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A study on MHD free convection flow past an oscillating porous plate

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Abstract

In this paper we study on MHD free convection flow past an infinite vertical oscillating porous plate in the presence of heat and mass transfer. The governing equation are solved by Laplace transform technique. The results are obtained for velocity, temperature, concentration, skin-friction and Nusselt number. The results have been shown graphically.

Key words : MHD, temperature, porous medium, oscillating plate, concentration, skin-friction, Nusselt number.

Introduction

The study on magnetohydrodynamic free convection flow past an oscillating porous plate has received the attention of many researchers, like Soundalgekar[19] has discussed free connection effects on the oscillatory flow past an infinite vertical porous plate with constant suction. Soundalgekar[20] presented free convection effects on the flow past a vertical oscillating plate. MHD oscillatory flow through porous medium was carried out by Gholizadeh[8]. Singh *et al.*[18] have analyzed unsteady free convective flow through a porous medium between two infinite vertical parallel oscillating porous plate with different amplitude. Soundalgekar *et al.*[21] presented MHD flow past an infinite vertical oscillating plate with mass transfer and constant heat flux. Deka *et al.*[6] discussed free convection effects on MHD flow past an infinite vertical oscillating plate with constant heat flux.

Revankar[13] has founded free convection effect on flow past an impulsively started or oscillating infinite vertical plate. Jaiswal and Soundelgekar[10] studied oscillating plate temperature effects on a flow past an infinite porous plate with constant suction and embedded in a porous medium. Heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity were

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depicted by Singh *et al.*[16]. Hayat *et al.*[9] derived the flow of a visco-elastic fluid on an oscillating plate. Sharma *et al.*[15] extended unsteady MHD flow and heat transfer over a continuous porous moving horizontal surface in the presence of an oscillating free stream and heat source. Singh and Gupta[17] have investigated MHD free convective flow of a viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime with mass transfer.

Sridhar *et al.*[22] investigated MHD free convective flow of an incompressible viscous dissipative fluid in an infinite vertical oscillating plate with constant heat flux. Muthucumaraswamy and Meenakshisundaram[11] presented theoretical study of chemical reaction effects on vertical oscillating plate with variable temperature. Radiation and mass transfer effects on two-dimensional flow past an impulsively started infinite vertical plate were depicted by Prasad *et al.*[12]. Ferdows *et al.*[7] have discussed MHD free convection and mass transfer flow in a porous media with simultaneous rotating fluid. Das *et al.*[5] derived mass transfer effects on free convective MHD flow of a viscous fluid bounded by an oscillating porous plate in the slip flow regime with heat source. Studied oscillatory chemically-reacting MHD free convection heat and mass transfer in a porous medium with soret and dufour effects : finite element modelling were founded by Bhargava *et al.*[2].

Ahmed[1] analyzed free and forced convective MHD oscillatory flow over an infinite porous surface in an oscillating free stream. Das and Jana[3] depicted heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium. Effect of heat source on MHD free convection flow past an oscillating porous plate in the slip flow regime were presented by Das *et al.*[4]. Saravana *et al.*[14] have presented mass transfer effects on MHD viscous flow past an impulsively started infinite vertical plate with constant mass flux.

Formulation of the problem :

We consider a two-dimensional of a viscous incompressible electrically conducting fluid along an infinite non-conducting vertical flat plate through a porous medium. Initially the plate and the fluid are at same temperature T_∞^* in the stationary condition with concentration level C_∞^* at all points. Taking X^*-axis along the plate in the vertically upward direction and taking y^*-axis normal to the plate. A magnetic field of uniform strength B_0 is applied in the direction of flow and the induced magnetic field is neglected, At $t > 0$, the plate starts oscillating in its own plane with a velocity $U_0 \cos\omega^* t^*$. Its temperature is raised to T_w^* and the concentration level at the plate is raised to C_w^*.

