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Estimation of HIV Infectivity Through Two Modes of Transmission

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Abstract

It is now known that the immunodeficiency virus (HIV) can be transmitted through a variety of contact mechanisms that include homosexual intercourse, heterosexual intercourse and needle sharing among intravenous drug abusers. The level of infectivity plays a major role in applying epidemic models to project the spread of the virus and in evaluating various intervention strategies. As a retrovirus HIV shows application error rate and leads to the creation of distinct viral genomes with different immunological properties. This character of HIV is called antigenic diversity. Every contact contributes to the transmission of some HIV which in turn contributes of the increase of antigenic diversity. Total damage crosses the threshold of any one transmission then the HIV infected person reaches the threshold level. Most of the behavior in the stochastic model for the expected time strongly depends on initial conditions. In this paper the expected time and variance of the estimated threshold level through two modes of transmission using three parameter Generalized Rayleigh Distribution has been derived. The analytical results are substantiated with suitable numerical illustrations.

Key words: Threshold, Three Parameter Generalized Rayleigh Distribution, Inter Arrival Time, Expected Time, Seroconversion, Antigenic Diversity.

I. Introduction

The exponential distribution does not provide a reasonable parametric fit for some practical applications where the underlying hazard rates are non constant, presenting monotone shapes. In recent years, in order to overcome such a problem, new classes of models were introduced based on modifications of the exponential distribution. The advantages from both stochastic models and statistical models are efficiently to estimate the unknown parameters and the numbers of infective people and AIDS cases to be found. The antigenic diversity threshold is a particular level of the antigenic diversity of the invading antigen beyond which the human immune system breaks down and a person become seropositive. The expected time to seroconversion is derived under the assumption that the antigenic diversity threshold comprises of two components namely the natural antigenic diversity threshold level of human immune system and the threshold components due to use of ART has been discussed by Palanivel *et.al.*[6].

The mathematical model is developed to obtain the expected time of breakdown point or the expected time to reach the threshold level, in the context of HIV/AIDS with the assumptions that the times between decision epochs are independent and identically distributed (i.i.d.) random variable, the number of exits at each period time for i.i.d. random variables and that the threshold level is a random variable following a exponential distribution has been discussed by Rajivgandhi *et.al.*[7]. For more details about the expected time to cross the threshold level we can see, Esary *et.al.,*[1], Sathiyamoorthi and Kannan[8], Elangovan and Ramajayam[2], Elangovan and Ramajayam[3]. In this paper the expected time and variance of the estimated threshold level through two modes of transmission using three parameter Generalized Rayleigh Distribution has been derived. The analytical results are substantiated with suitable numerical illustrations.

II. Assumptions of the Models :

(i) A person is exposed to HIV infection. At every epoch of contact with an infected there is some contribution to the antigenic diversity.

(ii) Anti Retroviral Therapy is administed to the infected.

(iii) There is a particular level of antigenic diversity of the invading, and it is called the antigenic diversity threshold. If antigenic diversity crosses this threshold the seroconversion takes place.

(iv) The interarrival times between the successive contacts are random variables which are identically independently distributed.

III. Notations

X_i : a continuous random variable denoting the amount of damage/depletion caused to the system due to the exit of persons on the i^{th} occasion of policy announcement, $i=1,2,3,\ldots k$ and $X'_i S$ are i.i.d and $X_i = X$ for all i.

Y_1,Y_2: continuous random variable denoting the threshold levels for the two grades which follows three parameter Generalized Rayleigh distribution.

$g(.)$: The probability density functions (p.d.f) of X_i

$g_k(.)$: The k- fold convolution of $g(.)$ *i.e.*, p.d.f. of $\sum_{i=1}^{k} X_i$

$g^*(.)$: Laplace transform of $g(.)$

$g_k^*(.)$: Laplace transform of $g_k(.)$

$h(.)$: The probability density function of random threshold level which has three parameter generalized Rayleigh distribution and $H(.)$ is the corresponding Probability generating functions.

U : a continuous random variable denoting the inter-arrival times between decision epochs.

$f\,(.)$: p.d.f. of random variable U with corresponding Probability Generating function.

$V_k(t)$: $F_k(t)-F_{k+1}(t)$

$F_k(t)$: Probability that there are exactly 'k' policies decisions in (0,t]

$S(.)$: The survivor function i.e. $P(T>t)$ $1-S(t)=L(t)$

IV. Model Description and Results

Any component exposed to shocks which cause damage to the immune system is likely to fail when the total cumulated damage exceed a level called threshold.

$$F(x,\alpha,\lambda,\mu)=[1-e^{-\lambda_1(x-\mu_1)^2}]^{\alpha}\,[1-e^{-\lambda_2(x-\mu_2)^2}]^{\alpha};$$
$$=1-e^{-\lambda_2(x-\mu_2)^2}-e^{-\lambda_1(x-\mu_1)^2}+ e^{-(\lambda_1(x-\mu_1)^2+\lambda_2(x-\mu_2)^2)}$$

$$\overline{H}(X) = e^{-\lambda_1(x-\mu_1)^2} + e^{-\lambda_2(x-\mu_2)^2}- e^{-(\lambda_1(x-\mu_1)^2+\lambda_2(x-\mu_2)^2)}$$

In general, assuming that the threshold Y follows a three parameter generalized Rayleigh distribution with parameter λ, μ it can be proved that

Transfer of system from to Y_1,Y_2 is also possible. We have the breakdown of the component is at $Y = \max\,(Y_1,Y_2)$

$$P[\max\,(Y_1,Y_2)=P[(Y_1<y)\cap(Y_2<y)]$$
$$= P(Y_1<y) \cap P(Y_2<y)$$

Now that Y_1 and Y_2 follow three parameter generalized Rayleigh distribution with parameter λ_1, λ_2,μ_1,μ_2

$$P(x_i<y)$$
$$=\int_o^{\infty} g_k(x)\overline{H}(X)dx$$
$$=\int_o^{\infty} g_k(x)[e^{-\lambda_1(x-\mu_1)^2} + e^{-\lambda_2(x-\mu_2)^2} -e^{-(\lambda_2(x-\mu_2)^2+\lambda_2(x-\mu_2)^2)}]dx$$

One is interested in an item for which there is a significant individual variation in ability to withstand shocks. There may be no practical way to inspect an individual item to determine its threshold y. In this case, the threshold must be a random variable. The shock survival probability are given by

$$P(x_i<y)=[g^*\lambda_1(1-\mu_1)^2]^k +[g^*\lambda_2(1-\mu_2)^2]^k -[g^*\lambda_1(1-\mu_1)^2 +\lambda_2(1-\mu_2)^2)]^k$$

Survival analysis is a class of statistical methods for studying the occurrence and timing of events.

