

Complete n- Domination Number of Union of n Copies of Certain Edge Deleted Subgraphs

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Abstract

The concept of complete graphs with real life application was introduced^{14,17}. A. Nellai Murugan *et. al.*, was introduced the concept of complete dominating number of a graph. In this paper, We introduce a new domination parameter called complete n-domination number of the union of n copies of certain edge deleted subgraphs of K_4 with edges in common, That is n images of complete graphs with edges in common. A subset S of V of a non trivial graph G is called a dominating set of G if every vertex in V-S is adjacent to at least one vertex in S. The domination number $\gamma(G)$ of G is the minimum cardinality taken over all dominating set in G. A subset S of V of a nontrivial graph G is said to be complete dominating set, If Choose $v \in V(G)$ and put $S = \{v\}$; For every v we have $N_G(S) = V - S$ denoted by S' is the complete dominating set. The minimum cardinality taken over all complete dominating set is called the complete domination number and is denoted by $\gamma_n(G)$ ¹⁴. A subset S of V of a nontrivial graph G is said to be complete n-domination number if every vertex in V is dominated by exactly n vertices of S and complete. The minimum cardinality of a complete n-dominating set is the complete n-domination number of G is denoted by $n\gamma_n(G)$. We determine this number for some standard graphs and obtain bounds for general graphs. Its relationship with other graph theoretical parameters are also investigated.

Keywords: Domination number, complete domination number, complete n-domination number.

Mathematics Subject Classification: 05C69

I Introduction

The concept of domination in graphs evolved from a chess board problem known as the Queen problem- to find the minimum number of queens needed on an 8x8 chess board such that each square is either occupied or attacked by a queen. C.Berge³ in 1958 and O. Ore⁸ in 1962 started the formal study on the theory of dominating sets. Thereafter several studies have been dedicated in obtaining variations of the concept. The authors in⁷ listed over 1200 papers related to domination in graphs in over 75 variation¹.

Throughout this paper, $G(V, E)$ a finite, simple, connected and undirected graph where V denotes its vertex set and E its edge set. Unless otherwise stated the graph G has n vertices and m edges. Degree of a vertex v is denoted by $d(v)$, the *maximum degree* of a graph G is denoted by $\Delta(G)$. Let C_n a cycle on n vertices, P_n a path on n vertices and a complete graph on n vertices by K_n . A graph is *connected* if any two vertices are connected by a path. A maximal connected subgraph of a graph G is called a *component* of G . The *number of components* of G is denoted by $\omega(G)$. The *complement* \bar{G} of G is the graph with vertex set V in which two vertices are adjacent iff they are not adjacent in G . A tree is a connected acyclic graph. A *bipartite graph* is a graph whose vertex set can be divided into two disjoint sets V_1 and another in V_2 . A *complete bipartite graph* is a bipartite graph with partitions of order $|V_1| = m$ and $|V_2| = n$, is denoted by $K_{m,n}$. A star denoted by $K_{1, n-1}$ is a *tree* with one root vertex and $n-1$

pendant vertices. A *bistar*, denoted by $B(m,n)$ is the graph obtained by joining the root vertices of the stars denoted by F_n can be constructed by identifying n copies of the cycle C_3 at a common vertex. A *wheel graph*² denoted by W_n is a graph with n vertices formed by connecting a single vertex to all vertices of C_{n-1} . A *Helm graph* denoted by H_n is a graph obtained from the wheel W_n by attaching a pendant vertex to each vertex in the outer cycle⁴⁻⁶ of W_n .

The *chromatic number* of a graph G denoted by $\chi(G)$ is the smallest number of colors needed to colour all the vertices of a graph G in which adjacent vertices receive different colours. For any real number x , $\lceil x \rceil$ denotes the largest integer greater than or equal to x and $\lfloor x \rfloor$ the smallest integer less than or equal to x . A Nordhaus - Gaddum – type result is a lower or upper bound on the sum or product of a parameter of a graph and its complement. Throughout this paper, we only consider undirected graphs with no loops. The basic definitions and concepts used in this study are adopted from¹¹.