The governing equation of the motion are given by

$$\frac{\partial u^*}{\partial t^*} = \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} + g\beta\left(T^* - T_\infty^*\right) + g\beta_c\left(C^* - C_\infty^*\right) - \frac{\sigma B_0^2 u^*}{\rho} - \frac{\upsilon u^*}{K^*} \tag{1}$$

$$\rho C_p \frac{\partial T^*}{\partial t^*} = \kappa \frac{\partial^2 T^*}{\partial y^{*2}} \tag{2}$$

$$\frac{1}{D}\frac{\partial C^*}{\partial t^*} = \frac{\partial^2 C^*}{\partial y^{*2}} \tag{3}$$

The initial and boundary conditions are

$$\left.\begin{array}{llll} u^* = 0, & T^* = T^*_\infty, & C^* = C^*_\infty, & y^*, t^* \le 0 \\ u^* = U_0 \cos\omega^* t^*, & T^* = T^*_w, & C^* = C^*_w, & y^* = 0, t^* > 0 \\ u^* = 0, & T^* = T^*_\infty, & C^* = C^*_\infty, & y^* \to \infty, t^* > 0 \end{array}\right\} \tag{4}$$

We introduce the following non-dimensional quantities.

$$u = \frac{u^*}{U_0}, \quad t = \frac{t^* U_0^2}{\upsilon}, \quad y = \frac{y^* U_0}{\upsilon}, \quad \omega = \frac{\omega^* \upsilon}{U_0^2},$$

$$S_c = \frac{\upsilon}{D}, \quad P_r = \frac{\mu C_p}{\kappa}, \quad M = \frac{\sigma B_0^2 \upsilon}{\rho U_0^2}, \quad K = \frac{U_0^2 K^*}{\upsilon^2},$$

$$G_r = \frac{\upsilon g \beta_t \left(T^*_w - T^*_\infty\right)}{U_0^3}, \quad G_m = \frac{\upsilon g \beta_c \left(C^*_w - C^*_\infty\right)}{U_0^3},$$

$$\theta = \frac{T^* - T^*_\infty}{T^*_w - T^*_\infty}, \quad \phi = \frac{C^* - C^*_\infty}{C^*_w - C^*_\infty} \tag{5}$$

From equations (1), (2) and (3), we have

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + G_r\theta + G_m\phi - \left(M + \frac{1}{K}\right)u \tag{6}$$

$$P_r \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

$$S_c \frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial y^2} \tag{8}$$

where

u^* - Velocity component in x^*-direction.
u - Dimensionless velocity component.
t^* - Time in x^*, y^* coordinate system.
t - Time in dimensionless coordinate.
υ - Kinematic viscosity.
β - Coefficient of volume expansion.
β_t - Coefficient of thermal expansion.
β_c - Coefficient of concentration expansion.
K^* - Permeability of medium.
C_P - Specific heat at constant pressure.

κ - Thermal conductivity of the fluid.
D - Mass diffusivity.
U_0 - Velocity of plate.
ω - Frequency of oscillation.
S_c - Schmidt number.
P_r - Prandtl number.
M - Magnetic parameter.
K - Permeability parameter.
G_r - Grashof number.
G_m - Modified Grashof number.

The initial and boundary conditions are

$$\left.\begin{array}{llll} u=0, & \theta=0, & \phi=0, & y,t\le 0 \\ u=\cos\omega t, & \theta=1, & \phi=1, & y=0,t>0 \\ u=0, & \theta=0, & \phi=0, & y\to\infty, t>0 \end{array}\right\} \tag{9}$$

Solution of the Problem :

