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Recursive Implementation of Type-IV Discrete Cosine Transform Using Type-II Discrete Sine Transform

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Abstract

This paper presents a novel recursive algorithm for implementation of type-IV discrete cosine transform (DCT-IV) of any general length using type-II discrete sine transform (DST-II). The recursive algorithms apply to arbitrary length algorithms and are suitable for VLSI implementation.

Key words: Discrete cosine transform; Discrete sine transform; Recursive algorithm.

1. Introduction

Discrete transforms play a significant role in digital signal processing. Discrete cosine transform (DCT) and discrete sine transform (DST) are used as key functions in many signal and image processing applications. There are four types of DCT and DST. Of these, the DCT-II, DST-II, DCT-IV, and DST-IV have gained popularity.

The original definition of the DCT introduced by Ahmed *et al*. in 1974[1] was one-dimensional (1-D) and suitable for 1-D digital signal processing. The DCT has found wide applications in speech and image processing as well as telecommunication signal processing for the purpose of data compression, feature extraction, image reconstruction, and filtering. Thus, many algorithms and VLSI architectures for the fast computation of DCT have been proposed[2,7]. Among those algorithms[6] and[7] are believed to be most efficient two-dimensional DCT algorithms in the sense of minimizing any measure of computational complexity.

The DST was first introduced to the signal processing by Jain[8] and several versions of this original DST were later developed by Kekre *et al.*[9], Jain[10] and Wang *et al.*[11]. Ever since the introduction of the first version of the DST, the different DST's have found wide applications in several areas in Digital signal processing (DSP), such as image processing[8,12,13], adaptive digital filtering[14] and interpolation[15]. The performance of DST can be compared to that of the DCT and it may therefore be considered as a viable alternative to the DCT. For images with high correlation, the DCT yields better results; however, for images with a low correlation of coefficients, the DST yields lower bit rates[16]. Yip and Rao[17] have proven that for large sequence length ($N \geq 32$) and low correlation coefficient ($\rho < 0.6$), the DST performs even better than the DCT.

In this paper, a new recursive algorithm for computation of DCT-IV of any general length from DST-II is presented.

The rest of the paper is organized as follows. The proposed recursive algorithm for DCT-II is presented in Section-2. An example for computation of DCT-IV using DST-II is given in Section-3. Conclusion is given in Section-4.

2. Proposed algorithm for DCT-IV:

The DCT-IV of an N-point input sequence $y(n)$ is given by

$$Y_{IV}^{c}(k) = \sum_{n=0}^{N-1} y(n)\cos\left[\left(n+\frac{1}{2}\right)\left(k+\frac{1}{2}\right)\frac{\pi}{N}\right] \quad (1)$$

for $k = 0, 1, 2, \ldots, N-1$.

Replacing k by k-1 in (1), we obtain

$$Y_{IV}^{c}(k-1) = \sum_{n=0}^{N-1} y(n)\cos\left[\left(n+\frac{1}{2}\right)\left(k-\frac{1}{2}\right)\frac{\pi}{N}\right] \quad (2)$$

Consider the following trigonometric relation

$$\cos B - \cos A = 2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \quad (3)$$

Subtracting (1) from (2), we get

$$Y_{IV}^{c}(k-1) - Y_{IV}^{c}(k) = \sum_{n=0}^{N-1} y(n)\left[\cos\left(n+\frac{1}{2}\right)\left(k-\frac{1}{2}\right)\frac{\pi}{N} - \cos\left(n+\frac{1}{2}\right)\left(k+\frac{1}{2}\right)\frac{\pi}{N}\right] \quad (4)$$

Using (3) in RHS of (4), we have

$$\begin{aligned} Y_{IV}^{c}(k-1) - Y_{IV}^{c}(k) &= \sum_{n=0}^{N-1} 2y(n)\sin\left[\left(n+\frac{1}{2}\right)\frac{k\pi}{N}\right]\sin\left[\left(n+\frac{1}{2}\right)\frac{\pi}{2N}\right] \\ &= \sum_{n=0}^{N-1} X(n)\sin\left[\left(n+\frac{1}{2}\right)\frac{k\pi}{N}\right] \end{aligned} \quad (5)$$

where

$$X(n) = 2y(n)\sin\left[\left(n+\frac{1}{2}\right)\frac{\pi}{2N}\right] \quad (6)$$