The survival function $S(t)$ is

$S(t) = P\,(T > t)$

$= \sum_{k=0}^{\infty}$ P{*there are exactly k contacts* $(0,t)$}

$*P${*the total cumulative threshold* $(0,\ t)$}

$$P(T>t) = \sum_{k=0}^{\infty} V_k(t)P(x_i < \max\,(Y_1,Y_2))$$

It may happen that successive shocks become increasingly effective in causing damage, even though they are independent. This means that $V_k(t)$, the distribution function of the k^{th} damage is decreasing in $k=1,2,\ldots$ or each t. It is also known from renewal process that

P (exactly k policy decisions in (0, t))

$= F_k(t) - F_{k+1}(t)$ with $F_0(t)=1$

$$P(T > t) = \sum_{k=0}^{\infty} V_k(t) P(x_i < y)^{(1)}$$

$$= \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\, [g^*\lambda_1(1-\mu_1)^2]^k$$

$$+ \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]\, [g^*\lambda_2(1-\mu_2)^2]^k$$

$$- \sum_{k=0}^{\infty} [F_k(t) - F_{k+1}(t)]$$

$$[g^*(\lambda_1(1-\mu_1)^2]^k + \lambda_2(1-\mu_2)^2)]^k \qquad (2)$$

$P(T < t) = L(t) =$

The distribution of Life time (T)

$$L(t) = 1 - S(t)$$

$$= 1 - \Big\{\sum_{k=0}^{\infty} [F_k(t) - F_{x+1}(t)]\, [g^*\lambda_1(1-\mu_1)^2]^k + \sum_{k=0}^{\infty} [F_k(t) - F_{x+1}(t)]\, [g^*\lambda_2(1-\mu_2)^2]^k - \sum_{k=0}^{\infty} [F_k(t) - F_{x+1}(t)]\, [g^*(\lambda_1(1-\mu_1)^2 + \lambda_2(1-\mu_2)^2)]^k\Big\}$$

$$= 1 - \left\{ \frac{[1 - g^*(\lambda_1 + \lambda_1\mu_1^2 - 2\lambda_1\mu_1)]\, f^*(s)}{[1 - g^*(\lambda_1 + \lambda_1\mu_1^2 - 2\lambda_1\mu_1)\, f^*(s)]} \right.$$

$$\left. + \frac{[1 - g^*(\lambda_2 + \lambda_2\mu_2^2 - 2\lambda_2\mu_2)]\, f^*(s)}{[1 - g^*(\lambda_2 + \lambda_2\mu_2^2 - 2\lambda_2\mu_2)\, f^*(s)]} \right.$$

$$\left. - \frac{\left[1 - g^*\left\{\begin{matrix}\lambda_1 + \lambda_1\mu_1^2 - 2\lambda_1\mu_1 \\ + \lambda_2 + \lambda_2\mu_2^2 - 2\lambda_2\mu_2\end{matrix}\right\}\right] f^*(s)}{\left[1 - g^*\left\{\begin{matrix}\lambda_1 + \lambda_1\mu_1^2 - 2\lambda_1\mu_1 \\ + \lambda_2 + \lambda_2\mu_2^2 - 2\lambda_2\mu_2\end{matrix}\right\} f^*(s)\right]} \right\}$$

Let the random variable U denoting inter arrival time which follows exponential with parameter c. Now $f^*(s) = \left(\frac{c}{c+s}\right)$, substituting in the above equation (3) we get,

$$= 1 - \left\{ \frac{c[1 - g^*(\lambda_1 + \lambda_1\mu_1^2 - 2\lambda_1\mu_1)]}{[c + s - g^*(\lambda_1 + \lambda_1\mu_1^2 - 2\lambda_1\mu_1)\, c]} \right.$$

$$\left. + \frac{c[1 - g^*(\lambda_2 + \lambda_2\mu_2^2 - 2\lambda_2\mu_2)]}{|c + s - g^*(\lambda_2 + \lambda_2\mu_2^2 - 2\lambda_2\mu_2)\, c|} \right.$$

$$\left. - \frac{c\left[1 - g^*\left\{\begin{matrix}\lambda_1 + \lambda_1\mu_1^2 - 2\lambda_1\mu_1 \\ + \lambda_2 + \lambda_2\mu_2^2 - 2\lambda_2\mu_2\end{matrix}\right\}\right]}{\left[c + s - g^*\left\{\begin{matrix}\lambda_1 + \lambda_1\mu_1^2 - 2\lambda_1\mu_1 \\ + \lambda_2 + \lambda_2\mu_2^2 - 2\lambda_2\mu_2\end{matrix}\right\} c\right]} \right\}$$

$E(T) = \frac{-d}{ds} l^*(s)$ Given S = 0

On simplification we get

$E(T)$

$$= \frac{1}{c[1 - g^*(\lambda_1 + \lambda_1\mu_1^2 - 2\lambda_1\mu_1)]}$$

$$+ \frac{1}{c[1 - g^*(\lambda_2 + \lambda_2\mu_2^2 - 2\lambda_2\mu_2)]}$$

$$-\frac{1}{c[1-g^*(\lambda_1+\lambda_1\mu_1^2-2\lambda_1\mu_1+\lambda_2+\lambda_2\mu_2^2-2\lambda_2\mu_2)]}$$

$$g^*_{\bullet}()\sim \exp(\mu),\ g^*(\lambda_1)=\frac{\mu_1}{\mu_1+\lambda_1}$$

$$,g^*(\lambda_2)=\frac{\mu_2}{\mu_2+\lambda_2},$$

$$g^*(\lambda_1\mu_1^2)=\frac{\mu_1}{\mu_1+\lambda_1\mu_1^2}$$

$$g^*(\lambda_2\mu_2^2)=\frac{\mu_2}{\mu_2+\lambda_2\mu_2^2},$$

$$g^*(2\mu_1\lambda_1)=\frac{\mu_1}{\mu_1+2\mu_1\lambda_1},$$

$$g^*(2\mu_2\lambda_2)=\frac{(3)\quad\mu_2}{\mu_2+2\mu_2\lambda_2}$$

$E(T)$

$$=\frac{1}{c\left\{1-\left[\frac{\mu_1}{\mu_1+\lambda_1}+\frac{\mu_1}{\mu_1+\lambda_1\mu_1^2}-\frac{\mu_1}{\mu_1+2\mu_1\lambda_1}\right]\right\}}$$

$$+\frac{1}{c\left\{1-\left[\frac{\mu_2}{\mu_2+\lambda_2}+\frac{\mu_2}{\mu_2+\lambda_2\mu_2^2}-\frac{\mu_2}{\mu_2+2\mu_2\lambda_2}\right]\right\}}$$

$$-\frac{1}{c\left\{1-\left[\begin{array}{l}\frac{\mu_1}{\mu_1+\lambda_1}+\frac{\mu_1}{\mu_1+\lambda_1\mu_1^2}-\frac{\mu_1}{\mu_1+2\mu_1\lambda_1}\\+\frac{\mu_2}{\mu_2+\lambda_2}+\frac{\mu_2}{\mu_2+\lambda_2\mu_2^2}-\frac{\mu_2}{\mu_2+2\mu_2\lambda_2}\end{array}\right]\right\}}\quad(4)$$

