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A study on MHD flow and heat transfer along a porous flat plate with mass transfer

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Abstract

In this paper we study on MHD flow and heat transfer along a porous flat plate in the presence of mass transfer. The governing equations are solved by finite difference technique. The results are obtained for velocity profiles, temperature profiles and concentration profiles of the flow field. The results have been shown graphically.

Key words : MHD, Hall current, flat plate, heat transfer, mass transfer, temperature and concentration.

Introduction

The study on magnetohydrodyamic flow and heat transfer along a porous flat plate with mass transfer has received the attention of many researchers, like Debnath *et al*.[6] have discussed effects of Hall current on hydromagnetic flow past a porous plate in a rotating fluid system. Sattar and Hossain[11] presented unsteady hydromagnetic free convection flow with Hall current and mass transfer along an accelerated porous plate with time dependent temperature and concentration. Sattar and Alam[10] analyzed MHD free convective heat and mass transfer flow with Hall current and constant heat flux through a porous medium. Hall current effects on the velocity and temperature field of an unsteady Hartmann flow was founded by Attia[1]. Dash *et al*.[5] depicted Hall effects on MHD flow along an accelerated porous flat plate with mass transfer and internal heat generation.

Sattar and Maleque[12] derived unsteady MHD natural convection flow along an accelerated porous plate with Hall current and mass transfer in rotating porous medium. Sharma and Mishra[13] considered effects of mass transfer on unsteady MHD flow and heat transfer past an infinite porous vertical moving plate. Attia and Ahmed[2] examined Hall effect on unsteady MHD Couette flow and heat transfer of a Bingham field with suction and injection. Hydromagnetic free convection and

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mass transfer flow with Joule heating, thermal diffusion, heat source and Hall current were carried out by Singh *et al.*[14]. Venkateswarlu and Rao[17] studied numerical solution of heat and mass transfer in MHD flow of a viscous fluid past a vertical plate under oscillatory suction velocity.

Chaudhary and Jain[4] investigated Hall effect on MHD mixed convection flow of a visco-elastic fluid past an infinite vertical porous plate with mass transfer and radiation. Singh *et al.*[15] discussed MHD free convection and mass transfer flow past a flat plate. Unsteady MHD flow with heat and mass transfer through porous medium past an oscillating porous horizontal plate in slip flow regime were extended by Singh *et al.*[16]. Ghosh *et al.*[7] presented Hall effects on MHD flow in a rotating system with heat transfer characteristics.

Poonia and Chaudhary[9] analyzed MHD free convection and mass transfer flow over an infinite vertical porous plate with viscous dissipation. Kumar and Chand[8] depicted effect of slip conditions and Hall current on unsteady MHD flow of a visco-elastic fluid past an infinite vertical porous plate through porous medium. Babu and Reddy[3] derived mass transfer effects on MHD mixed convective flow from a vertical surface with ohmic heating and viscous dissipation.

Formulation of the problem :

We are considering a two-dimensional of a viscous incompressible electrically conducting fluid along an infinite vertical porous plate. Taking x*-axis along the plate in upward direction and taking y*-axis normal to the plate. At time $t^*\geq 0$, the temperature is raised to T_w^* and the species concentration at the plate raised to C_w^*. The species thermal diffusion and diffusion thermal energy effects are neglected due to assuming the level of species concentration is very low. A magnetic field of uniform strength is applied transversely to the porous plate and the magnetic Reynolds number is very small so that the induced magnetic field can be neglected.

The equation of conservation of electric charge $\Delta.J=0$, which gives j_y = constant, where $J=\left(j_x, j_y, j_z\right)$. It is also assumed that the plate is non-conducting, which gives $j_y = 0$ at the plate and hence zero everywhere.