The solution of equation (6), (7) and (8) by Laplace transform technique, we have

$$\frac{d^2\bar{u}}{dy^2}-\left(p+M^*\right)\bar{u}=-G_r\bar{\theta}-G_m\bar{\phi} \tag{10}$$

where $M^* = M+\frac{1}{K}$

$$\frac{d^2\bar{\theta}}{dy^2}-pP_r\bar{\theta}=0 \tag{11}$$

$$\frac{d^2\bar{\phi}}{dy^2}-pS_c\bar{\phi}=0 \tag{12}$$

The boundary conditions equation (9) are transformed to

$$\left.\begin{array}{llll} \bar{u}=\frac{p}{p^2+\omega^2}, & \bar{\theta}=\frac{1}{p}, & \bar{\phi}=\frac{1}{p}, & y=0,t>0 \\ \bar{u}=0, & \bar{\theta}=0, & \bar{\phi}=0, & y\to\infty,t>0 \end{array}\right\} \tag{13}$$

On solving equation (10), (11) and (12) under the boundary conditions equation (13), we have

$$\bar{u}(y,p)=\frac{pe^{-y\sqrt{(p+M^*)}}}{\left(p^2+\omega^2\right)}+\frac{G_r}{p(P_r-1)\left(p-\frac{M^*}{P_r-1}\right)}\{e^{-y\sqrt{(p+M^*)}}-e^{-y\sqrt{pP_r}}\}$$
$$+\frac{G_m}{p(S_c-1)\left(p-\frac{M^*}{(S_c-1)}\right)}\left\{e^{-y\sqrt{(p+M^*)}}-e^{-y\sqrt{pS_c}}\right\} \tag{14}$$

$$\bar{\theta}(y,p)=\frac{e^{-y\sqrt{pP_r}}}{p} \tag{15}$$

$$\bar{\phi}(y,p)=\frac{e^{-y\sqrt{pS_c}}}{p} \tag{16}$$

Taking inverse Laplace transform of equation (14), (15) and (16), we have

$$
\begin{aligned}
u(y,t) = {} & \frac{1}{4} e^{i\omega t}\left\{ e^{-2\eta\sqrt{(M^*+i\omega)t}}\ \operatorname{erfc}\left(\eta - \sqrt{(M^*+i\omega)t}\right) + e^{2\eta\sqrt{(M^*+i\omega)t}}\ \operatorname{erfc}\left(\eta + \sqrt{(M^*+i\omega)t}\right)\right\} \\
& + \frac{1}{4} e^{-i\omega t}\left\{ e^{-2\eta\sqrt{(M^*-i\omega)t}}\ \operatorname{erfc}\left(\eta - \sqrt{(M^*-i\omega)t}\right) + e^{2\eta\sqrt{(M^*-i\omega)t}}\ \operatorname{erfc}\left(\eta + \sqrt{(M^*-i\omega)t}\right)\right\} \\
& - \frac{1}{2}\left(\frac{G_r + G_m}{M^*}\right)\left\{ e^{-2\eta\sqrt{M^* t}}\ \operatorname{erfc}\left(\eta - \sqrt{M^* t}\right) + e^{2\eta\sqrt{M^* t}}\ \operatorname{erfc}\left(\eta + \sqrt{M^* t}\right)\right\} \\
& + \frac{G_r}{2M^*} e^{\left(\frac{M^* t}{P_r - 1}\right)}\left[e^{-2\eta\sqrt{\left(\frac{M^* P_r t}{P_r - 1}\right)}}\left\{ \operatorname{erfc}\left(\eta - \sqrt{\left(\frac{M^* P_r t}{P_r - 1}\right)}\right)\right.\right. \\
& \left. - \operatorname{erfc}\left(\eta\sqrt{P_r} - \sqrt{\left(\frac{M^* t}{P_r - 1}\right)}\right)\right\} + e^{2\eta\sqrt{\left(\frac{M^* P_r t}{P_r - 1}\right)}}\left\{ \operatorname{erfc}\left(\eta + \sqrt{\left(\frac{M^* P_r t}{P_r - 1}\right)}\right)\right. \\
& \left.\left. - \operatorname{erfc}\left(\eta\sqrt{P_r} + \sqrt{\left(\frac{M^* t}{P_r - 1}\right)}\right)\right\}\right] + \frac{G_m}{2M^*} e^{\left(\frac{M^* t}{S_c - 1}\right)}\left[e^{-2\eta\sqrt{\left(\frac{M^* S_c t}{S_c - 1}\right)}}\right. \\
& \times\left\{ \operatorname{erfc}\left(\eta - \sqrt{\left(\frac{M^* S_c t}{S_c - 1}\right)}\right) - \operatorname{erfc}\left(\eta\sqrt{S_c} - \sqrt{\left(\frac{M^* t}{S_c - 1}\right)}\right)\right\} \\
& \left. + e^{2\eta\sqrt{\left(\frac{M^* S_c t}{S_c - 1}\right)}}\left\{ \operatorname{erfc}\left(\eta + \sqrt{\left(\frac{M^* S_c t}{S_c - 1}\right)}\right) - \operatorname{erfc}\left(\eta\sqrt{S_c} + \sqrt{\left(\frac{M^* t}{S_c - 1}\right)}\right)\right\}\right] \\
& + \frac{G_r}{M^*}\operatorname{erfc}\left(\eta\sqrt{P_r}\right) + \frac{G_m}{M^*}\operatorname{erfc}\left(\eta\sqrt{S_c}\right) \qquad (17)
\end{aligned}
$$