Let,

I_1

$$=\frac{1}{c\left\{1-\left[\frac{\mu_1}{\mu_1+\lambda_1}+\frac{\mu_1}{\mu_1+\lambda_1\mu_1^2}-\frac{\mu_1}{\mu_1+2\mu_1\lambda_1}\right]\right\}}$$

$$=\frac{\begin{array}{c}(\lambda_1^2+\mu_1^2+2\mu_1^3+\lambda_1\mu_1^3+2\mu_1^3\lambda_1^2\\+\mu_1^4+2\lambda_1\mu_1^4+2\lambda_1\mu_1^2)\end{array}}{c[\mu_1^2+2\lambda_1^2\mu_1^3-2\mu_1^3+2\lambda_1\mu_1^3+\mu_1^4}\quad(5)$$

I_2

$$=\frac{1}{c\left\{1-\left[\frac{\mu_2}{\mu_2+\lambda_2}+\frac{\mu_2}{\mu_2+\lambda_2\mu_2^2}-\frac{\mu_2}{\mu_2+2\mu_2\lambda_2}\right]\right\}}$$

$$=\frac{\begin{array}{c}(\lambda_2^2+\mu_2^2+2\mu_2^3+\lambda_2\mu_2^3+2\mu_2^3\lambda_2^2\\+\mu_2^4+2\lambda_2\mu_2^4+2\lambda_2\mu_2^2)\end{array}}{c[\mu_2^2+2\lambda_2^2\mu_2^3-2\mu_2^3+2\lambda_2\mu_2^3+\mu_2^4}\quad(6)$$

I_3

$$=-\frac{1}{c\left\{1-\left[\begin{array}{l}\frac{\mu_1}{\mu_1+\lambda_1}+\frac{\mu_1}{\mu_1+\lambda_1\mu_1^2}-\frac{\mu_1}{\mu_1+2\mu_1\lambda_1}\\+\frac{\mu_2}{\mu_2+\lambda_2}+\frac{\mu_2}{\mu_2+\lambda_2\mu_2^2}-\frac{\mu_2}{\mu_2+2\mu_2\lambda_2}\end{array}\right]\right\}}$$

On simplification we get

$$\begin{aligned}I_{31}=&\lambda_1\lambda_2+\lambda_1\mu_2+\lambda_1\lambda_2^2\mu_2+\lambda_1\lambda_2\mu_2^2\\&+\lambda_12\lambda_2^2+2\lambda_1\lambda_2\mu_2+2\lambda_1\lambda_2^3\mu_2\\&\qquad+2\lambda_2^2\lambda_1\mu_2^2\\&+\mu_1\lambda_2+\mu_1\mu_2+\mu_1\lambda_2^2\mu_2+\mu_1\lambda_2\mu_2^2\\&+2\mu_1\lambda_2^2+2\mu_1\mu_2\lambda_2+2\mu_1\mu_2\lambda_2^3\\&\qquad+2\mu_1\mu_2^2\lambda_2^2\\&+\lambda_1^2\lambda_2\mu_1+\mu_1\mu_2\lambda_1^2+\mu_1\mu_2\lambda_1^2\lambda_2^2\\&+\lambda_1^2\mu_1\lambda_2\mu_2^2+\lambda_1^2\,2\lambda_2^2\mu_1+2\mu_1\mu_2\lambda_2\lambda_1^2\\&+2\mu_1\mu_2\lambda_1^2\lambda_2^3+2\lambda_1^2\mu_1\lambda_2^2\mu_2^2\\&+\lambda_1\lambda_2\mu_1^2+\lambda_1\mu_2\mu_1^2+\lambda_1\mu_2\mu_1^2\lambda_2^2\\&+\lambda_1\lambda_2\mu_2^2\mu_1^2+2\mu_1^2\lambda_1\lambda_2^2+2\mu_1^2\lambda_1\mu_2\lambda_2\\&+2\mu_1^2\lambda_1\lambda_2^3\mu_2+4\lambda_1^2\lambda_2^3\mu_2\\&\qquad+2\mu_1^2\lambda_1\mu_2^2\lambda_2^2\\&+2\lambda_1^2\lambda_2+2\lambda_1^2\mu_2+2\lambda_1^2\lambda_2^2\mu_2\end{aligned}$$

$+2\lambda_1^2\lambda_2\mu_2^2+4\lambda_1^2\lambda_2^2+4\lambda_1^2\lambda_2\mu_2$
$+4\lambda_1^2\mu_2^2\lambda_2^2$
$+2\lambda_1\lambda_2\mu_1+2\lambda_1\mu_1\mu_2+2\lambda_1\mu_1\lambda_2^2\mu_2$
$+2\lambda_1\mu_1\lambda_2\mu_2^2+4\lambda_1\mu_1\lambda_2^2+4\lambda_1\mu_1\lambda_2\mu_2$
$+4\lambda_1\mu_1\lambda_2^3\mu_2+4\lambda_1\mu_1\mu_2^2\lambda_2^2$
$+2\lambda_1^3\mu_1\lambda_2+2\lambda_1^3\mu_1\mu_2+2\lambda_1^3\mu_1\lambda_2^2\mu_2$
$+4\lambda_1^3\mu_1\lambda_2^2+2\lambda_1^3\mu_1\lambda_2\mu_2^2+4\lambda_1^3\mu_1\lambda_2\mu_2$
$+4\lambda_1^3\mu_1\lambda_2^3\mu_2+4\lambda_1^3\mu_1\mu_2^2\lambda_2^2$
$+2\lambda_1^2\mu_1^2\lambda_2$
$+2\lambda_1^2\mu_1^2\mu_2+2\lambda_1^2\mu_1^2\lambda_2^2\mu_2$
$+2\lambda_1^2\mu_1^2\lambda_2\mu_2^2+4\lambda_1^2\mu_1^2\lambda_2^2$
$+4\lambda_1^2\mu_1^2\lambda_2\mu_2$
$+4\lambda_1^2\mu_1^2\lambda_2^3\mu_2+4\lambda_1^2\mu_1^2\mu_2^2\lambda_2^2$