Given a graph $G = (V(G), E(G))$, the cardinality $|V(G)|$ of the vertex set $V(G)$ is the order of G is n . The distance $d_G(u, v)$ between two vertices u and v of G is the length of the *shortest path* joining u and v . If $d_G(u, v) = 1$, u and v are said to be adjacent.

For a given vertex v of a graph G , The open neighbourhood of v in G is the set $N_G(v)$ of all vertices of G that are adjacent to v .

The degree $\deg_G(v)$ of v refers

to $|N_G(v)|$, and $\Delta(G) = \max \{deg_G(v) : v \in V(G)\}$. The closed neighbourhood of v is the set $N_G[v] = N_G(v) \cup v$ for $S \subseteq V(G)$, $N_G(S) = \bigcup_{v \in S} N_G(v)$ and $N_G[v] = N_G(S) \cup S$. If $S = V(G)$, then S is a dominating set in G . The minimum cardinality among dominating sets in G is called the *domination number* of G and is denoted by $g(G)$.

A dominating set S in a graph G is an independent dominating set if for every pair of distinct vertices u and v in S , u and v are non adjacent in G . The minimum cardinality $\gamma_i(G)$ of an independent dominating set in G is called the *independent domination number* of G . Given a graph G , choose $v_1 \in V(G)$ and put $S_1 = \{v_1\}$. If $N_G[S_1] \neq V(G)$, choose $v_2 \in V(G)$, $|N_G(v_2)| = |N_G(S_1)|$ and put if $S_2 = \{v_1, v_2\}$, If $N_G[S_2] \neq V(G)$ where possible $k \geq 3$, choose $v_k \in V(G)$; $|N_G(v_k)| = |N_G(S_{k-1})|$ and put $S_k = \{v_1, v_2, \dots, v_k\}$, there exists a positive integer k such that $N_G[S_k] = V(G)$. A dominating set obtained in this way above is called a *degree equitable dominating set*. The minimum cardinality of a degree equitable dominating set is called the *degree equitable domination number* denoted by $\gamma_N(G)$ ¹⁴. Given a graph G , choose $v \in V(G)$ and put $S = \{v\}$; For every v we have $N_G(S) = V - S$ denoted by S' is the complete dominating set. The smallest cardinality of a *complete dominating set* is called the *complete domination number of G* is denoted by $\gamma_N(G)$ ¹⁵. The following results are observed.

II. Main Results

Definition : 2.0 :

A set $S \subseteq V$ is a complete n -dominating set of G if every vertex in V is dominated by exactly n vertices of S and complete. The minimum cardinality of a complete n -dominating set is the complete n -domination number of G is denoted by $n\gamma_n(G)$.

Example 2.1: For any graph $G = n K_n$ is a complete n -dominating set with one edge in common is $n\gamma_n(G) = \lfloor \frac{n}{2} \rfloor$ for $n \leq 5$

Example 2.2: For any graph $G = n K_n$ is a complete n -dominating set with two edge in common is $n\gamma_n(G) = \lfloor \frac{n}{2} \rfloor$ for $n = 6$

Observation 2.3: We observe^{9,10} that each of these types has number of vertices in n copies of edge deleted subgraph of $K_4 = n + i \binom{n}{2}$ for all $i = 1, 2, \dots$. Where i is the image of any complete graphs and n is complete graph of $G = K_4$ with one edge in common and $\Delta(G) \leq [(i+1)(n-1) - i]$

Result 2.4: For any graph $G = n(K_4 - e)$ with one edge in common is not complete n -dominating set where $uv \in E$. If $S = \{u, v\}$, $V - S = N(S) = \{u_i, v_i\}$, where u_i, v_i are neighbor of u and v respectively.

since $N(u_i) = \{u_i, v_i\}$ and $N(v_i) = \{v_i, u_i\}$. Hence $\{N(u_i) \cap N(v_i)\} \neq S$ by definition it is not a complete n -domination.

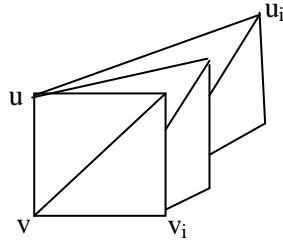


Fig 1. Not a complete n-dominating set

Result 2.5: For any graph $G = n(K_4 - e)$ with one edge in common is complete n- dominating set where $uv \in E$. If $S = \{u, v\}$, $N(S) = \{v_i\}$, since $N(v_i) = \{u, v\} = S$ by definition it is a complete n-domination and it is complete 2-dominated, Since $|N(v_i)| = |\{u, v\}| = 2$.

