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LRS Bianchi Type - V String Dust Cosmological Model in General Relativity

VARUN HUMAD[1] and SAPNA SHRIMALI[2]

[1]Research Scholar, Mewar University, Chittorgarh (Rajasthan), (INDIA)
[2]Department of Mathematics, Pacific Academy of Higher Education & Research University,Udaipur (Rajasthan) (INDIA)

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Abstract

We have investigated LRS Bianchi Type - V cosmological model with string dust as the source of gravitational field. For the complete determination of the model, we assume that rest energy density (ξ) is equal to the string tension density (λ). Some physical and geometrical features of the model are also discussed.

Key words: Bianchi Type-V String Dust, Cosmology, General Relativity.

1. Introduction

The string theory play an important role in the study of physical situation at the very early stages of the formation of the universe and the study is more interesting as these models contain isotropic special cases and permit arbitrary small anisotropy levels at some point of time. This property makes them suitable as model of our universe. The homogeneous and isotropic Friedman-Robertson-Walker (FRW) cosmological models, which are particular case of Bianchi Type I, V and IX universe, according to whether the constant curvature of the physical three space, t = constant, is zero, negative or positive. Bianchi Type V cosmological models are the natural generalization of FRW models with negative curvature.

Homogeneous cosmological models representing matter and electromagnetic field have been investigated by Vaijk and Eltgroth[1], Damiao Soares and Assad[2], Dunn and Tupper[3] Roy and Singh[4] have obtained a Bianchi Type V universe with stiff fluid and a source free electromagnetic field. Coley[5] has investigated Bianchi Type V imperfect fluid cosmological models in general relativity. Banerjee *et al.*[6] have investigated an axially symmetric Bianchi Type I string dust cosmological model in presence and absence of magnetic field. Bali and Upadhaya[7] have investigated LRS Bianchi

Type I string dust magnetized cosmological models. Pradhan *et al.*[8] have presented the generation of Bianchi Type V cosmological models with varying Λ term. Bali and Anjali[9] and Bali[10] have investigated Bianchi Type I and type V string cosmological models in general relativity. Bali and Upadhaya[11] have investigated LRS (Locally Rotationally Symmetric) Bianchi Type I string dust magnetized cosmological models using the condition that σ (shear) is proportional to the expansion (θ). Roy and Banerjee[12] have presented some LRS Bianchi Type II string cosmological models for cloud of geometrical and massive strings using the condition $\xi = \lambda$ and (σ/θ) constant respectively. Recently, Baysal *et al.*[13], Kilinc and Yavuz[14], Pradhan[15], Pradhan *et al.*[16,17] and Yadav *et al.*[18] have obtained some string cosmological models in cylindrically symmetric inhomogeneous universe. Singh and Tyagi[19-21] investigated various Bianchi Type cosmological models with variable cosmological and gravitational constant in presence and absence of magnetic field. The perfect and bulk viscous fluids are considered as source of matter.

Motivated by aforesaid, we have investigated LRS Bianchi Type – V cosmological model with string dust as the source of gravitational field. To get a determinate solution, we have assumed an equation of state $\xi = \lambda$. The physical and geometrical aspects of the model are also discussed.

2. Metric and Field Equation :

We consider an LRS Bianchi Type - V metric of the form

$$ds^2 = -dt^2 + A^2dx^2 + B^2e^{2x}(dy^2 + dz^2) \quad (1)$$

Where A and B are function of time t.

The energy momentum tensor for string dust is given b

$$T_i^j = \xi v_i v^j - \lambda x_i x^j \quad (2)$$

With $\quad v_i v^i = -x_i x^i = -1 \quad (3)$

And $\quad x_i v^i = 0 \quad (4)$

where ξ is the rest energy density for a cloud of strings with particles attached to them, λ is the cloud strings tension density, v^i the four velocity vector and x^i is the direction of strings.

The expression for scalar of expansion θ and shear scalar σ are

$$\theta = u^i_{;i} = \frac{\dot{A}}{A} + \frac{2\dot{B}}{B} \quad (5)$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} = \frac{1}{3}\left(\theta^2 - \frac{2\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2}\right) \quad (6)$$

The Einstein's field equation (in gravitational units c = 1, $8\pi G = 1$) for a system of string

$$R_i^j - \frac{1}{2}Rg_i^j = -T_i^j \quad (7)$$

For the metric (1), Einstein's field equation's can be written as

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{1}{A^2} = \lambda \quad (8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{1}{A^2} = 0 \quad (9)$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} - \frac{3}{A^2} = \xi \qquad (10)$$

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = 0 \qquad (11)$$

Where an over dot stands for the first and double dot for the second derivative with respect to cosmic time t.

From equation (11), we have

$$A = m\,B \qquad (12)$$

Where m is the constant of integration.

From equation (12), without loss of generality we can take m = 1

$$A = B \qquad (13)$$

3. Solution of the Field Equations :

The field equations (8) - (10) are three equations in four unknowns A, B, λ and ξ. In order to obtain a determinate solution, we assume a relation

$$\xi = \lambda \qquad (14)$$

Using the condition given by (14) in (8) and (10), we have

$$\frac{\dot{A}\dot{B}}{AB} - \frac{\ddot{B}}{B} - \frac{1}{A^2} = 0 \qquad (15)$$

Now, using equation (13) in (9) and (15),we have

$$\frac{2\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{1}{A^2} = 0 \qquad (16)$$

$$\frac{\dot{A}^2}{A^2} - \frac{\ddot{A}}{A} - \frac{1}{A^2} = 0 \qquad (17)$$

From equation (16) and (17), we have

$$\ddot{A} = 0 \qquad (18)$$

Now integrating equation (18), we obtain

$$A = at + b \qquad (19)$$

Where a and b are constant of integration.

From equation (13), We get

$$B = at + b \qquad (20)$$

Hence the model (1) is reduce to

$$ds^2 = -dt^2 + (at+b)^2\, dx^2 + (at+b)^2 e^{2x}\,(dy^2 + dz^2) \qquad (21)$$

4. Some physical and geometrical aspects of the model :

For the model (21), the physical and geometrical parameters can be easily obtained. The rest energy density (ξ), the string tension density (λ), the scalar of expansion (θ) and the shear scalar (σ) are respectively given by

$$\xi = \frac{a^2 - 1}{(at+b)^2} \qquad (22)$$

$$\lambda = \frac{a^2 - 1}{(at+b)^2} \qquad (23)$$

$$\theta = \frac{3a}{(at+b)} \qquad (24)$$

$$\sigma = \frac{\sqrt{2}a}{(at+b)} \qquad (25)$$

$$\frac{\sigma}{\theta} = \frac{\sqrt{2}}{3} = constant \qquad (26)$$

5. Conclusions

(i) As the time t increase, the rate of expansion θ decrease.

(ii) $\frac{\sigma}{\theta} = \frac{\sqrt{2}}{3} = constant$, therefore model does not approach isotropy for large value of t.

(iii) As $t \to 0$, the scalar of expansion θ tends to infinitely large and when $T \to \infty$, the scalar of expansion $\theta \to 0$. Also at $t \to 0$ shear scalar σ tends to infinity and when $t \to \infty$, shear scalar σ tends to zero. Hence the model describes a shearing, non- rotating and expanding universe with a big- bang start.

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