$I_{32}= \lambda_1\lambda_2+2\lambda_2^2\lambda_1\mu_2+\lambda_1\lambda_2\mu_2^2$
$+2\lambda_1\lambda_2\mu_2+2\lambda_1\lambda_2^3\mu_2+2\lambda_1^2\lambda_2\mu_1$
$+2\lambda_1^2\lambda_2^2\mu_1\mu_2+2\lambda_1^2\lambda_2\mu_1\mu_2^2$
$+4\lambda_1^2\lambda_2\mu_1\mu_2+4\lambda_1^2\lambda_2^3\mu_1\mu_2$
$+2\lambda_1^2\lambda_1^2\mu_1\mu_2^2$
$+\lambda_1\lambda_2\mu_1^2+\lambda_1\lambda_2\mu_1^2\mu_2^2+2\lambda_1\mu_1^2\lambda_2^2$
$+2\lambda_1\lambda_2\mu_1^2\mu_2+2\lambda_1\lambda_2^3\mu_1^2\mu_2$
$+2\lambda_1\lambda_2\mu_1\mu_2$
$+4\lambda_2^2\lambda_1\mu_1+4\lambda_1\mu_1\lambda_2^3\mu_2+2\lambda_1^3\mu_1\lambda_2$
$+2\lambda_1^3\mu_1\mu_2+4\lambda_1^3\lambda_2^2\mu_1\mu_2+2\lambda_1^3\lambda_2\mu_1\mu_2^2$
$+4\lambda_1^3\lambda_2^2\mu_1+4\lambda_1^3\mu_1\mu_2\lambda_2+4\lambda_1^3\lambda_2^3\mu_1\mu_2$
$+2\lambda_1^2\mu_1^2\mu_2-2\lambda_1^2\lambda_2\mu_2^2-2\lambda_1\mu_1\lambda_2$
$-4\lambda_1\mu_1\mu_2-4\lambda_1\lambda_2^2\mu_1\mu_2-4\lambda_1\lambda_2\mu_1\mu_2^2$
$-12\lambda_1\mu_1\lambda_2^2-12\lambda_1\lambda_2\mu_1\mu_2-8\lambda_1\lambda_2^3\mu_1\mu_2$
$-8\lambda_1\mu_1\lambda_2^3\mu_2^2-2\lambda_1^2\mu_1^2\mu_2+\lambda_1^2\mu_1\mu_2$
$+2\lambda_1^2\lambda_2^2\mu_1-\mu_1\mu_2-2\lambda_1^2\mu_2-2\mu_1\mu_2\lambda_1^3$
$-2\mu_1^2\mu_2\lambda_1^2-2\mu_1\lambda_2^2-2\lambda_2^2\lambda_1\mu_1^2$
$-4\lambda_2^2\lambda_1^2-4\lambda_2^2\lambda_1^3\mu_1-4\lambda_2^2\lambda_1^2\mu_1^2$
$-4\lambda_2^2\lambda_1\mu_2-4\mu_1\mu_2\lambda_2-4\mu_1\mu_2\lambda_1^2\lambda_2$
$-4\lambda_2\lambda_1\mu_1^2\mu_2-8\lambda_1^2\lambda_2\mu_2-8\lambda_1^3\mu_1\mu_2\lambda_2$
$-8\lambda_2\mu_2\mu_1^2\lambda_1^2-2\lambda_2^2\mu_2^2\mu_1$
$-4\lambda_2^2\mu_2^2\lambda_1^2$
$-4\lambda_2^2\mu_2^2\lambda_1^2\mu_1^2+\lambda_2^2\mu_1\mu_2+\lambda_2^2\lambda_1^2\mu_2\mu_1$
$+2\lambda_1\lambda_2^2\mu_1^2\mu_2+2\lambda_2^2\lambda_1^2\mu_2$
$+2\lambda_2^2\lambda_1^2\mu_2\mu_1^2$

$$I_3 = -\frac{I_{31}}{c\left(I_{32}\right)} \tag{7}$$

Equations (5), (6), (7) substituting in the above equation (4) we get

$$E(T) = I_1 + I_2 - I_3$$

$$E(T^2) = \frac{-d^2}{ds^2} l^*(s) \text{ Given S=0}$$

$$E(T^2)$$

$$= \frac{2}{c^2\left(1-\left[\frac{\mu_1}{\mu_1+\lambda_1}+\frac{\mu_1}{\mu_1+\lambda_1\mu_1^2}-\frac{\mu_1}{\mu_1+2\mu_1\lambda_1}\right]\right)^2}$$

$$+\frac{2}{c^2\left(1-\left[\frac{\mu_2}{\mu_2+\lambda_2}+\frac{\mu_2}{\mu_2+\lambda_2\mu_2^2}-\frac{\mu_2}{\mu_2+2\mu_2\lambda_2}\right]\right)^2}$$

$$-\frac{2}{c^2\left(1-\left[\begin{array}{l}\frac{\mu_1}{\mu_1+\lambda_1}+\frac{\mu_1}{\mu_1+\lambda_1\mu_1^2}-\frac{\mu_1}{\mu_1+2\mu_1\lambda_1}\\+\frac{\mu_2}{\mu_2+\lambda_2}+\frac{\mu_2}{\mu_2+\lambda_2\mu_2^2}-\frac{\mu_2}{\mu_2+2\mu_2\lambda_2}\end{array}\right]\right)^2} \tag{8}$$

Let,

I_1

$$= \frac{2}{c^2\left[1-\left(\frac{\mu_1}{\mu_1+\lambda_1}+\frac{\mu_1}{\mu_1+\lambda_1\mu_1^{2}}-\frac{\mu_1}{\mu_1+2\mu_1\lambda_1}\right)\right]^2}$$

$$= \frac{{}_{2}(\lambda_1^{2}+\mu_1^{2}+2\mu_1^{3}+\lambda_1\mu_1^{3}+2\mu_1^{3}\lambda_1^{2}+\mu_1^{42} + 2\lambda_1\mu_1^{4}+2\lambda_1\mu_1^{2})}{c^2\left[\mu_1^{2}+2\lambda_1^{2}\mu_1^{3}-2\mu_1^{3}+2\lambda_1\mu_1^{3}+\mu_1^{4}\right]^2} \quad (9)$$

I_2

$$= \frac{2}{c^2\left[1-\left(\frac{\mu_2}{\mu_2+\lambda_2}+\frac{\mu_2}{\mu_2+\lambda_2\mu_2^{2}}-\frac{\mu_2}{\mu_2+2\mu_2\lambda_2}\right)\right]^2}$$

$$= \frac{{}_{2}(\lambda_2^{2}+\mu_2^{2}+2\mu_2^{3}+\lambda_2\mu_2^{3}+2\mu_2^{3}\lambda_2^{2^2} + \mu_2^{4}+2\lambda_2\mu_2^{4}+2\lambda_2\mu_2^{2})}{c^2\left[\mu_2^{2}+2\lambda_2^{2}\mu_2^{3}-2\mu_2^{3}+2\lambda_2\mu_2^{3}+\mu_2^{4}\right]^2} \quad (10)$$