The general equation of ohm's law, when the strength of magnetic field is very large and in the absence of electric field is given by

$$J+\frac{\omega_e\tau_e}{B_0}JB=\sigma\left(\mu_e VB+\frac{1}{en_e}\nabla P_e\right) \quad (1)$$

Under the assumption the electron pressures for weakly ionized gas, the thermo-electric pressure and ion-slip are neglected.

The equation (1) can be written as

$$\left.\begin{aligned} j_x &= \frac{\sigma\mu_e B_0}{1+m^2}\left(mu-w\right)\\ j_z &= \frac{\sigma\mu_e B_0}{1+m^2}\left(u+mw\right)\end{aligned}\right\} \quad (2)$$

where

ω_e - Electron frequency.

τ_e - Electron collision time.

σ - Electric conductivity.
μ_e - Magnetic permeability.
V - Velocity vector.
e - Electron charge.
n_e - Number density of the electron.
P_e - Electron pressure.
u - x-component of velocity vector.
w - z-component of velocity vector.
m - Hall parameter.

The governing equations of motion are given by

$$\frac{\partial v^*}{\partial y^*} = 0 \tag{3}$$

$$\frac{\partial u^*}{\partial t^*} + v^* \frac{\partial u^*}{\partial y^*} = \upsilon \frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma \mu_e^2 B_0^2}{\rho(1+m^2)}\left(u^* + m w^*\right) + g\beta\left(T^* - T_\infty^*\right) + g\beta_c\left(C^* - C_\infty^*\right) \tag{4}$$

$$\frac{\partial w^*}{\partial t^*} + v^* \frac{\partial w^*}{\partial y^*} = \upsilon \frac{\partial^2 w^*}{\partial y^{*2}} - \frac{\sigma \mu_e^2 B_0^2}{\rho(1+m^2)}\left(w^* - m u^*\right) \tag{5}$$

$$\frac{\partial T^*}{\partial t^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{\kappa}{\rho C_p} \frac{\partial^2 T^*}{\partial y^{*2}} \tag{6}$$

$$\frac{\partial C^*}{\partial t^*} + v^* \frac{\partial C^*}{\partial y^*} = D \frac{\partial^2 C^*}{\partial y^{*2}} \tag{7}$$

The initial and boundary conditions are

$$\left.\begin{array}{llllll} t^* \le 0: & u^* = 0, & w^* = 0, & T^* = T_\infty^*, & C^* = C_\infty^* & \text{for all } y^* \\ t^* > 0: & u^* = 0, & w^* = 0, & T^* = T_w^*, & C^* = C_w^* & \text{at } y^* = 0 \\ t^* > 0: & u^* = 0, & w^* = 0, & T^* = T_\infty^*, & C^* = C_\infty^* & \text{as } y^* \to \infty \end{array}\right\} \tag{8}$$

We introduce the following non-dimensional quantities.

$$(u, v, w) = \frac{\left(u^*, v^*, w^*\right)}{U_0}, \quad t = \frac{t^* U_0^2}{\upsilon}, \quad y = \frac{y^* U_0}{\upsilon},$$

$$M = \frac{\sigma \mu_e^2 B_0^2 \upsilon}{\rho U_0^2}, \quad P_r = \frac{\mu_e C_p}{\kappa}, \quad S_c = \frac{\upsilon}{D},$$

$$G_m = \frac{\upsilon g\beta\left(T_w^* - T_\infty^*\right)}{U_0^3}, \quad G_{m*} = \frac{\upsilon g\beta_c\left(C_w^* - C_\infty^*\right)}{U_0^3},$$

$$\theta = \frac{T^* - T_\infty^*}{T_w^* - T_\infty^*}, \quad C = \frac{C^* - C_\infty^*}{C_w^* - C_\infty^*} \tag{9}$$

By using the above dimensionless quantities into equation (3) to (7), we get

$$\frac{\partial v}{\partial y} = 0 \tag{10}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - \frac{M}{1+m^2}(u + mw) + G_m\theta + G_{m*}C \tag{11}$$

$$\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial y} = \frac{\partial^2 w}{\partial y^2} - \frac{M}{1+m^2}(w - mu) \tag{12}$$

$$\frac{\partial \theta}{\partial t} + v\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} \tag{13}$$

$$\frac{\partial C}{\partial t} + v\frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} \tag{14}$$

where

υ - Kinematic viscosity.