The DST-II for input sequence $X(n)$ is defined by

$$X_{II}^{s}(k)=\sqrt{\frac{2}{N}}C_k\sum_{n=0}^{N-1}X(n)\sin\left[k\left(n+\frac{1}{2}\right)\frac{\pi}{N}\right] \tag{7}$$

for k = 1,2,….,N

where

$$C_k=\begin{cases}\frac{1}{\sqrt{2}}, & if\ k=N\\ 1, & if\ k=1,2,.....,N-1\end{cases}$$

Without loss of generality, the scale factor $\sqrt{\frac{2}{N}}C_k$ in (7) may be ignored in the rest of the paper. Using (7) after ignoring its scale factor in (5), we get the following recursive relation.

$$Y_{IV}^{c}(k-1)-Y_{IV}^{c}(k)=X_{II}^{s}(k)$$

$$\Rightarrow Y_{IV}^{c}(k-1)=Y_{IV}^{c}(k)+X_{II}^{s}(k) \tag{8}$$

Putting $k=N$ in (1), we have

$$Y_{IV}^{c}(N)=\sum_{n=0}^{N-1}y(n)\cos\left[\left(n+\frac{1}{2}\right)\left(N+\frac{1}{2}\right)\frac{\pi}{N}\right]$$

$$=\sum_{n=0}^{N-1}y(n)\cos\left[\left(n+\frac{1}{2}\right)\pi+\left(n+\frac{1}{2}\right)\frac{\pi}{2N}\right]$$

$$=y(0)\cos\left(\frac{\pi}{2}+\frac{\pi}{4N}\right)+y(1)\cos\left(\frac{3\pi}{2}+\frac{3\pi}{4N}\right)y(2)\cos\left(\frac{5\pi}{2}+\frac{5\pi}{4N}\right)+........$$

$$=-y(0)\sin\left(\frac{\pi}{4N}\right)+y(1)\sin\left(\frac{3\pi}{4N}\right)-y(2)\sin\left(\frac{5\pi}{4N}\right)+........ \tag{9}$$

Putting $k=N-1$ in (1), we get

$$Y_{IV}^{c}(N-1)=\sum_{n=0}^{N-1}y(n)\cos\left[\left(n+\frac{1}{2}\right)\left(N-\frac{1}{2}\right)\frac{\pi}{N}\right]$$

$$=\sum_{n=0}^{N-1} y(n)\cos\left[\left(n+\frac{1}{2}\right)\pi-\left(n+\frac{1}{2}\right)\frac{\pi}{2N}\right]$$

$$=y(0)\cos\left(\frac{\pi}{2}-\frac{\pi}{4N}\right)+y(1)\cos\left(\frac{3\pi}{2}-\frac{3\pi}{4N}\right)+y(2)\cos\left(\frac{5\pi}{2}-\frac{5\pi}{4N}\right)+......$$

$$=y(0)\sin\left(\frac{\pi}{4N}\right)-y(1)\sin\left(\frac{3\pi}{4N}\right)+y(2)\sin\left(\frac{5\pi}{4N}\right)-......... \quad (10)$$

From (9) and (10), we have

$$Y_{IV}^{c}(N)=-Y_{IV}^{c}(N-1) \quad (11)$$

Putting $k = N$ in (8), we have

$$Y_{IV}^{c}(N-1)-Y_{IV}^{c}(N)=X_{II}^{s}(N) \quad (12)$$

Using (11) in (12), we get

$$Y_{IV}^{c}(N-1)=\frac{1}{2}X_{II}^{s}(N) \quad (13)$$

The DCT-IV can be realized from DST-II using the recursive relation (8) along with (13).

3. Example for computation of DCT-IV from DST-II for N=8.

For $N = 8$, we have $k = 0,1,2,3,4,5,6,7$ for DCT-IV.
Putting $k = 1,2,3,4,5,6,7$ successively in (8), we get

$$Y_{IV}^{c}(0)=Y_{IV}^{c}(1)+X_{II}^{s}(1)$$
$$Y_{IV}^{c}(1)=Y_{IV}^{c}(2)+X_{II}^{s}(2)$$
$$Y_{IV}^{c}(2)=Y_{IV}^{c}(3)+X_{II}^{s}(3)$$
$$Y_{IV}^{c}(3)=Y_{IV}^{c}(4)+X_{II}^{s}(4) \quad (14)$$
$$Y_{IV}^{c}(4)=Y_{IV}^{c}(5)+X_{II}^{s}(5)$$
$$Y_{IV}^{c}(5)=Y_{IV}^{c}(6)+X_{II}^{s}(6)$$
$$Y_{IV}^{c}(6)=Y_{IV}^{c}(7)+X_{II}^{s}(7)$$