I_3

$$= - \frac{2}{c^2\left[1-\left(\frac{\mu_1}{\mu_1+\lambda_1}+\frac{\mu_1}{\mu_1+\lambda_1\mu_1^{2}}-\frac{\mu_1}{\mu_1+2\mu_1\lambda_1} + \frac{\mu_2}{\mu_2+\lambda_2}+\frac{\mu_2}{\mu_2+\lambda_2\mu_2^{2}}-\frac{\mu_2}{\mu_2+2\mu_2\lambda_2}\right)\right]^2}$$

On simplification we get

$I_{31}=\lambda_1\lambda_2+\lambda_1\mu_2+\lambda_1\lambda_2^{2}\mu_2+\lambda_1\lambda_2\mu_2^{2}$
$+\lambda_1 2\lambda_2^{2}+2\lambda_1\lambda_2\mu_2+2\lambda_1\lambda_2^{2}\mu_2$
$+2\lambda_2^{2}\lambda_2\mu_2^{2}$
$+\mu_1\lambda_2+\mu_1\mu_2+\mu_1\lambda_2^{2}\mu_2+\mu_1\lambda_2\mu_2^{2}$
$+2\mu_1\lambda_2^{2}+2\mu_1\mu_2\lambda_2+2\mu_1\mu_2\lambda_2^{3}$
$+2\mu_1\mu_2^{2}\lambda_2^{2}+\lambda_1^{2}\lambda_2\mu_1+\mu_1\mu_2\lambda_1^{2}\mu_2$
$+\mu_1\mu_2\lambda_1^{2}\lambda_2^{2}+\lambda_1^{2}\mu_1\lambda_2\mu_2^{2}+\lambda_1^{2}2\lambda_2^{2}+\mu_1$
$+2\mu_1\mu_2\lambda_2\lambda_1^{2}+2\mu_1\mu_2\lambda_1^{2}\lambda_2^{3}$
$+2\lambda_1^{2}\mu_1\lambda_2^{2}\mu_2^{2}$
$+\lambda_1\lambda_2\mu_1^{2}+\lambda_1\mu_2\mu_1^{2}+\lambda_1\mu_2\mu_1^{2}\lambda_2^{2}$
$+\lambda_1\lambda_2\mu_2^{2}\mu_1^{2}+2\mu_1^{2}\lambda_1\lambda_2^{2}+2\mu_1^{2}\lambda_1\mu_2\lambda_2$
$+2\mu_1^{2}\lambda_1\lambda_2^{3}\mu_2+4\lambda_1^{2}\lambda_2^{3}\mu_2$
$+2\mu_1^{2}\lambda_1\mu_2^{2}\lambda_2^{2}$
$+2\lambda_1^{2}\lambda_2+2\lambda_1^{2}\mu_2+2\lambda_1^{2}\lambda_2^{2}\mu_2$
$+2\lambda_1^{2}\lambda_2\mu_2^{2}+4\lambda_1^{2}\lambda_2^{2}+4\lambda_1^{2}\lambda_2\mu_2$
$+4\lambda_1^{2}\mu_2^{2}\lambda_2^{2}+2\lambda_1\lambda_2\mu_1+2\lambda_1\mu_1\mu_2$
$+2\lambda_1\mu_1\lambda_2^{2}\mu_2+2\lambda_1\mu_1\lambda_2\mu_2^{2}+4\lambda_1\mu_1\lambda_2^{2}$
$+4\lambda_1\mu_1\lambda_2\mu_2+4\lambda_1\mu_1\lambda_2^{3}\mu_2$
$+4\lambda_1\mu_1\mu_2^{2}\lambda_2^{2}$
$+2\lambda_1^{3}\mu_1\lambda_2+2\lambda_1^{3}\mu_1\mu_2+2\lambda_1^{3}\mu_1\lambda_2^{2}\mu_2$
$+4\lambda_1^{3}\mu_1\lambda_2^{2}+2\lambda_1^{3}\mu_1\lambda_2\mu_2^{2}+4\lambda_1^{3}\mu_1\lambda_2\mu_2$
$+4\lambda_1^{3}\mu_1\lambda_2^{3}\mu_2+4\lambda_1^{3}\mu_1\mu_2^{2}\lambda_2^{2}$
$+2\lambda_1^{2}\mu_1^{2}\lambda_2$
$+2\lambda_1^{2}\mu_1^{2}\mu_2+2\lambda_1^{2}\mu_1^{2}\lambda_2^{2}\mu_2$
$+2\lambda_1^{2}\mu_1^{2}\lambda_2\mu_2^{2}$
$+4\lambda_1^{2}\mu_1^{2}\lambda_2^{2}+4\lambda_1^{2}\mu_1^{2}\lambda_2\mu_2$
$+4\lambda_1^{2}\mu_1^{2}\lambda_2^{3}\mu_2+4\lambda_1^{2}\mu_1^{2}\mu_2^{2}\lambda_2^{2}$

$I_{32}=\lambda_1\lambda_2+2\lambda_2^{2}\lambda_1\mu_2+\lambda_1\lambda_2\mu_2^{2}$
$+2\lambda_1\lambda_2\mu_2$
$+2\lambda_1\lambda_2^{3}\mu_2+2\lambda_1^{2}\lambda_2\mu_1+2\lambda_1^{2}\lambda_2^{2}\mu_1\mu_2$
$+2\lambda_1^{2}\lambda_2\mu_1\mu_2^{2}+4\lambda_1^{2}\lambda_2\mu_1\mu_2+4\lambda_1^{2}\lambda_2^{3}\mu_1\mu_2$
$+2\lambda_2^{2}\lambda_1^{2}\mu_1\mu_2^{2}+\lambda_1\lambda_2\mu_1^{2}+\lambda_1\lambda_2\mu_1^{2}\mu_2^{2}$
$+2\lambda_1\mu_1^{2}\lambda_2^{2}+2\lambda_1\lambda_2\mu_1^{2}\mu_2+2\lambda_1\lambda_2^{3}\mu_1^{2}\mu_2$
$+2\lambda_1\lambda_2\mu_1\mu_2^{2}+4\lambda_2^{2}\lambda_1\mu_1+4\lambda_1\mu_1\lambda_2^{3}\mu_2$
$+2\lambda_1^{3}\mu_1\lambda_2+2\lambda_1^{3}\mu_1\mu_2+4\lambda_1^{3}\lambda_2^{2}\mu_1\mu_2$
$+2\lambda_1^{3}\lambda_2\mu_1\mu_2^{2}+4\lambda_1^{3}\lambda_2^{2}\mu_1+4\lambda_1^{3}\mu_1\mu_2\lambda_2$