ρ - Density of the fluid.

β - Coefficient of thermal expansion.

β_c - Coefficient of species concentration expansion.

T^* - Temperature of the fluid within the boundary layer.

T^*_∞ - Temperature of the fluid far away from the boundary layer.

C^* - Species concentration.

C^*_∞ -Species concentration far away from the boundary layer.

κ - Thermal conductivity of the fluid.

C_p - Specific heat at constant pressure.

D - Chemical molecular diffusivity.

U_0 - Reference velocity.

M - Magnetic field parameter.

P_r - Prandtl number.

S_c - Schmidt number.

G_m - Modified Grashof number for heat transfer.

G_{m*} - Modified Grashof number for mass transfer.

The non-dimensional boundary conditions are

$$\left.\begin{array}{lllllll} t \le 0: & u = 0, & w = 0, & T = 0, & C = 0 & \text{for all } y \\ t > 0: & u = 0, & w = 0, & \theta = 1, & C = 1 & \text{at} & y = 0 \\ t > 0: & u = 0, & w = 0, & \theta = 0, & C = 0 & \text{as} & y \to \infty \end{array}\right\} \tag{15}$$

It is clear from equation (10), v is either constant or a function of time t and solution of equation (11) to (14) under the boundary conditions equation (15) exist only if

$$v = -\lambda t^{-1/2} \tag{16}$$

Here λ is transpiration parameter. From above equation it is clear that the assumption is suitable only for small value of time (t).

Solution of the problem :

An implicit finite difference technique of Crank-Nicolson Method is used to solve our problem.

On introducing forward and central difference approximation for the first and second derivatives of the dependent variable u, w, θ and C as

$$\frac{\partial u}{\partial t} = \frac{u_{i,j+1} - u_{i,j}}{\Delta t}, \qquad \frac{\partial w}{\partial t} = \frac{w_{i,j+1} - w_{i,j}}{\Delta t},$$

$$\frac{\partial \theta}{\partial t} = \frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t}, \qquad \frac{\partial C}{\partial t} = \frac{C_{i,j+1} - C_{i,j}}{\Delta t},$$

$$\frac{\partial u}{\partial y} = \frac{u_{i,j+1} - u_{i,j}}{\Delta y}, \qquad \frac{\partial w}{\partial y} = \frac{w_{i,j+1} - w_{i,j}}{\Delta y},$$

$$\frac{\partial\theta}{\partial y}=\frac{\theta_{i,j+1}-\theta_{i,j}}{\Delta y},\quad \frac{\partial C}{\partial y}=\frac{C_{i,j+1}-C_{i,j}}{\Delta y},\quad \frac{\partial^2 C}{\partial y^2}=\frac{C_{i,j+1}-2C_{i,j}+C_{i+i,j}}{(\Delta y)^2} \tag{17}$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{u_{i,j+1}-2u_{i,j}+u_{i+1,j}}{(\Delta y)^2},$$

$$\frac{\partial^2 w}{\partial y^2}=\frac{w_{i,j+1}-2w_{i,j}+w_{i+1,j}}{(\Delta y)^2},$$

$$\frac{\partial^2 \theta}{\partial y^2}=\frac{\theta_{i,j+1}-2\theta_{i,j}+\theta_{i+i,j}}{(\Delta y)^2},$$

