

## MHD Free convective Flow Past an Inclined Plate with Chemical reaction in Porous Medium

JAI SINGH\*\*

Department of Mathematics, University of Allahabad, Allahabad-211002 (INDIA)  
jai\_au@rediffmail.com

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### Abstract

In this paper, the free convective flow, with chemical reaction and magnetic field, past an inclined plate has been studied numerically. The governing partial differential equations are reduced to a system of ordinary differential equations using similarity transformations. The non-linear similarity equations are solved numerically by applying the computer supported method. The effects of various parameters entering into the problem on the flow field have been discussed and presented graphically.

*Key words:* Chemical reaction, MHD, Free convection, heat and mass transfer, inclined plate.

### Nomenclature :

$u$ – Velocity in X-direction	$T_{\infty}$ - Temperature of the fluid away from the plate
$v$ - Velocity in Y-direction	$C_w$ - Species concentration near the plate
$\alpha$ - measure of angle	$C_{\infty}$ - Species concentration away from the plate
$\nu$ - Kinematic viscosity	$\beta$ – Coefficient of volume expansion
$g$ - Acceleration due to gravity	$\beta^*$ – Coefficient of volume expansion with concentration
$c_p$ - Specific heat at constant pressure	$M$ - Magnetic field parameter
$Sc$ - Schmidt number	$\gamma$ - Chemical reaction parameter
$Pr$ - Prandtl Number	$D$ - Chemical molecular diffusivity
$Gr$ - Thermal Grashof Number	$\delta$ - Dimensionless permeability parameter
$Gm$ - Mass Grashof number	$K$ - Permeability of porous medium
$T_w$ - Temperature of the fluid near the plate	

## Introduction

The problem of convective heat and mass transfer has received quite intensified interest of many researchers due to its occurrence in various physical phenomena and its wide applications in geophysics, geothermal reservoirs, exploration of petroleum products and astrophysical studies *etc.*

The steady state natural convection on a vertical plate using similarity variables was studied by B. Gebhart, L. Pera<sup>1</sup>. The problem of free convection effects on stokes problem for a vertical plate under the action of transversely applied magnetic field was studied by Soundalgekar *et al.*<sup>2</sup>. Sacheti *et al.*<sup>3</sup> obtained an exact solution for unsteady magneto hydrodynamics free convection flow on an impulsively started vertical plate with constant heat flux. Shanker and Kishan<sup>4</sup> have analysed the MHD flow past an impulsively started vertical plate. Helmy<sup>5</sup> presented an un steady two dimensional laminar free convection flow of an incompressible, viscous electrically conducting fluid through a porous medium bounded by an infinite vertical plane surface of constant temperature. Hossian *et. al.*<sup>6</sup> studied the free convection flow from an isothermal plate inclined at a small angle to the horizontal. The effect of mass transfer on the MHD flow past on an impulsively started isothermal inclined plate have been studied by Ganesan and Palani<sup>7</sup>. MHD free convection heat and mass transfer flow past an inclined surface with heat generation were studied by M.A. Sattar *et. al.*<sup>8</sup>. Analysis of natural convection heat and mass transfer past an inclined surface has

been done by S. Sivasankaran *et.al.*<sup>9</sup> applying Lie groups. Heat and mass transfer in MHD flow by natural convection from a permeable inclined surface with variable wall temperature and concentration were studied by C.H. Chen *et. al.*<sup>10</sup>. Effect of variable electric conductivity and non-uniform heat source (or sink) on convective micropolar fluid flow along an inclined flat plate with surface heat flux were studied by M.M.Rahman *et.al.*<sup>11</sup>. Effect of chemical reaction, Heat and mass transfer on non- linear MHD laminar boundary -layer flow over a wedge with suction or injection were studied by S.P.Anjali Devi and R. kandasamy<sup>12</sup>.

In this paper, we have investigated the steady MHD free convective flow past an inclined plate in porous medium. The buoyancy effect is due to the variation of temperature and concentration across the boundary layer. The first order chemical reactions are assumed to take place and heat generated in this process is neglected.