$$+4\lambda_1^3\lambda_2^3\mu_1\mu_2+2\lambda_1^2\mu_1^2\mu_2-2\lambda_1^2\lambda_2\mu_2^2$$
$$-2\lambda_1\mu_1\lambda_2-4\lambda_1\mu_1\mu_2-4\lambda_1\lambda_2^2\mu_1\mu_2$$
$$-4\lambda_1\lambda_2\mu_1\mu_1^2-12\lambda_1\mu_1\lambda_2^2-12\lambda_1\lambda_2\mu_1\mu_2$$
$$-8\lambda_1\lambda_2^3\mu_1\mu_2-8\lambda_1\mu_1\lambda_2^3\mu_2^2-2\lambda_1^2\mu_1^2\mu_2$$
$$+\lambda_1^2\mu_1\mu_2+2\lambda_1^2\lambda_2^2\mu_1-\mu_1\mu_2$$
$$-2\lambda_1^2\mu_2-2\mu_1\mu_2\lambda_1^3-2\mu_1^2\mu_2\lambda_1^2$$
$$-2\mu_1\lambda_2^2-2\lambda_2^2\lambda_1\mu_1^2-4\lambda_1^2\lambda_1^2$$
$$-4\lambda_2^2\lambda_1^3\mu_1-4\lambda_2^2\lambda_1^2\mu_1^2-4\lambda_2^2\lambda_1\mu_2$$
$$-4\mu_1\mu_2\lambda_2-4\mu_1\mu_2\lambda_1^2\lambda_2-4\lambda_2\lambda_1\mu_1^2\mu_2$$
$$-8\lambda_1^2\lambda_2\mu_2-8\lambda_1^3\mu_1\mu_2\lambda_2-8\lambda_2\mu_2\mu_1^2\lambda_1^2$$
$$-2\lambda_2^2\mu_2^2\mu_1-4\lambda_2^2\mu_2^2\lambda_1^2$$
$$-4\lambda_2^2\mu_2^2\lambda_1^2\mu_1^2$$
$$+\lambda_2^2\mu_1\mu_2+\lambda_2^2\lambda_1^2\mu_2\mu_1+2\lambda_1\lambda_2^2\mu_1^2\mu_2$$
$$+2\lambda_2^2\lambda_1^2\mu_2+2\lambda_2^2\lambda_1^2\mu_2\mu_1^2$$

$$J_3 - - \frac{2\left(J_{31}\right)^2}{c^2\left(J_{32}\right)^2} \tag{11}$$

Equations (9), (10), (11) substituting in the above equation (8) we get

$$E(T^2) = J_1 + J_2 - J_3 \tag{12}$$
$$V(T) = E(T^2) - [E(T)]^2$$

On simplification we get,

$$= (I_1 + I_2 + I_3)$$
$$- (I_1 + I_2 + I_3)^2 \tag{13}$$

V. Numerical Examples :

The behavior of $E(T)$ and $V(T)$ due to the changes in the different parameters associated with the distribution of the random variables in the model is explained by taking the numerical examples.

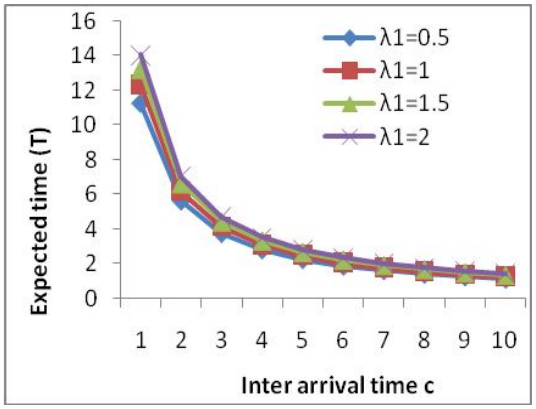

Figure 1a. Variation in $E(T)$ for Changes c

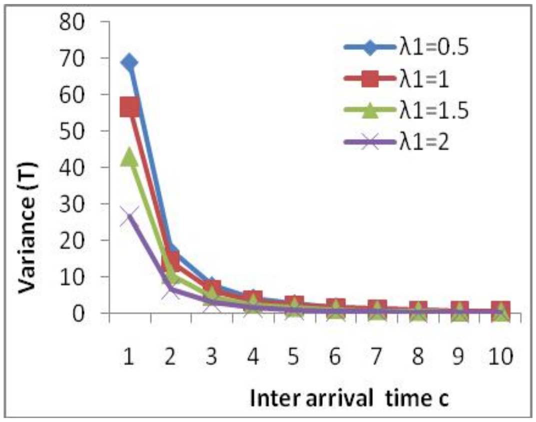

Figure 1b. Variation in $V(T)$ for Changes c

When λ_2, μ_1, μ_2 are kept fixed with other parameters increasing λ_1 and inter-arrival time 'c'. The inter-arrival time follows exponential distribution, which is an increasing parameter. Therefore, the value of the expected time $E(T)$ to cross the threshold is decreasing, for all cases of the parameter value λ_1=0.5,1.0,1.5,2.0. When the value of the parameter λ_1 increases, the expected time is decreasing, this is observed in figure1a.Similar findings are observed in $V(T)$ which showed in figure 1b.

Table 1. Variation in *E*(*T*) and V(T) for changes in the inter arrival times between successive Contacts, keeping $\lambda_2=0.2$, $\mu_1=0.4$, $\mu_2=0.3$ fixed

c	$\lambda_1=0.5$		$\lambda_1=1.0$		$\lambda_1=1.5$		$\lambda_1=2.0$	
	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
1	11.283	69.010	12.339	56.713	13.255	43.024	14.058	26.632
2	5.6414	17.252	6.1698	14.178	6.6277	10.756	7.0293	6.6580
3	3.7612	7.6678	4.1132	6.3015	4.4184	4.7805	4.6862	2.9591
4	2.8207	4.3131	3.0849	3.5446	3.3138	2.6890	3.5146	1.6645
5	2.2566	2.7604	2.4679	2.2685	2.6510	1.7209	2.8117	1.0652
6	1.8805	1.9169	2.0566	1.5753	2.2092	1.1951	2.3431	0.7397
7	1.6118	1.4083	1.7628	1.1574	1.8936	0.8780	2.0083	0.5435
8	1.4103	1.0782	1.5424	0.8861	1.6569	0.6722	1.7573	0.4161
9	1.2536	0.8519	1.3710	0.7001	1.4728	0.5311	1.5620	0.3287
10	1.1283	0.6901	1.2339	0.5671	1.3255	0.4302	1.4058	0.2663