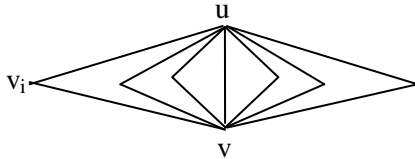


Fig. 2. $ny_n(G) = [n - (n - 2)] = 2$

Result 2.6: For any graph $G = n(K_4 - 2e)$ with one edge in common is not a complete n- dominating set. If $S = \{u, v\}$, $V - S = N(S) = \{v_i\}$. Then the set S is a complete dominating set but it is not a complete n-dominating set. Since $N(v_i) = \{u, v\}$ and $uv \notin E$.

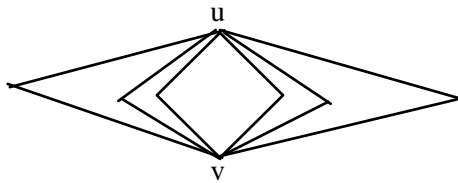


Fig. 3. Not a complete n-dominating set

Result 2.7: For any graph $G = nK_4$ with one edge in common is a complete n- dominating set where $uv \in E$. If $S = \{u, v\}$, $N(S) = \{u_i, v_i\}$, since $N(u_i) = \{u_i, v, v_i\}$ and $N(v_i) = \{u_i, v, u_i\}$. Hence $\{N(u_i) \cap N(v_i)\} = S$ and $|S| = 2$ by definition it is a complete n-domination is complete 2-dominated.

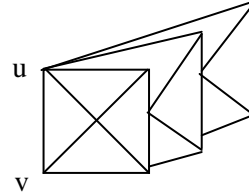


Fig 4. $ny_n(G) = [n - (n - 2)] = 2$

Result 2.8: For any graph $G = n(K_4 - e)$ with one edge in common is a complete n- dominating set where $uv \in E$. If $S = \{u, v\}$, $N(S) = \{u_i, v_i\}$, for all $i = 1, 2, \dots, n$ since $N(u_i) = \{u, v\}$ and $N(v_i) = \{u, v\}$. Hence $\{N(u_i) \cap N(v_i)\} = S$ and $|S| = 2$ by definition it is a complete n-domination is complete 2-dominated.

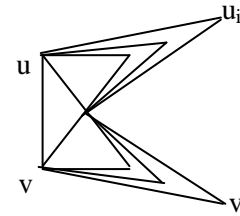


Fig. 5. $ny_n(G) = [n - (n - 2)] = 2$

Result 2.9: For any graph $G = n(K_4 - 3e)$ or $n(K_3)$ with one edge in common is complete n- dominating set. If $S = \{u, v\}$, $V - S = N(S) = \{w_i\}$, $N(w_i) = \{u, v\}$ and $|N(w_i) \cap S| = 2$ is a complete n-dominating set is a complete 2-dominating set^{12,13}.

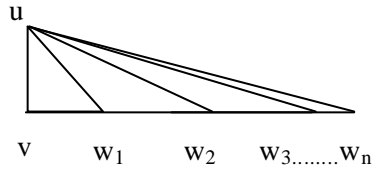


Fig 6. $\gamma_n(G) = [n - (n - 2)] = 2$

Result 2.10: For any graph $G = n(K_{4-e})$ with no edge in common with $uv \notin E$. If $S = \{u, v\}$, $N(S) = \{v_i\}$ for all $i = 1, 2, \dots, n$ since $\{N(v_i)\} = \{u, v\} = S$ by definition it is a complete dominating set but it is not a complete n- dominating set.

Result 2.11: For any graph $G = n(K_{4-2e})$ with one edge in common. If $S = \{u, v\}$, $V - S = N(S) = \{u_i, v_i\}$ for all $i = 1, 2, \dots, n$ and $N(u_i) = \{u, v_i\}$, $N(v_i) = \{v_i, u_i\}$ which implies $\{N(u_i) \cap N(v_i)\} = \emptyset$ with $uv \in E$ then the graph G is not a complete dominating set and it is not a complete n-dominating set^{16,18}.

