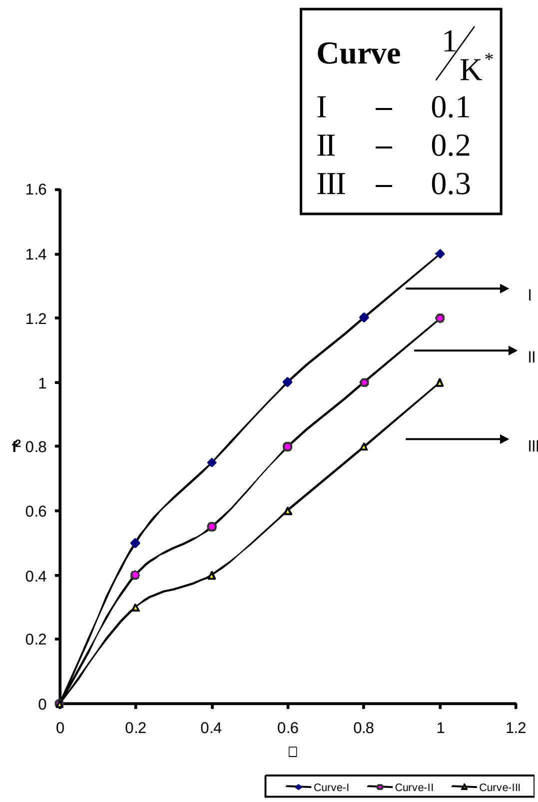

Fig. 2. Effects of porosity parameter on f_2 when $R_c = 0.05$, $E = 0.02$, $\Omega = 5.0$

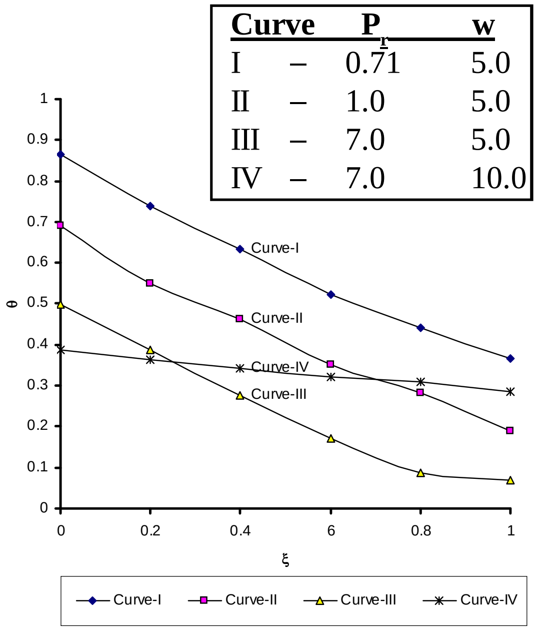

Fig. 3. Effects P_r and w on the temperature when R_c=0.05, $\frac{1}{K^*}$ = 0.1, E=0.02, Ω = 5.0

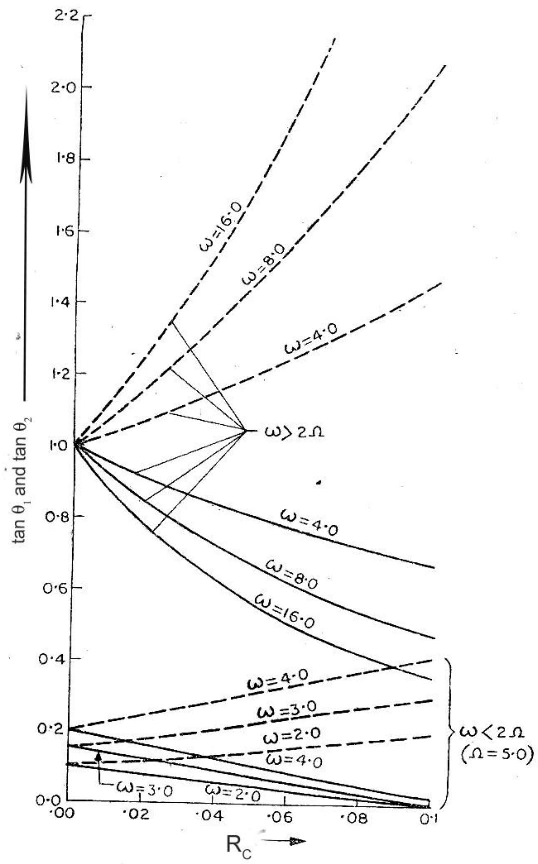

Fig. 4. Graphs of tan θ_1 and tan θ_2 against R_c

The numerical values of the rate of heat transfer presented in Table-3 explain that for fixed w, the rate of heat transfer decreases with increase in R_c while for fixed R_c, the rate of heat transfer increases with increase in w. It is also marked that the rate of heat transfer is less in the case of visco-elastic fluid ($R_c \neq 0$) than those of Newtonian fluid (R_c=0). Similar results were obtained by Datta and Jana[6] without the imposition of a porous media. The only difference is observed that the porosity reduces the velocity of flow which, in turn, results in the production of less friction between the fluid layers and thereby transmission of low thermal energy.

5. Conclusions

Following conclusions are gleaned from the results obtained from our investigation.

i) Exact solutions of the constitutive equations are arrived at.

ii) The velocity of flow has been reduced due to the action of the permeability of the porous medium.

iii) The amplitudes of the shear stresses τ_{zx} and τ_{zy} increase with increase in R_c.

iv) The phase (tan θ_1) of the skin-friction τ_{zx} decreases with increase in R_c and the phase (tan θ_2) of the skin-friction τ_{zy} increases with increase in R_c.

v) The rate of heat transfer decreases with increase in R_c while for fixed R_c, the Nusselt number increases with increase in w.

vi) The rate of heat transfer is less in case of non-Newtonian fluid ($R_c \neq 0$) than those of Newtonian fluid ($R_c=0$).

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