Table 2. Variation in *E*(*T*) and V(T) for changes in the inter arrival times between successive Contacts, keeping $\lambda_1=0.4$, $\lambda_2=0.2$, $\mu_2=0.3$ fixed

C	$\mu_1=0.5$		$\mu_1=1.0$		$\mu_1=1.5$		$\mu_1=2.0$	
	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
1	11.057	74.280	11.447	78.309	10.853	78.248	10.187	78.220
2	5.5285	18.570	5.7236	19.577	5.4267	19.562	5.0937	19.555
3	3.6857	8.2533	3.8157	8.7010	3.6178	8.6943	3.3958	8.6912
4	2.7642	4.6425	2.8618	4.8943	2.7133	4.8905	2.5468	4.8888
5	2.2114	2.9712	2.2894	3.1323	2.1707	3.1299	2.0375	3.1288
6	1.8428	2.0633	1.9078	2.1752	1.8089	2.1735	1.6979	2.1728
7	1.5795	1.5159	1.6353	1.5981	1.5505	1.5969	1.4553	1.5963
8	1.3821	1.1606	1.4309	1.2235	1.3566	1.2226	1.2734	1.2222
9	1.2285	0.9170	1.2719	0.9667	1.2059	0.9660	1.1319	0.9656
10	1.1057	0.7428	1.1447	0.7830	1.0853	0.7824	1.0187	0.7822

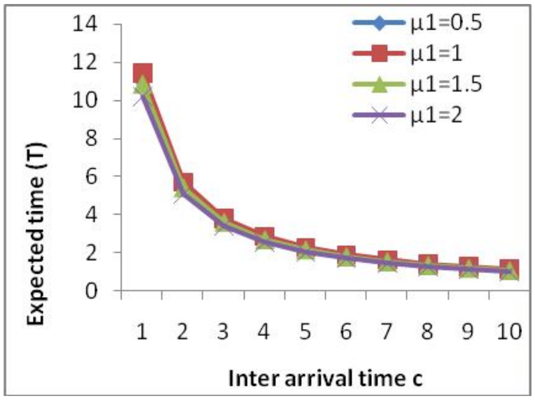

Figure 2a. Variation in $E(T)$ for Changes c

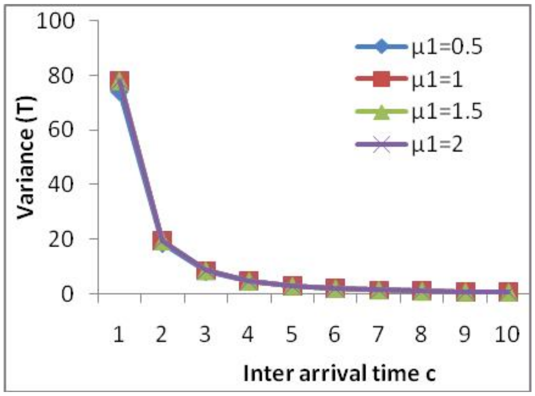

Figure 2b. Variation in $V(T)$ for Changes c

When λ_1, λ_2, μ_2 are kept fixed with other parameters increasing μ_1 and inter-arrival time 'c'. As the inter-arrival time increases, the value of the expected time E (T) to cross the threshold is found to be decreasing, in all the cases of the parameter value μ_1=0.5,1.0,1.5,2.0. when the value of the parameter μ_1 increases, the expected time is decreasing. This is indicated in figure 2a. Similar findings are observed in $V(T)$ which showed in figure 2b.

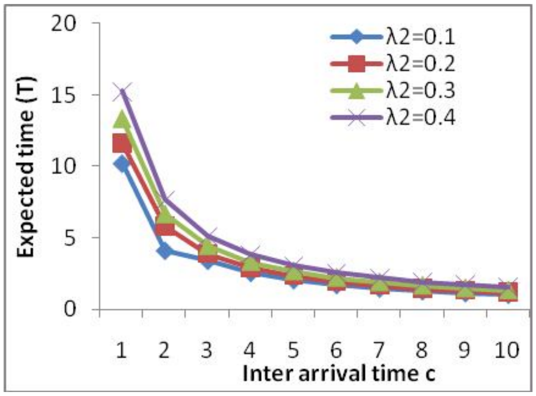

Figure 3a. Variation in $E(T)$ for Changes c

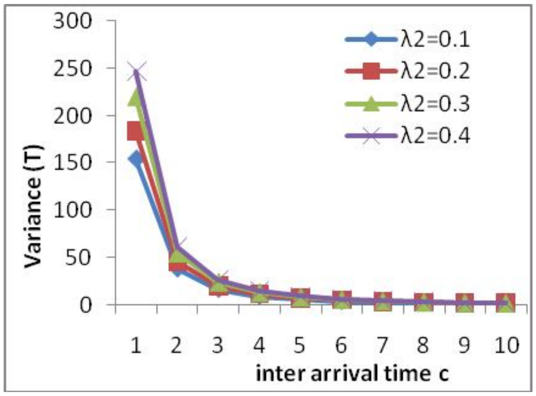

Figure 3b. Variation in $V(T)$ for Changes c

When λ_1, μ_1, μ_2 are kept fixed with other parameter λ_2 and inter-arrival time 'c' increasing. As the inter-arrival time increases, the value of the expected time E (T) to cross the threshold is found to be decreasing, in all the cases of the parameter value λ_2=0.1,0.2,0.3,0.4. when the value of the parameter λ_2 increases, the expected time is decreasing. This is indicated in figure 3a. Similar findings are observed in $V(T)$ which showed in figure 3b.

Table 3. Variation in $E(T)$ and V(T) for changes in the inter arrival times between successive Contacts, keeping $\lambda_1=0.4$, $\mu_1=0.3$, $\mu_2=0.2$ fixed

C	$\lambda_2=0.1$		$\lambda_2=0.2$		$\lambda_2=0.3$		$\lambda_2=0.4$	
	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
1	10.221	154.58	11.661	184.03	13.350	220.08	15.255	247.15
2	4.1105	38.646	5.8309	46.009	6.6751	55.020	7.6276	61.788
3	3.4070	17.176	3.8872	20.448	4.4501	24.453	5.0850	27.461
4	2.5552	9.6616	2.9154	11.502	3.3375	13.755	3.8138	15.447
5	2.0442	6.1834	2.3323	7.3614	2.6700	8.8032	3.0510	9.8861
6	1.7035	4.2940	1.9436	5.1121	2.2250	6.1133	2.5425	6.8653
7	1.4601	3.1548	1.6659	3.7558	1.9071	4.4914	2.1793	5.0439
8	1.2776	2.4154	1.4577	2.8755	1.6687	3.4387	1.9069	3.8617
9	1.1356	1.9084	1.2957	2.2720	1.4833	2.7170	1.6950	3.0512
10	1.0221	1.5458	1.1661	1.8403	1.3350	2.2008	1.5255	2.4715