Result 2.12: For any graph $G = n(K_{4-3e})$ with one edge in common. If $S = \{u, v\}$, $V - S = N(S) = \{u, v_i\}$, for all $i = 1, 2, \dots, n$ and $N(u_i) = \{u\}$, $N(v_i) = \{v\}$ which implies $\{N(u_i) \cap N(v_i)\} = \emptyset$ with $uv \in E$ then the graph G is not a complete dominating set and it is not a complete n-dominating.

Result 2.13: For any graph $G = n(K_{2,n})$ with no edge in common with $uv \notin E$. If $S = \{u, v\}$, $N(S) = \{u_i\}$ for all $i = 1, 2, \dots, n$ since $N(u_i) = \{u, v\} = S$ by definition it is not a complete n-dominating set but it is independent complete dominating set. If $uv \in E$ then $G = n(K_{2,n})$ is complete n-

dominating set also it is complete 2- dominating set.

Result 2.14: For any graph $G = n(K_{4,n})$ with no edge in common with $uvw \notin E$. If $S = \{u, v, w\}$, $N(S) = \{u_i\}$ for all $i = 1, 2, \dots, n$ since $N(u_i) = \{u, v, w\} = S$ by definition it is not a complete n-dominating set but it is independent complete dominating set.

Lemma 2.15: For any graph G. Let S be a complete n-dominating set if there exists vertices u and v such that $uv \in E$ and $\{N(u) \cap N(v)\} = V - S$ is the complete n- dominating set.

Proof: For any graph G if there exists vertices u and v such that $uv \in E$ and $N(u) = \{u_i, v_i\}$ and $N(v) = \{u_i, v_i\}$ and $\{N(u) \cap N(v)\} = \{u_i, v_i\} = V - S$ then $S = \{u, v\}$ is a complete n-dominating set.

Corollary 2.16 : A graph G with one edge in common is a complete n-dominating set if there exists $u_i, v_i \in V$ such that $\{N(u_i) \cap N(v_i)\} = S$.

Proof: By the definition of complete n-dominating set.

Lemma 2.17: For any graph G. Let S is not a complete n-dominating set if there exists vertices u and v such that $uv \in E$ and $\{N(u) \cap N(v)\} = V - S$.

Proof: For any graph G. If there exists vertices u and v such that $uv \notin E$ and $N(u) = \{u_i, v_i\}$ and $N(v) = \{u_i, v_i\}$ and $\{N(u) \cap N(v)\} = \{u_i, v_i\}$. Hence $N(S) = V - S$ is complete dominating

set, since $uv \notin E$ is not a complete n -dominating set.

Theorem 2.18: Let S and S' be any two complete n dominating sets in a graph G with one edge in common, then their domination numbers are (i) $n\gamma_n(G) = [n - (n - 2)] = 2$ if G is nK_4 with one edge in common (ii) $n\gamma_n(G) = [n - (n - 1)] = 1$ if G is a star graph (iii) $n\gamma_n(G) = [n - (n - 2)] = 2$ if G is $K_{2,n} + e$ with one edge in common (iv) $n\gamma_n(G) = [n - (n - 2)] = 2$ if G is $n(K_4 - e)$ with one edge in common (v) $n\gamma_n(G) = [n - (n - 2)] = 2$ if G is $n(K_4 - 3e)$ with one edge in common (vi) $n\gamma_n(G) = [n - (n - 1)] = 1$ if G is $n(K_4 - e)$ with one edge in common and independent.

Proof: Given S and S' be any two complete dominating sets in a graph G .