$$\theta(y,t) = \operatorname{erfc}\left(\eta\sqrt{P_r}\right) \qquad (18)$$

$$\phi(y,t) = \text{erfc}\left(\eta\sqrt{S_c}\right) \tag{19}$$

Skin-friction

We can calculate from the velocity field and skin-friction is non-dimensional form.

$$\tau = -\mu \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} \tag{20}$$

In non-dimensional form, the skin-friction can be reduce to

$$\tau = -\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{21}$$

$$\tau = \frac{1}{2\sqrt{t}}\left\{e^{i\omega t}\sqrt{\left(M^* + i\omega\right)t}\ \text{erf}\left(\sqrt{\left(M^* + i\omega\right)t}\right) + e^{-i\omega t}\sqrt{\left(M^* - i\omega\right)t}\ \text{erf}\left(\sqrt{\left(M^* - i\omega\right)t}\right)\right\}$$
$$+\frac{1}{\sqrt{\pi t}}e^{-M^*t} - \left(\frac{G_r + G_m}{M^*}\right)\sqrt{M^*}\ \text{erf}\sqrt{M^*t}$$
$$+\frac{G_r}{M^*}\sqrt{\left(\frac{M^*P_r}{P_r - 1}\right)}\,e^{\left(\frac{M^*t}{P_r-1}\right)}\left\{\text{erf}\sqrt{\left(\frac{M^*P_r t}{P_r - 1}\right)} - \text{erf}\sqrt{\left(\frac{M^*t}{P_r - 1}\right)}\right\}$$
$$+\frac{G_m}{M^*}\sqrt{\left(\frac{M^*S_c}{S_c - 1}\right)}\,e^{\left(\frac{M^*t}{S_c-1}\right)}\left\{\text{erf}\sqrt{\left(\frac{M^*S_c t}{S_c - 1}\right)} - \text{erf}\sqrt{\left(\frac{M^*t}{S_c - 1}\right)}\right\} \tag{22}$$

Nusselt number :

We can calculate from the temperature field and Nusselt number is non-dimensional form.

$$N_u = -\left(\frac{\partial \theta}{\partial y}\right)_{y=0} = \sqrt{\frac{P_r}{\pi t}} \tag{23}$$

