Ultra Scientist Vol. 26(2)B, 139-142 (2014). ISSN 2231-3478 (Print)
www.ultrascientist.org 2319-8052 (Online)

Bianchi Type - I String Cosmological Models with Bulk Viscous Fluid in General Relativity

VARUN HUMAD[1] and SAPNA SHRIMALI[2]

[1]Research Scholar of Mewar University, Chittorgarh (Rajasthan), (INDIA)
[2]Department of Mathematics, Pacific Academy of Higher Education & Research University, Udaipur (Rajasthan) (INDIA)

(Acceptance Date 16th August, 2014)

Abstract

We have investigated Bianchi Type - I string cosmological model with Bulk Viscous fluid in general relativity. To obtain a determinate model of the universe, we assumed that (i) $3\xi\theta = \rho$, where ξ is the coefficient of bulk viscosity, θ is the scalar of expansion and ρ is the energy density (ii) relation between metric potential $B = A^n$. Some physical properties of the model are also discussed.

Key words: Bianchi Type-I models, bulk viscosity, scalar of expansion, energy density.

1. Introduction

Even today, it is a basic problem in cosmology is to know that how the formation of large scalar structure of the universe. String cosmological models are widely studied in recent times because of their important role in the description of the evolution of the early phase of universe.

Collins *et al.*[1] investigated Bianchi Type I spatially homogeneous perfect fluid cosmological models, in which shear bears a constant ratio to the expansion in the model. Benerjee *et al.*[2] have investigated an axially symmetric Bianchi Type I string dust cosmological model in presence and absence of magnetic field. The string cosmological models with a magnetic field are also discussed by Chakraborty[3], Tikekar and patel[4,5]. Bali and Tyagi[6] have obtained a cylindrically symmetric inhomogeneous cosmological model with electromagnetic field for perfect fluid distribution. The influence of intergalactic magnetic field on cosmological enolution has been studied for over four decades from theoretical and observational point of view[7,8]. Bali *et al.*[9,10,11] have investigated Bianchi Type I magnetized string cosmological models. Saha *et al.*[12] and Saha[13] have studied Bianchi Type I cosmological model in presence of magnetic flux in different contexts. Rao *et al.*[14] have studied Bianchi Type I string cosmological models in bimetric theory of gravitation. Tripathy *et al.*[15], Pawar *et al.*[16] have investigated about different aspects of plane symmetric Bulk viscous fluid string dust magnetized cosmological model in general relativity. Recently Tyagi *et al.*[17] investigated Bulk viscous fluid plane symmetric string dust magnetized cosmological model in general relativity.

In this paper, we have investigated Bianchi Type - I String cosmological models with bulk viscous fluid in general relativity. An equation $3\xi\theta = \rho$ and a relation between metric potentials $B = A^n$ are assumed. The physical and geometric aspects of the model are also discussed.

2. Metric and Field Equation :

We consider Bianchi Type - I metric of the form

$$ds^2 = -dt^2 + A^2(dx^2 + dy^2) + B^2 dz^2 \quad (1)$$

where A and B are function of time only. The energy momentum tensor for a cloud of string with bulk viscosity is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(\rho u_i u_j + g_{ij}) \quad (2)$$

where $\rho = \rho_\rho + \lambda$ is the rest energy density of the cloud of string with particles attached to them with ρ_ρ is the rest energy density of particle and λ is the tension density of the cloud of strings, $\theta = u^i_{;i}$ is the scalar of expansion and ξ is the coefficient of bulk viscosity. The vector u^i describes the flow velocity vector and x^i represent a direction of anisotropy *i.e.* the direction of strings, satisfy the standard relation.

$$u^i u_i = -x^i x_i = -1, u^i x_i = 0 \quad (3)$$

The expression for scalar of expansion θ and shear scalar σ are

$$\theta = u^i_{;i} = \frac{2\dot{A}}{A} + \frac{\dot{B}}{B} \quad (4)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{2}\left(\frac{2\dot{A}^2}{A^2} + \frac{\dot{B}^2}{B^2}\right) - \frac{\theta^2}{6} \quad (5)$$

The Einstein's field equation (in gravitational units c = 1, $8\pi G = 1$) for a system of string

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij} \quad (6)$$

For the metric (1), Einstein's field equation's can be written as

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \xi\theta \quad (7)$$

$$\frac{2\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} = \lambda + \xi\theta \quad (8)$$

$$\frac{\dot{A}^2}{A^2} + \frac{2\dot{A}\dot{B}}{AB} = \rho \quad (9)$$

Where an over dot stands for the first and double dot for the second derivative with respect to cosmic time t.

3. Solution of the Field Equations :

The field equations (7) to (9) are three equations in five unknowns parameters A, B, ξ, ρ and λ. In order to obtain a determinate solution, we assume a relation between ξ (coefficient of bulk viscosity), θ (scalar of expansion) and ρ (energy density) *i.e.*

$$3\xi\theta = \rho \tag{10}$$

And the relation between metric potentials

$$B = A^n \tag{11}$$

Where n is constant.