Case (i) Since $G = nK_4$ is a graph with one edge is common. There exists vertices u and v such that $uv \in E$ which is common to all nK_4 and $N(u) = \{v, u_i, v_i\}$, $N(v) = \{u, u_i, v_i\}$, $\{N(u) \cap N(v)\} = \{u_i, v_i\} = V - S$ for all $i = 1, 2, \dots$. Hence $S = \{u, v\}$ and $V - S = \{u_i, v_i\}$ is the complete n -dominating sets also $n\gamma_n(G) = [n - (n - 2)] = 2$

Case (ii) and Case (iii) Given a graph $G = K_{2,n} + e$ is a complete bipartite graph with one edge in common, Since G is a complete bipartite graph there exists a vertices $u, v \in G$ which is common to all $K_{2,n} + e$ and $N(u) = \{v, u_{ij}\}$, $N(v) = \{u, v_{ij}\}$, $\forall, i = 1, 2, \dots, m, j = 1, 2, \dots, n$. Hence $S = \{u, v\}$ and $V - S = \{u_{ij}, v_{ij}\}$ is the complete n dominating set and $n\gamma_n(G) =$

$[n - (n - 2)] = 2$ Similarly for a graph $G = K_{1,n}$ is a complete n dominating set with $n\gamma_n(G) = [n - (n - 1)] = 1$

Case (iv) : Given a graph $G = n(K_4 - e)$ with one edge in common, There exists $u, v \in G$ such that $uv \in E$. Which is common to all $n(K_4 - e)$ and $N(u) = \{v, u_i, v_i\}$, $N(v) = \{u, u_i, v_i\}$ and $\{N(u) \cap N(v)\} = \{u_i, v_i\} = V - S$ is the complete n - dominating set with $n\gamma_n(G) = [n - (n - 2)] = 2$

Case (v): Suppose $G = n(K_4 - 2e)$, $N(u) = \{u_i, v_i, v\}$ but $N(v) = \{u, v_i\}$ and $\{N(u) \cap N(v)\} \neq \{u_i, v_i\} \neq V - S$ is not a complete dominating set also it is not a complete n -dominating set.

Sub case (i): Suppose $N(u) = \{v, u_i\}$ and $N(v) = \{u, v_i\}$ such that $\{N(u) \cap N(v)\} = \emptyset \neq V - S$ is not a complete dominating set also it is not a complete n - dominating set but $\{N(u) \cup N(v)\} = V - S$. Hence $S = \{u, v\}$ is a dominating set.

Case (vi): Given a graph $G = n(K_4 - 3e)$ with one edge in common, There exists $u, v \in G$ such that $uv \in E$. Which is common to all $n(K_4 - 3e)$ with $N(u) = \{v, w_i\}$ and $N(v) = \{u, w_i\}$ and $\{N(u) \cap N(v)\} = \{w_i\} = V - S$. Hence $S = \{u, v\}$ and $V - S = \{w_i\}$ is the complete n - dominating set with $n\gamma_n(G) = [n - (n - 2)] = 2$

Sub case (i): Suppose $N(u) = \{v, u_i\}$ and $N(v) = \{u, v_i\}$ such that $\{N(u) \cap N(v)\} \neq \{u_i, v_i\} \neq V - S$ is not a complete dominating set also it is not a complete n - dominating set.

Theorem 2.19: For any graph G is complete n- dominating set iff it is complete dominating set and there exists $u, v \in S$ such that $uv \in E$.

Proof: Given G be a complete n- dominating set, that is there exists vertices u and v such that $uv \in E$, which is common to all nK_n and $N(u) = \{v, u_i, v_i\}$ and $N(v) = \{u, u_i, v_i\}$ is the complete n- dominating set. By the definition of complete n- dominating set, for every $u, v \in S$ such that $N(u) \cap N(v) = V - S = S$. Hence S is the complete dominating set of G. Conversely, Given S is the complete dominating set $S = \{u, v\}$, $\{N(u) \cap N(v)\} = \{u_i, v_i\}$, Hence $N(u) = N(v) = \{u_i, v_i\} = V - S$, for all $i = 1, 2, \dots, n$. Hence \exists vertices $u, v \in S$ such that $uv \in E$. Hence S is the complete n-dominating set $\gamma_n(G) = [n - (n - 2)] = 2$

Theorem 2.20: Every complete n- dominating set is a degree equitable dominating set.

Proof: Given G is a complete n- dominating set, that is there exists vertices u and v such that $uv \in E$ which is common to all nK_n and $N(u) = \{v, u_i, v_i\}$ and $N(v) = \{u, u_i, v_i\}$ and $\{N(u) \cap N(v)\} = \{u_i, v_i\} = V - S$. Hence $S = \{u, v\}$ and $V - S = \{u_i, v_i\}$ and $|N(u)| = |N(v)|$ where $|N(u)| = n$ and $|N(v)| = n$ which gives $\frac{|N(u)|}{|N(v)|} = 1$, $N[S] = V(G)$ Which implies that $|d(u) - d(v)| = 0$. By the definition of degree equitable dominating set. The given complete n-dominating set is a degree equitable dominating set.