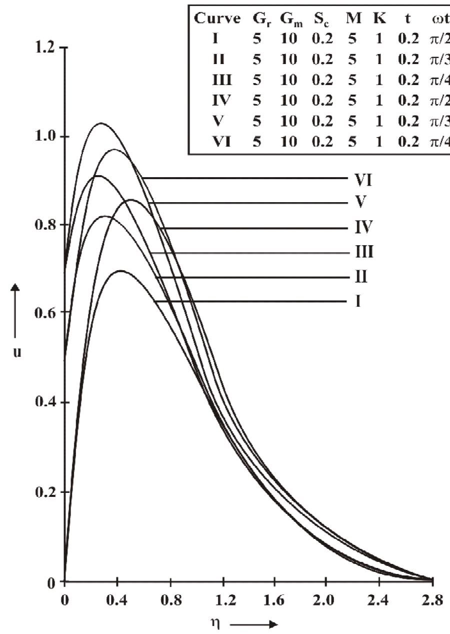

Fig. 1. Velocity profiles

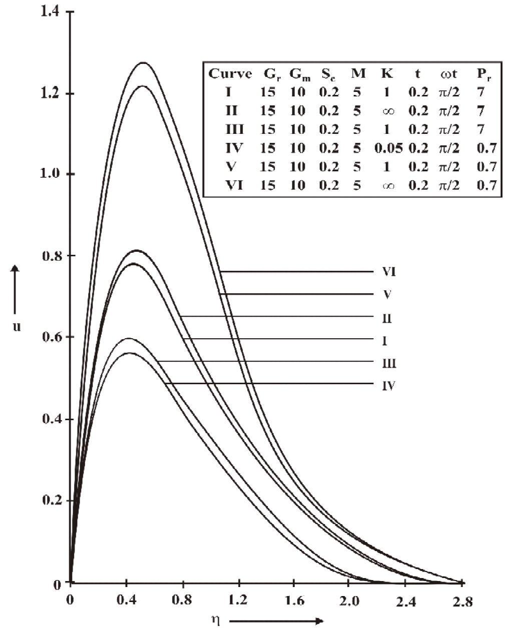

Fig. 3. Velocity profiles

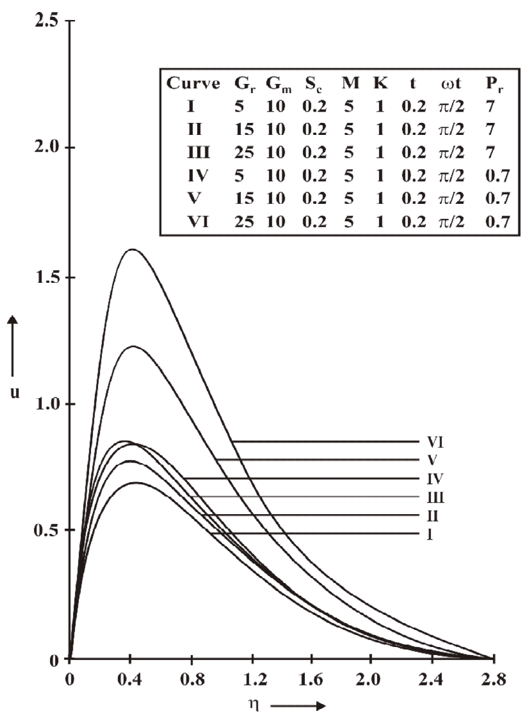

Fig. 2. Velocity profiles

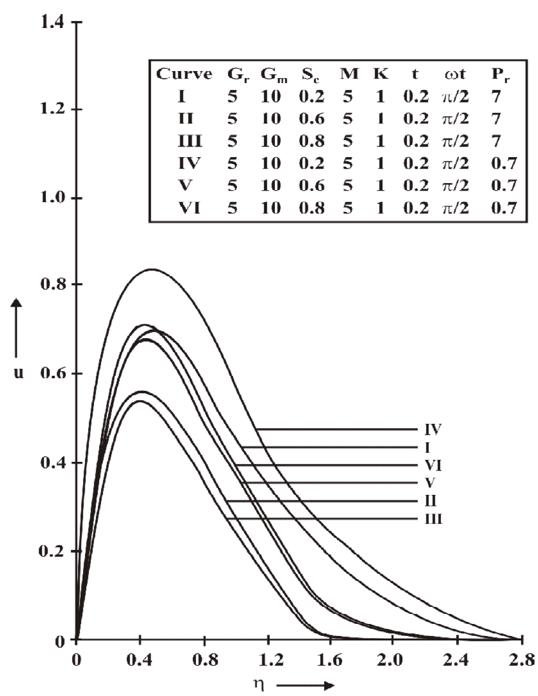

Fig. 4. Velocity profiles

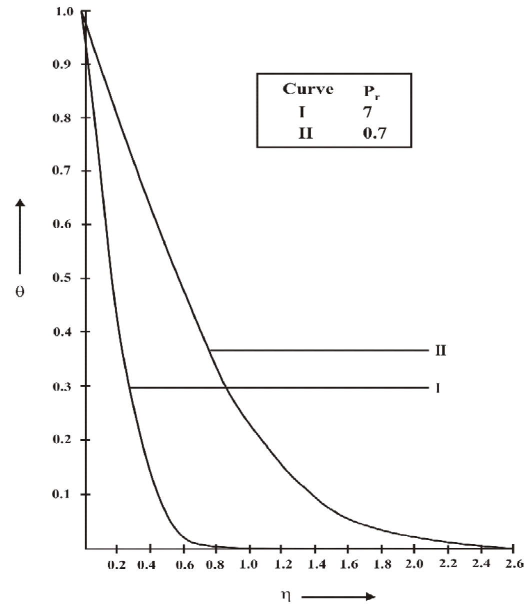

Fig. 5. Temperature profiles

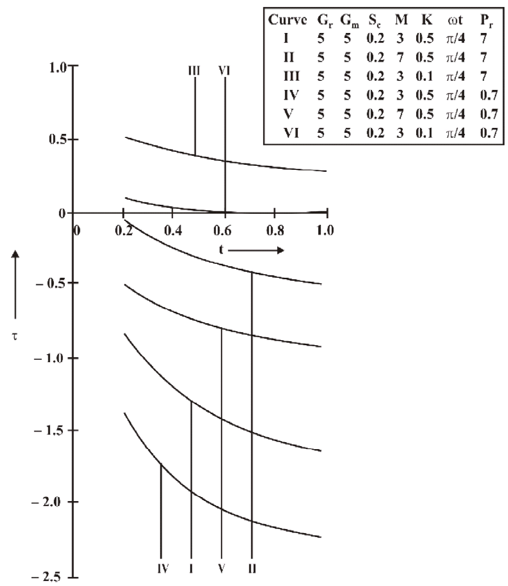

Fig.7. Skin-friction profiles

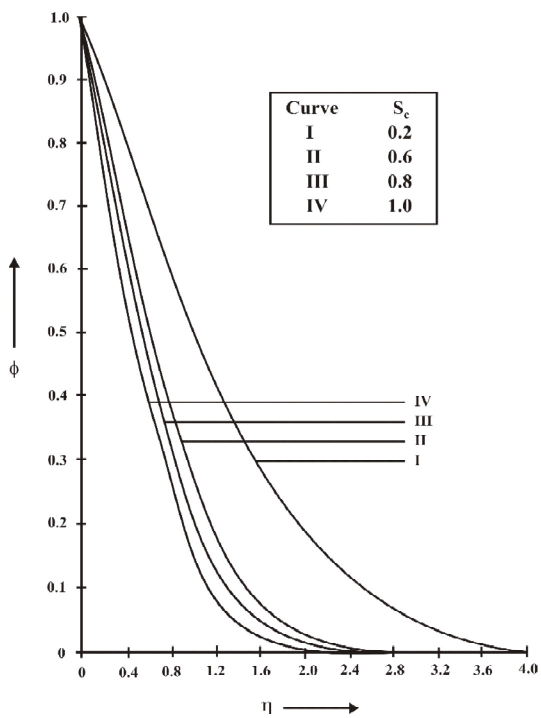

Fig. 6. Concentration profiles

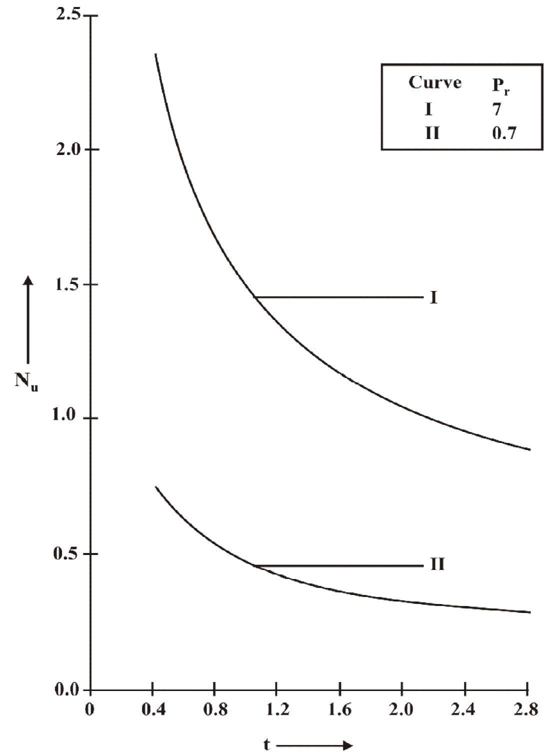

Fig. 8. Nusselt number

Results and Discussion

The variation of velocity profiles are shown in graph of figure (1) for the case of water (P_r = 7.0) and air (P_r = 0.7) due to the variations in Phase angle (ωt). It is evident from the figure that the velocity near the plate exceeds at the plate. The magnitude of the velocity decreases with increasing in phase angle (ωt) for both water (P_r = 7.0) and air (P_r=0.7). The velocity for air (P_r=0.7) is greater than water (P_r=7.0). Graph of figure (2) depicts the velocity variations with Grashof number (G_r), modified Grashsof number (G_m) and Prandtl number (P_r) in cases of cooling and heating of the surface respectively. It