### *Mathematical formulation :*

Let us consider a steady two dimensional hydromagnetic flow of a viscous, incompressible fluid along an inclined plate with an acute angle  $\alpha$  to the vertical. The x-axis is measured along the plate and the y-axis is measured normal to the plate. A magnetic field of uniform strength  $B_0$  is introduced normal to the direction of the flow.. The chemical reactions of first order with rate constant  $k_1$  are taking place in the flow and physical properties are constant throughout the fluid. The plate temperature and concentration are assumed to be a power function of  $x$

i.e.  $T_w = T_\infty + Ax^n$  and  $C_w = C_\infty + Bx^m$ , where  $a, b, m$  and  $n$  are constants. The plate is maintained at a temperature  $T_w$ , which is higher than the temperature  $T_\infty$  of the surrounding fluid and concentration  $C_w$  is greater than the concentration  $C_\infty$ . Here the convective flow under consideration takes place due to buoyancy forces which arise from a combination of thermal and species concentration effects. Then under the usual Boussinesq's and boundary layer approximations, the governing equations are given by

$$\left. \begin{aligned} \eta &= y \left( \frac{U_\infty}{\nu x} \right)^{\frac{1}{2}} \\ \psi &= \sqrt{\nu U_\infty x} f(\eta) \text{ such that } u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \\ \text{Thus, } u &= U_\infty f'(\eta) \text{ and } v = \frac{1}{2} \sqrt{\frac{U_\infty \nu}{x}} (\eta f' - f) \\ \theta(\eta) &= \frac{T - T_\infty}{T_w - T_\infty} \\ \phi(\eta) &= \frac{C - C_\infty}{C_w - C_\infty} \end{aligned} \right\} \quad (6)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty) \cos \alpha + g\beta^*(C - C_\infty) \cos \alpha - \frac{\nu}{K} u - \frac{\sigma B_0^2}{\rho} u \quad (2)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} \quad (3)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - k_1 C \quad (4)$$

with boundary conditions

$$\left. \begin{aligned} u = 0, \quad v = 0, \quad T = T_w, \quad C = C_w, \quad y = 0 \\ u = 0, \quad T = T_\infty, \quad C = C_\infty, \quad y \rightarrow \infty \end{aligned} \right\} \quad (5)$$

In order to obtain a similarity solution of the problem, we introduce the following non-dimensional variables:

Using equation (6), equations (2) to (4) become

$$f''' + \frac{1}{2} f f'' + Gr \theta \cos \alpha + Gm \phi \cos \alpha - \delta f' - M f'^2 = 0 \quad (7)$$

$$\theta'' + \frac{1}{2} Pr f \theta' + Pr \theta f' = 0 \quad (8)$$

$$\phi'' + \frac{1}{2} Sc f \phi' - Sc \phi f' - \gamma Sc \phi = 0 \quad (9)$$

Where

$$Gr = \frac{g\beta(T_w - T_\infty)x}{U_\infty^2}$$

$$Gm = \frac{g\beta^*(C_w - C_\infty)x}{U_\infty^2}$$

$$Pr = \frac{\nu \rho c_p}{k}$$

$$Sc = \frac{\nu}{D}$$

$$M = \frac{\sigma B_0^2 x}{\rho U_\infty}$$

$$\delta = \frac{vx}{KU_{\infty}}$$

and  $U_{\infty}$  is some reference velocity.

The transformed boundary conditions are given by

$$\left. \begin{array}{llll} f=0 & f'=0 & \theta=1 & \varphi=1 & \text{at } \eta=0 \\ f'=0 & \theta=0 & \varphi=0 & & \text{at } \eta \rightarrow \infty \end{array} \right\} \quad (10)$$

## Result and Discussion

The equations (7)-(9) together with the boundary conditions (10) are solved numerically using BVP4C method. We assume  $n = 1, m = 1$  in these equations. The numerical results are displayed with the help of graphical illustrations.

The effect of Prandtl number on the velocity, temperature and concentration fields are shown in figs. 1-3 respectively. It is observed from figures 1 and 2 that the increase in the Prandtl number has an adverse effect on the velocity and temperature profiles of the fluid flow but it has opposite effect on the concentration profile of the fluid along the inclined plate as is clear by Fig. 3. In other words, as we increase the Prandtl no, concentration profile has increasing trend.

Figs. 4-6 show that the effect of Schmidt number  $Sc$  on velocity, temperature and concentration of the air ( $Pr = 0.71$ ) boundary layer. We observe that as we go on increasing the Schmidt number parameter, both the velocity and concentration layer thickness decrease while that of temperature increases.

From Fig. 7-9, we find the effect of inclination of the plate on velocity, temperature and concentration fields respectively. From Fig.