Theorem 2.21: Every complete n- dominating set is not an independent dominating set

Proof: Given G is a complete n- dominating set, There exists vertices u, v such that $uv \in E$ which is common in all nK_n and $N(u) = \{u_i, v_i\}$ and $N(v) = \{u_i, v_i\}$. Hence $S = \{u, v\}$ and $V - S = \{u_i, v_i\}$. Since S is a complete n- dominating set and $uv \in E$, S is not an independent dominating set.

Lemma 2.22: Every complete 2- dominating set is 2 dominated if $\{\cup N(S)\} = V$.

Proof: Given a graph G is complete 2-dominating set If there exists vertices u and v such that $uv \in E$ and $N(u) = \{v, u_i, v_i\}$ and $N(v) = \{u, u_i, v_i\}$ and $\{\cup N(S)\} = \{u, v, u_i, v_i\} = V$ and $N(u_i) = \{u, v\}$ and $N(v_i) = \{u, v\}$, Hence $|N(u_i) \cap N(v_i)| = \{u, v\} = |S| = 2$, Hence S is the 2-dominating set. Suppose $uv \notin E$, $N(u) = \{u_i, v_i\}$, $N(v) = \{u_i, v_i\}$, $\{\cup N(S)\} = \{u_i, v_i\} \neq V$.

Lemma 2.23: For any graph G with one edge in common is a complete n-dominating number if $\exists u_i, v_i \in V$ such that $|N(u_i) \cap N(v_i)| = n$ and $\{\cup N(S)\} = V$.

Proof: By the definition of complete n-dominating set.

Lemma 2.24: For any graph G with $\leq n$ edges in common is a complete n-dominating number if $\exists v_i \in V$ such that $|N(v_i)| = n$ and every v_i is adjacent

Proof: Obviously by the definition of complete n-dominating set.

Lemma 2.25 : For any graph G with one edge in common is a complete n-dominating number if $\exists u_i, v_i \in V$ such that $\{N(u_i) \cap$

$N(v_i) \cap S = \emptyset$.

Proof: Prof is as obviously by the definition of S.

Theorem 2.26: For any graph G with m edges in common is a complete n-domination number if $\exists u_i \in V, \forall i=1,2,\dots,n$ such that $|\bigcap N(u_i)| \leq n$, with $m < n$.

Proof: For any graph G with m edges in common if $\exists u_i \in V, \forall i=1,2,\dots,n, N(u_i) = \{U_j\}, \forall j=1,2,\dots,m, U_j \in S, \{\bigcap N(u_i)\} = \{U_j\}$ and every U_j is adjacent. Which implies that $|\bigcap N(u_i)| = |U_j| \leq n, \forall j=1,2,\dots,m$ with $m < n$.

Theorem 2.27 : Every complete 3-dominating set is 3 dominated. If $\forall U_j \in S$ is adjacent.

Proof: Given a graph G is complete 3-dominated if $\exists U_j \in S, \forall j=1,2,3$ such that $\forall U_j$ is adjacent and $N(U_j) = \{u_i\} = V - S = S', \forall i=1,2,\dots,n$ and $N(u_i) = U_j$. Hence $|\bigcap N(u_i)| = |U_j| = 3$. Hence S is 3-dominated.

Theorem 2.28: Every complete n-dominating set is n-dominated if $\forall U_j \in S$ is adjacent.

Proof: Given a graph G is complete n-dominating set if $\exists U_j \in S \forall j=1,2,\dots,n$ such that $\forall U_j$ is adjacent and $N(U_j) = \{u_i\} = V - S = S'$ and $N(u_i) = \{U_j\}$. Hence $|\bigcap N(u_i)| = |U_j| = n$. Hence S is n-dominated.

Corollary 2.29 : Every complete n-dominating set is not an n-dominated if \exists at

least one $U_j \in S$ is non adjacent.