The finite difference form of the equation (11) to (14) are given by

$$ru_{i-1,j+1}-2(r+1)\,u_{i,j+1}+ru_{i+1,j+1}=A_{i,j} \tag{18}$$

$$rw_{i-1,j+1}-2(r+1)\,w_{i,j+1}+rw_{i+1,j+1}=B_{i,j} \tag{19}$$

$$r\theta_{i-1,j+1}-2(r+1)\,\theta_{i,j+1}+r\theta_{i+1,j+1}=D_{i,j} \tag{20}$$

$$rC_{i-1,j+1}-2(r+1)\,C_{i,j+1}+rC_{i+1,j+1}=E_{i,j} \tag{21}$$

where

$$A_{i,j}=-ru_{i-1,j}+2\left(r+\frac{M\kappa}{1+m^2}-1-rhv_{i,j}\right)u_{i,j}+r\left(2hv_{i,j}-1\right)u_{i+1,j}+\frac{2mM\kappa}{1+m^2}w_{i,j}$$
$$-2\kappa\left(G_m\theta_{i,j}+G_{m*}C_{i,j}\right)$$

$$B_{i,j}=-rw_{i-1,j}+2\left(r+\frac{M\kappa}{1+m^2}-1-rhv_{i,j}\right)w_{i,j}+r\left(2hv_{i,j}-1\right)w_{i+1,j}-\frac{2mM\kappa}{1+m^2}u_{i,j}$$

$$D_{i,j}=-r\theta_{i-1,j}+2\left(r-P_r-rhP_rv_{i,j}\right)\theta_{i,j}+r\left(2hP_rv_{i,j}-1\right)\theta_{i+1,j}$$

$$E_{i,j}=-rC_{i-1,j}+2\left(r-S_c-rhS_cv_{i,j}\right)C_{i,j}+r\left(2hS_cv_{i,j}-1\right)C_{i+1,j}$$

here $r=\frac{\kappa}{h^2}$, Index i refers to space and j refers to time.

The boundary conditions equation (15) transformed to

$$\left.\begin{array}{lllll} u(i,0)=0, & w(i,0)=0, & \theta(i,0)=0, & C(i,0)=0 & \text{for all } i \\ u(0,j)=0, & w(0,j)=0, & \theta(0,j)=1, & C(0,j)=1 & \text{for all } j \\ u(100,j)=0, & w(100,j)=0, & \theta(100,j)=1, & C(100,j)=0 & \text{for all } j \end{array}\right\} \tag{22}$$

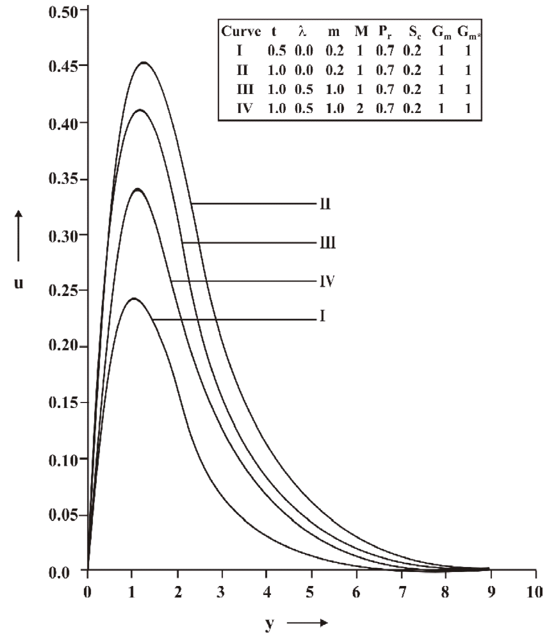

Fig. 1. Velocity profiles.