7 we observed that the fluid (air) velocity decreases for an increase in angle  $\alpha$ . The fluid has higher velocity when the surface is vertical ( $\alpha = 0$ ) than when inclined because the convective flow under consideration takes place due to the interaction of gravity and density differences and in the inclined position the effective gravity force is less than what it is when the plate vertical. On the other hand, both the temperature and concentration profiles increase with an increase of  $\alpha$ .

The effect of the chemical reaction parameter  $\gamma$  on the velocity, temperature and concentration profiles is depicted in figs.10-12. We observe that the chemical reaction parameter has increasing effect on the velocity as shown in fig.10. But the temperature decreases if the chemical reaction parameter is increased as clear from fig. 11. It is observed from fig. 12 that concentration profile decreases with increasing chemical reaction parameter.

The effects of permeability parameter  $\delta$  on the velocity, temperature and concentration profiles are depicted in figs. 13-15. From Fig.13, it is observed that velocity of fluid decreases with an increase in the permeability parameter while, from Fig. 14 and Fig. 15, we find that temperature and concentration profiles increase with an increase in the permeability parameter.

In Fig. 16, the effect of different values of magnetic field parameter  $M$  on the velocity profile is shown. From this figure, we see that an increase in magnetic parameter leads to a corresponding decrease in the velocity profiles. On the other hand, from Fig.17-18, we see that an increase in the magnetic parameter leads to a corresponding rise in both the temperature and concentration profiles.

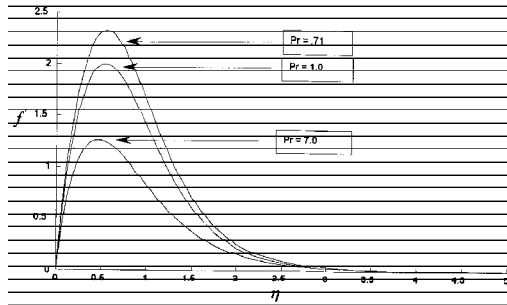


Fig. 1. Velocity profiles for different values of Prandtl number with  $Gr=10$ ,  $Gm=5$ ,  $Sc=0.22$ ,  $\delta=1.0$ ,  $\gamma=1.0$  and  $\alpha=20^0$

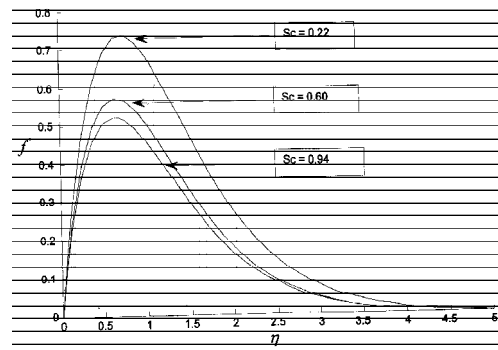


Fig. 4. Velocity profiles for different values of Shmidt number with  $Gr=10$ ,  $Gm=5$ ,  $Pr=0.71$ ,  $\delta=1.0$ ,  $\gamma=1.0$  and  $\alpha=20^0$

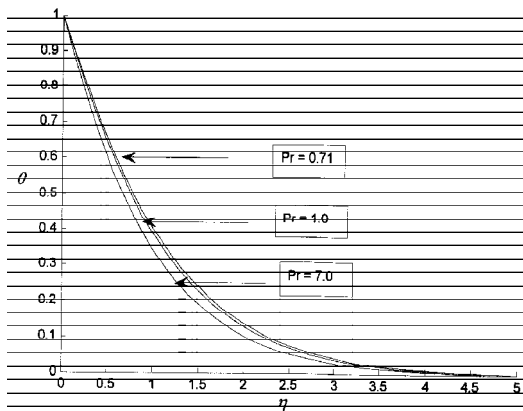


Fig. 2. Temperature profiles for different values of Prandtl number with  $Gr=10$ ,  $Gm=5$ ,  $Sc=0.22$ ,  $\delta=1.0$ ,  $\gamma=1.0$  and  $\alpha=20^0$