For $N = 8$, we have from (13)

$$Y_{IV}^{c}(7)=\frac{1}{2}X_{II}^{s}(8) \quad (15)$$

From the input data $y(n)$, $X(n)$ are found out using (6). Then $X(n)$ are transformed to DST-II, $X_{II}^{s}(k)$, using (7). The output DCT-IV components are realized from DST-II using the recursive relations (14) along with (15).

The architecture for recursive implementation of DCT-IV from DST-II for $N = 8$ is shown in Fig.1. It is made from (6),(7),(14) and (15).

4. Conclusion

In this paper, a novel recursive algorithm for realizing DCT-IV of any general length from DST-II is derived. An architecture

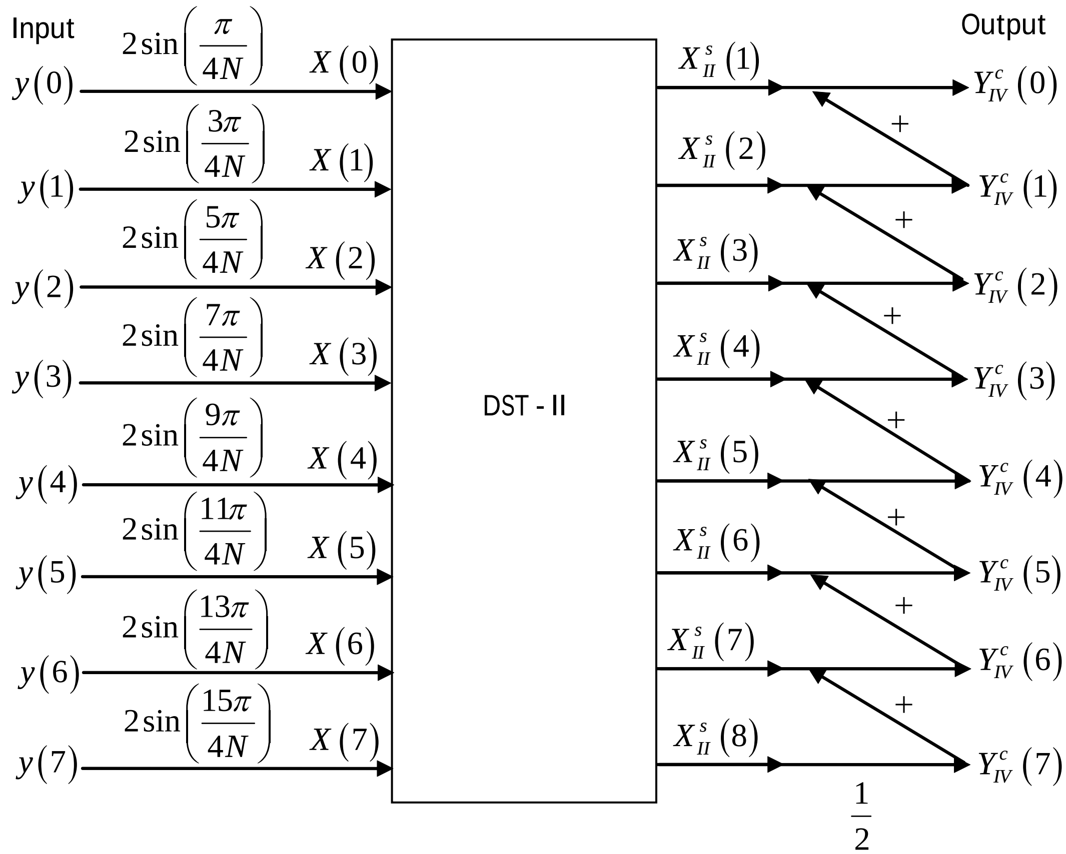

Figure 1. Architecture for computation of DCT-IV from DST-II for $N = 8$

for recursive implementation of DCT-IV using DST-II has been presented. The recursive structures require less memory and are suitable for parallel VLSI implementation.

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