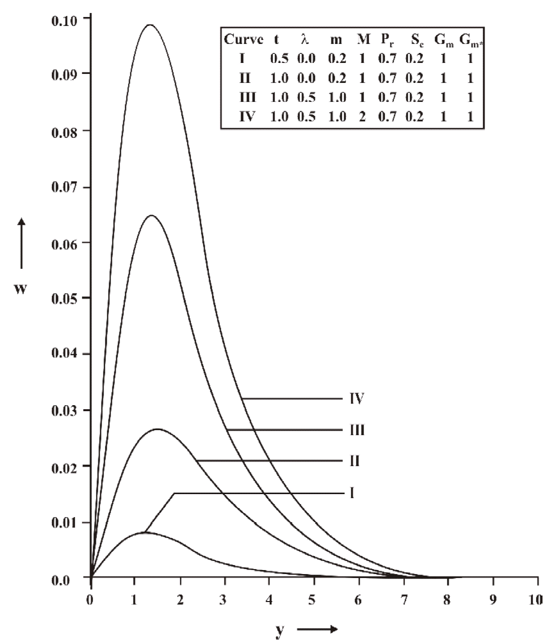

Fig. 2. Velocity profiles.

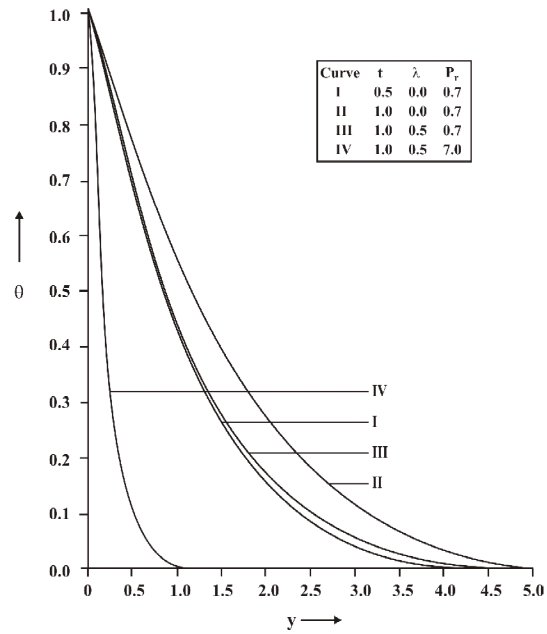

Fig. 3. Temperature profiles.

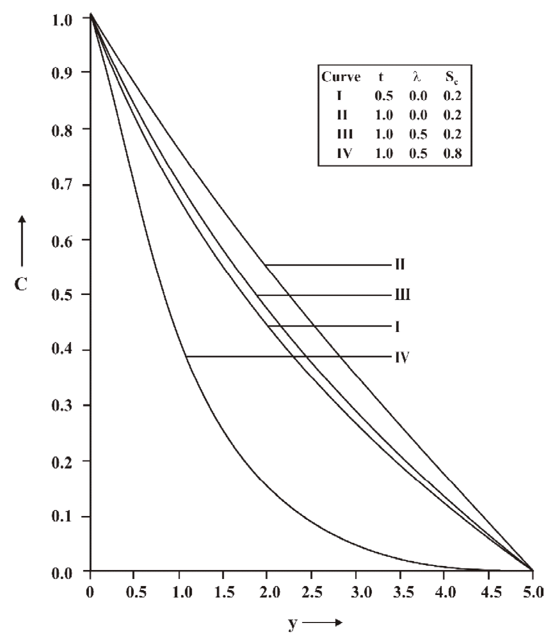

Fig. 4. Concentration profiles.

Results and Discussion

Graph of figure (1) depicts that the fluid velocity component (u) increases with an increasing value of time (t) and Hall parameter (m), but decreases owing with an increasing value of transpiration parameter (λ) and magnetic field parameter (M).

We see from Graph of figure (2) that the fluid velocity component (w) increases with an increasing value of time (t) and Hall parameter (m), but decreases owing with an increasing value of transpiration parameter (λ) and magnetic field parameter (M).

Graph of figure (3) represents that the temperature of the fluid field increases with an increasing value of time (t), but decreases with an increasing value of transpiration parameter (λ) and Prandtl number (P_r).

We see from Graph of figure (4) that the species concentration (C) increases with an increasing value of time (t), but decreases with an increasing value of transpiration parameter (λ) and Schmidt number (S_c).

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