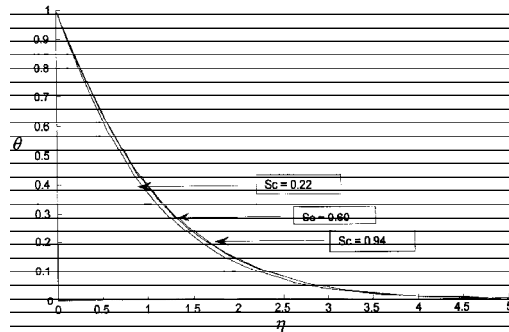


Fig. 5. Temperature profiles for different values of Schmidt number with  $Gr=10$ ,  $Gm=5$ ,  $Pr=0.71$ ,  $\delta=1.0$ ,  $\gamma=1.0$  and  $\alpha=20^0$

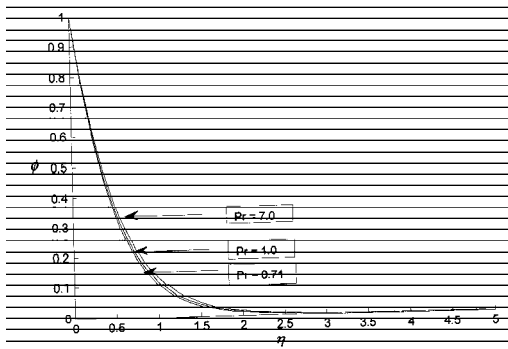


Fig. 3. Concentration profiles for different values of Prandtl number with  $Gr=10$ ,  $Gm=5$ ,  $Sc=0.22$ ,  $\delta=1.0$ ,  $\gamma=1.0$  and  $\alpha=20^0$

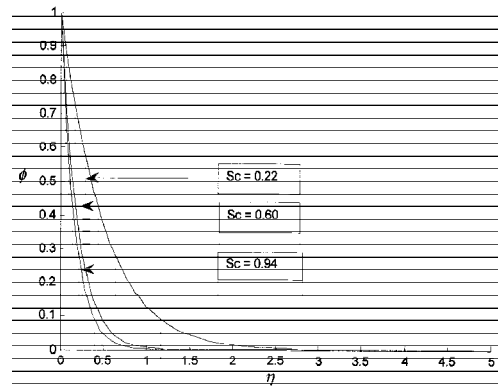


Fig. 6. Concentration profiles for different values of Schmidt number with  $Gr=10$ ,  $Gm=5$ ,  $Pr=0.71$ ,  $\delta=1.0$ ,  $\gamma=1.0$  and  $\alpha=20^0$

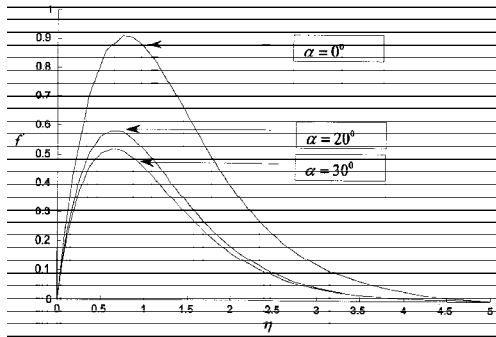


Fig. 7. Velocity profiles for different values of  $\alpha$  with  $Gr=10$ ,  $Gm=5$ ,  $Pr=0.71$ ,  $Sc=0.22$   $\delta=1.0$ ,  $\gamma=1.0$

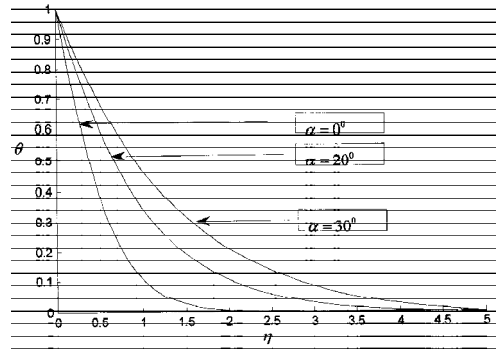


Fig. 8. Temperature profiles for different values of  $\alpha$  with  $Gr=10$ ,  $Gm=5$ ,  $Pr=0.71$ ,  $Sc=0.22$ ,  $\delta=1.0$ ,  $\gamma=1.0$

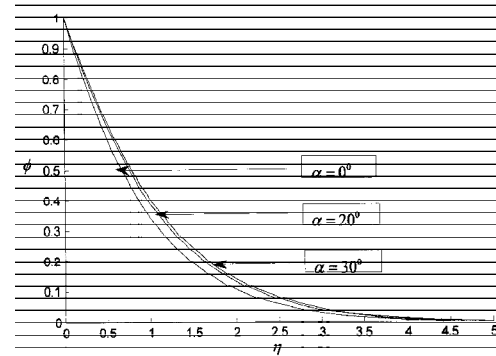


Fig. 9. Concentration profiles for different values of  $\alpha$  with  $Gr=10$ ,  $Gm=5$ ,  $Pr=0.71$ ,  $Sc=0.22$ ,  $\delta=1.0$ ,  $\gamma=1.0$