is observed that greater cooling of surface an increasing value of G_r and an increasing value of G_m results in an increase in the velocity for both water (P_r=7.0) and air (P_r=0.7). It is due to fact increase in the values of Grashof number (G_r) and modified Grashof number (G_m) tends to increase the thermal and mass buoyancy effect[7].

We see from graph of figure (3) that the influences of magnetic parameter (M), permeability parameter (K), and Prandtl number (P_r) in case of cooling and heating of the plate respectively. In case of cooling of the plate, the velocity near the plate is greater than at the plate. The maximum velocity near the plate and is in the neighbourhood of the point $\eta(= 0.4)$. After η (>0.4), the velocity decreases with increasing magnetic parameter (M) for both water (P_r =7.0) and air (P_r = 0.7). Graph of figure (4) depicts the effects of Schmidt number (S_c), Prandtl number (P_r) and time (t) on the velocity profiles for the case Grashof number (G_r > 0) and modified Grashof number (G_m > 0). In case of cooling of the plate, the velocity near the plate increases owing to the presence of water (P_r = 7.0) and air (P_r = 0.7) in the flow field[7].

We see from graph of figure (5) that the temperature profiles against η far away from the plate, magnitude of temperature is maximum at the plate and then tends to zero. The temperature for air (P_r = 0.7) is greater than water (P_r = 7.0). This is due to the fact that thermal conductivity of the fluid decreases with increasing value of Prandtl number (P_r), Resulting a decrease in thermal boundary layer thickness. Graph of figure (6) shows the effect of Schmidt number (S_c) on the concentration is maximum at the surface and falls exponentially. The concentration decreases with an increasing value of Schmidt number (S_c).

Graph of figure (7) depicts skin-friction (τ) against time (t) for different values of G_r, G_m, S_c, M, K due to water (P_r = 7.0) and air (P_r = 0.7). The skin-friction decreases with increasing value of time (t). The magnitude of skin-friction for air (P_r=0.7) is less than water (Pr= 7.0). Graph of figure (8) represents Nusselt number (N_u) against time (t). It is clear from the figure Nusselt number (N_u) decreases with increasing value of time (t). Nusselt number (N_u) for water (P_r = 7.0) is higher than air (P_r = 0.7).

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