From equation (7), (9) and (10), we get

$$\frac{3\ddot{A}}{A}+\frac{3\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\dot{A}^2}{A^2}=0 \tag{12}$$

Now, using equation (11) in (12),we get

$$\dot{A}=\frac{k}{A^{\alpha}} \tag{13}$$

Where k is a constant of integration and

$$\alpha = \frac{3n^2-2n-1}{3n+3} \tag{14}$$

Now integrating equation (13), we obtain

$$A = (k_1 t + k_2)^{\frac{1}{\alpha+1}} \tag{15}$$

Where $k_1 = k\,(\alpha+1)$ and $k_2 = (\alpha+1)\beta$

Where β is a constant of integration.
From equation (11) and (15), we get

$$B = (k_1 t + k_2)^{\frac{n}{\alpha+1}} \tag{16}$$

Hence the model (1) is reduce to

$$ds^2 = -dt^2 + (k_1 t + k_2)^{\frac{2}{\alpha+1}}(dx^2+dy^2) + (k_1 t + k_2)^{\frac{2n}{\alpha+1}} dz^2 \tag{17}$$

After using a suitable transformation of coordinates the model (17) reduce to

$$dS^2 = -\frac{dT^2}{k_1^2} + (T)^{\frac{2}{\alpha+1}}(dX^2+dY^2) + (T)^{\frac{2n}{\alpha+1}} dZ^2 \tag{18}$$

Where $(k_1 t + k_2)=T$, x = X, y = Y, z = Z

4. Some physical and geometrical aspects of the model :

For the model of equation (18), the other physical and geometrical parameters can be easily obtained. The energy density ρ, the string tension density λ, the coefficient of bulk viscosity ξ, the scalar of expansion θ and the shear scalar σ are respectively given by

$$\rho = (2n+1)\left(\frac{k_1}{\alpha+1}\right)^2 \frac{1}{T^2} \tag{19}$$

$$\lambda = \left(\frac{2-6\alpha-2n}{3}\right)\left(\frac{k_1}{\alpha+1}\right)^2 \frac{1}{T^2} \tag{20}$$

$$\xi = \frac{(2n+1)}{3(n+2)}\left(\frac{k_1}{\alpha+1}\right)\frac{1}{T} \tag{21}$$

$$\theta = (n+2)\left(\frac{k_1}{\alpha+1}\right)\frac{1}{T} \tag{22}$$

$$\sigma = \frac{(n-1)}{\sqrt{3}}\left(\frac{k_1}{\alpha+1}\right)\frac{1}{T} \tag{23}$$

$$\frac{\sigma}{\theta} = \frac{(n-1)}{\sqrt{3}(n+2)} = constant \tag{24}$$

4. Conclusion

(i) The energy condition $\rho \geq 0$ in the presence of bulk viscous fluid leads to

$$(2n+1)\left(\frac{k_1}{\alpha+1}\right)^2 \frac{1}{T^2} \geq 0$$

(ii) When n = -2, then scalar of expansion θ becomes zero.

(iii) $\frac{\sigma}{\theta} = \frac{(n-1)}{\sqrt{3}(n+2)} = constant$, therefore model does not approach isotropy for large value of T. Model isotropize for n = 1.

(iv) As $T \rightarrow 0$, the scalar of expansion θ tends to infinitely large and when $T \rightarrow \infty$, the scalar of expansion $\theta \rightarrow 0$. Also at $T \rightarrow 0$ shear scalar σ tends to infinity and when $T \rightarrow \infty$ shear scalar σ tends to zero. The energy density $\rho \rightarrow \infty$ when $T \rightarrow 0$ and $\rho \rightarrow 0$ when $T \rightarrow \infty$, therefore the model describes a shearing, non- rotating, continuously expanding universe with a big-bang start.

5. References

1. Collins,C.B.,Glass, E.N. and Wilkinson, D.A., *GRG.12*, 10, 305 (1980).
2. A. Banerjee, A.K. Sanyal and S. Chakraborty, Pramana-*J. Phys.* 34, 1 (1990).
3. S. Chakraborty, *Ind. J. pure Appl. Phys. 29*, 31 (1980).
4. R. Tikekar and L.K. Patel, *Gen. Rel. Grav. 24*, 397 (1992).
5. R. Tikekar and L.K. Patel, *Pramana-J. Phys. 42*, 483 (1994).
6. Bali, R. and Tyagi, A. *Gen. Re. Grav. 21*, (1989).
7. M. Giovannini, The magnetized universe, Astro- Ph/03102614 (2003).
8. J. T. Harwood and J. Wainwright, *Gen. Relativ. Gravit. 36*, 799 (2004).
9. R. Bali and R.D. Upadhaya, *Astrophys. Space Sci.* 283, 97 (2003).
10. R. Bali and Anjali, Astrophys. *Space Sci. 302*, 201 (2006).
11. R. Bali, U.K. Pareek and A. Pradhan, Chin. *Phys. Lett. 24*, 585 (2007).
12. B. Saha and M. Visinescu, *Astrophys. Space Sci. 315*, 99 (2008).
13. B. Saha, *Astrophys. Space Sci. 299*, 149 (2005)
14. V.U.M. Rao, T. Vinutha and K.V.S. Sireesha, *Astrophys. Space Sci. 317*, 79 (2008).
15. Tripathy, S.K., Nayak, S.K., Sahu, S.K. and Routrary, T.R., *Astrophys. Space Sci. 318*, 125-131 (2008).
16. Pawar, D.D., Bhaware, S.W. and Deshmukh, A.G., *Int. J. Theor. Phys. 47*, 599-605 (2008).
17. Tyagi, A. and Sharma, K. Ind. Acad. of Mathematics. 32, 2 (2010).