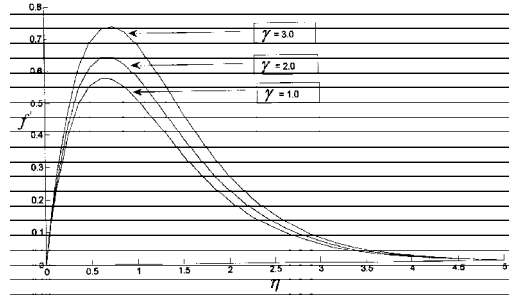


Fig. 10. Velocity profiles for different values of Chemical reaction parameter with  $Gr=10$ ,  $Gm=5$ ,  $Pr=0.71$ ,  $Sc=0.22$   $\delta=1.0$  and  $\alpha=20$

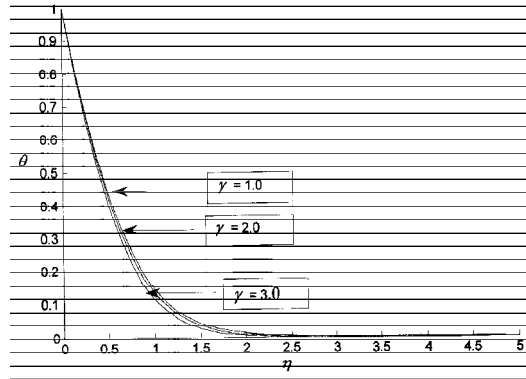


Fig. 11. Temperature profiles for different values of Chemical reaction parameter with  $Gr=10$ ,  $Gm=5$ ,  $Pr=0.71$ ,  $Sc=0.22$   $\delta=1.0$  and  $\alpha=20$

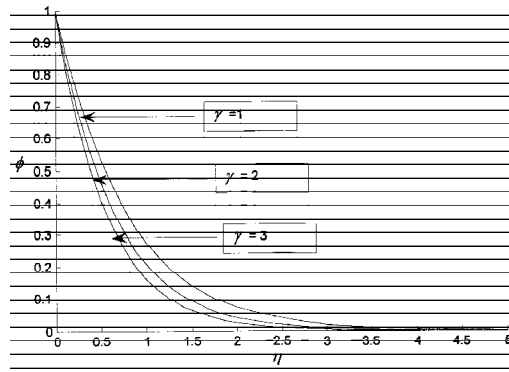


Fig. 12. Concentration profiles for different values of Chemical reaction parameter with  $Gr=10$ ,  $Gm=5$ ,  $Pr=0.71$ ,  $Sc=0.22$   $\delta=1.0$  and  $\alpha=20$

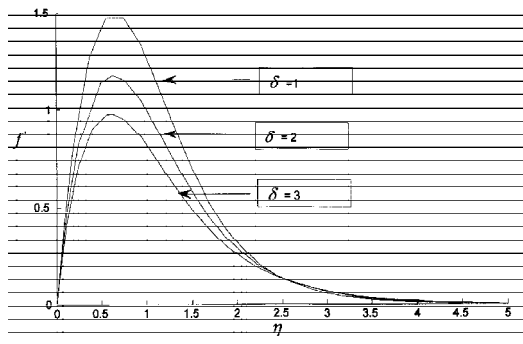


Fig. 13. Velocity profiles for different values of permeability parameter with  $Gr=10$ ,  $Gm=5$ ,  $Pr=0.71$ ,  $Sc=0.22$   $\gamma=1.0$   $\delta=1.0$  and  $\alpha=20$

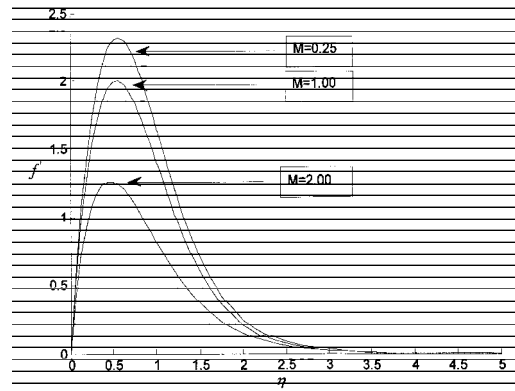


Fig. 16. Velocity profiles for different values of  $M$  with  $Gr=12$ ,  $Gm=6$ ,  $Sc=0.22$  and  $\alpha=30^0$