Proof: Obvious from the previous theorem.

Theorem 2.30: Suppose S and S' be the complete n-dominating set of G such that S and S' are not complete n-dominating set of $V(G \circ H)$.

Proof: Given S and S' be the complete n-dominating set of G, By the definition of corona graph $V(G \circ H)$ either $V(G)$ or $V(H)$ is a dominating set, If there exists some $v_1 \in S$ such that $N_S(v_1) = H_{v_1}$, similarly If there exists some $v_2 \in S$ such that $N_S(v_2) = H_{v_2}$, For every positive integer k such that $v_k \in S$ such that $N_S(v_k) = H_{v_k}$, where $S' = \{H_{v_1}, H_{v_2}, \dots, H_{v_k}\} = V - S$ and $\{\bigcap N_S(v_k)\} = \emptyset$, where $k=1,2,\dots,n$, which implies that S and S' are not complete n-dominating set.

III. Relationships with other Graph Theoretical Parameters:

Theorem 3.1: For any graph $G = nK_n$ with $n \geq 3$ vertices and one edge in common, $n\gamma_n(G) + \chi(G) \leq n+2$ and the bound is sharp iff $G \cong nK_n$, for all $n \geq 3$.

Proof: Let G be n copies of complete graph with one edge in common and $n \geq 3$ vertices. We know that $\chi(G) \leq n$ and by theorem:2.18 $n\gamma_n(G) \leq [n - (n-2)]$. Hence $n\gamma_n(G) + \chi(G) \leq n+2$. Suppose G is isomorphic to K_4 then clearly $n\gamma_n(G) + \chi(G) = n+2$. Conversely, Let $n\gamma_n(G) + \chi(G) = n+2$ this is

possible only if $n\gamma_n(G) \leq [n-(n-2)]$ and $\chi(G) = n$. Since $\chi(G) = n$, G is isomorphic to K_4 for which $n\gamma_n(G) = 2 = [n-(n-2)]$. Hence $G \cong nK_n$, for all $n \geq 3$.

Theorem 3.2: For any graph $G = nK_n$ with $n \geq 3$ vertices and one edge in common $n\gamma_n(G) + \kappa(G) \leq n+1$ and the bound is sharp iff $G \cong nK_n$ for all $n \geq 3$.

Proof: Let G be n copies of complete graph with one edge in common and $n \geq 3$ vertices. We know that $\kappa(G) \leq n-1$ and by theorem:2.18 $n\gamma_n(G) \leq [n-(n-2)]$. Hence $n\gamma_n(G) + \kappa(G) \leq n+1$. Suppose G is isomorphic to K_4 then clearly $n\gamma_n(G) + \kappa(G) = n+1$. Conversely, Let $n\gamma_n(G) + \kappa(G) = n+1$ this is possible only if $n\gamma_n(G) = [n-(n-2)]$ and $\kappa(G) = n-1$. Since $\kappa(G) = n-1$, G is isomorphic to nK_n for which $n\gamma_n(G) = 2 = [n-(n-2)]$. Hence $G \cong nK_n$, for all $n \geq 3$.

Theorem 3.3: For any graph $G = nK_n$ with $n \geq 3$ vertices and one edge in common, $n\gamma_n(G) + \Delta(G) \leq [n-(n-2)] + [(i+1)(n-1)-i]$, where i is the number of images of G and the bound is sharp iff $G \cong nK_n$, for all $n \geq 3$.

Proof: Let G be n copies of complete graph with one edge in common and $n \geq 3$ vertices. We know that $\Delta(G) \leq [(i+1)(n-1)-i]$ and by theorem 2.18 $n\gamma_n(G) \leq [n-(n-2)]$, Hence $n\gamma_n(G) + \Delta(G) \leq [n-(n-2)] + [(i+1)(n-1)-i]$, that is $n\gamma_n(G) + \Delta(G) \leq 2n-1$ for $i=1$ and nK_n , for all $n \geq 3$, the bound is sharp.

IV. Conclusion

In this paper we found an upper bound for the complete n-domination number and relationship between complete n-domination numbers of graphs and characterized the corresponding extremal graphs. Similarly complete n-domination numbers with other graph theoretical parameters can be considered.

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