Table 4. Variation in $E(T)$ and V(T) for changes in the inter arrival times between successive Contacts, keeping $\lambda_1=0.4$, $\lambda_2=0.2$, $\mu_1=0.3$ fixed

C	$\mu_2=0.1$		$\mu_2=0.2$		$\mu_2=0.3$		$\mu_2=0.4$	
	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)	E(T)	V(T)
1	15.155	184.03	12.305	1345.2	11.661	66.105	11.591	17.631
2	7.5775	46.009	6.1527	336.29	5.8309	16.526	5.7955	4.4079
3	5.0517	20.448	4.1018	149.46	3.8872	7.3450	3.8636	1.9590
4	3.7887	11.502	3.0763	84.074	2.9154	4.1315	2.8977	1.1019
5	3.0310	7.3614	2.4611	53.807	2.3323	2.6442	2.3182	0.7052
6	2.5258	5.1121	2.0509	37.366	1.9436	1.8362	1.9318	0.4897
7	2.1650	3.7558	1.7579	27.453	1.6659	1.3490	1.6558	0.3598
8	1.8944	2.8755	1.5381	21.018	1.4577	1.0329	1.4488	0.2754
9	1.6839	2.2720	1.3672	16.607	1.2957	0.8161	1.2878	0.2176
10	1.5155	1.8403	1.2305	13.452	1.1661	0.6610	1.1591	0.1763

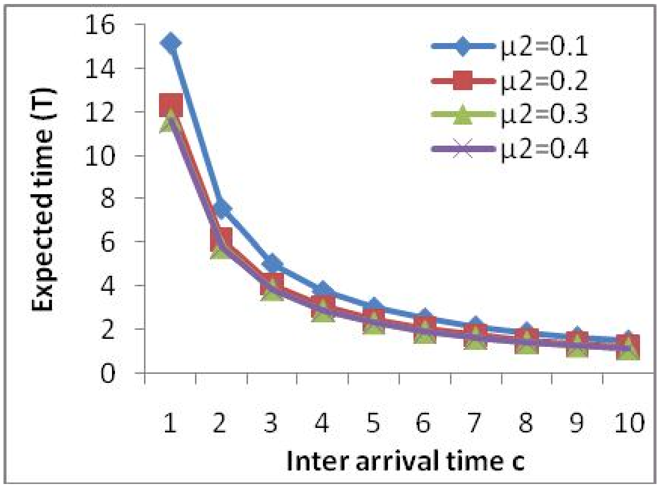

Figure 4a. Variation in $E(T)$ for Changes c

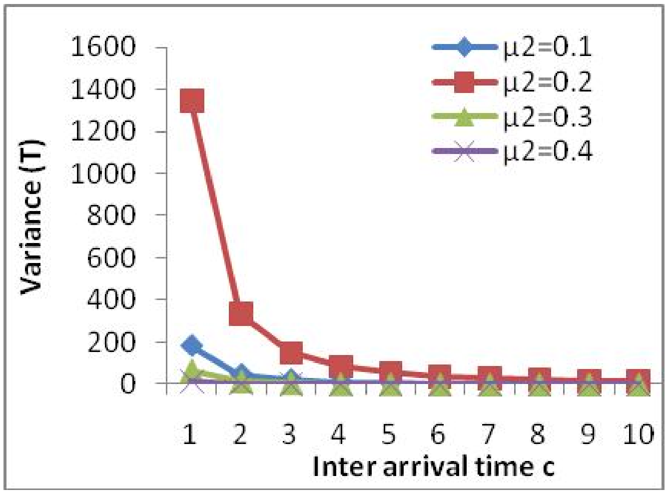

Figure 4b. Variation in V(T) for Changes c

When λ_1, λ_2, μ_1 are kept fixed with other parameter increasing μ_2 and inter-arrival time 'c'. As the inter-arrival time increases, the value of the expected time E (T) to cross the threshold is found to be decreasing, in all the cases of the parameter value μ_2=0.1, 0.2, 0.3, 0.4. when the value of the parameter μ_2 increases, the expected time is also found decreasing. This is indicated in figure 4a. Similar findings are observed in $V(T)$ which showed in figure 4b.

VI. Conclusion

It is observed that from table 1 the parameter denoting the antigenic diversity threshold λ_1, increases and the threshold parameter μ_1, μ_2 are kept fixed, the simulated results shows that as the inter arrival time follows exponential distribution c takes the value 1,2,...,10, the expected time to cross the antigenic diversity threshold is decreases and variance is also decreases which is depicted in figure 1a and figure 1b. This is due to fact that the successive contacts between the infected partners increases expected time to cross the antigenic diversity threshold is decreases. From table 2 it is observed that the threshold parameter μ_1 takes the inputted data values μ_1=0.5,1.0,1.5 and 2.0 keeping λ_1=0.4, λ_1=0.2 and μ_2=0.3 are kept fixed, the results shows that as inter arrival time follows exponential distribution c increases the expected time to cross the antigenic diversity threshold and it variance decreases as depicted in figure 2a and figure 2b. From table 3 it is observed that the threshold parameter μ_1=0.3 and μ_2=0.2 and the parameter of the antigenic diversity threshold λ_1=0.4 are kept fixed, as the inter contacts time c increases the expected time the antigenic diversity and it variance decreases. From table 4 it is observed that the chances in the inter arrival time between successive contacts and threshold parameter μ_2 increases, keeping the antigenic diversity threshold parameters λ_1,λ_2 and threshold parameter μ_1 kept fixed, then $E(T)$ and $V(T)$ shows a decreases as depicted in figure 4a and 4b. To analyze HIV/AIDS epidemiological data, many parametric distributions have been assumed for the HIV infection and seroconversion without due regard to the dynamics of the HIV epidemic and the biological and clinical features of the HIV. Since the discover of HIV and its implication in AIDS, can only be achieved by collecting real life data by assuming appropriate distribution and test for the goodness of fit can be used to validate the model. More information about the mechanisms adopted by HIV to render immune response ineffective and how genetic predisposition could affect susceptibility will assist in the development of HIV/AIDS epidemic. To enhance the projection of the spread of the epidemic, local surveillance systems on AIDS data collection must be encouraged and sustained, this will helps in improving the quality of the data necessary for statistical analysis and projection in this area of study. The major use of mathematical models of the transmission dynamics of HIV at present is to focus attention on the epidemiological parameters that need to be measured to predict future trends and to help to assess how different methods of control will influence the incidence of AIDS[4-5].

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