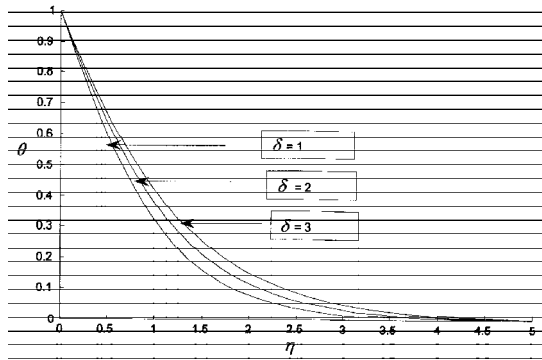


Fig. 14. Temperature profiles for different values of permeability parameter with  $Gr=10$ ,  $Gm=5$ ,  $Pr=0.71$ ,  $Sc=0.22$   $\gamma=1.0$   $\delta=1.0$  and  $\alpha=20$

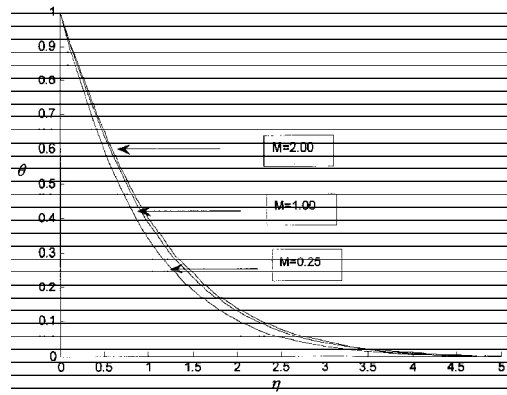


Fig. 17. Temperature profiles for different values of  $M$  with  $Gr=12$ ,  $Gm=6$ ,  $Sc=0.22$  and  $\alpha=30^0$

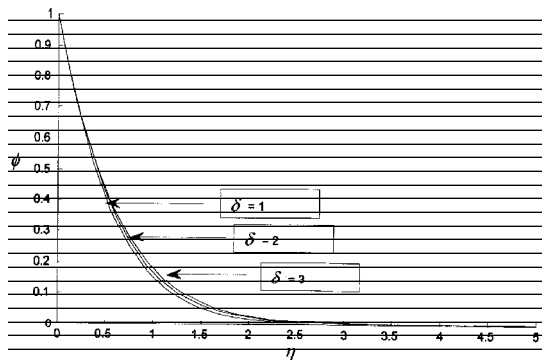


Fig. 15. Concentration profiles for different values of permeability parameter with  $Gr=10$ ,  $Gm=5$ ,  $Pr=0.71$ ,  $Sc=0.22$   $\gamma=1.0$   $\delta=1.0$  and  $\alpha=20$

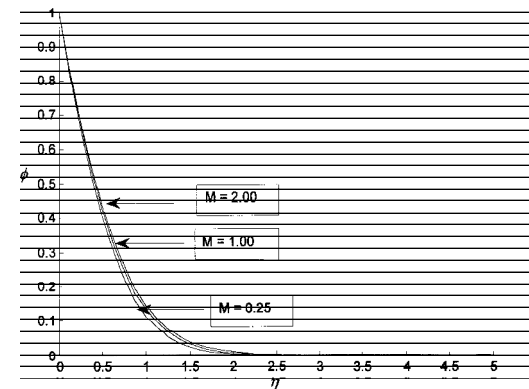


Fig. 18. Concentration profiles for different values of  $M$  with  $Gr=12$ ,  $Gm=6$ ,  $Sc=0.22$  and  $\alpha=30^0$

## Conclusions

- It is observed that an increase in Prandtl number has the effect of decreasing both velocity and temperature while, at the same time, concentration of the fluid increases.
- An increase in Schmidt number results in lowering the concentration and velocity while temperature of the fluid increases.
- By an increase in the angle of inclination, velocity decreases while temperature and concentration increases.
- Concentration and temperature decreases while velocity increases with increase in chemical reaction parameter.
- Velocity decreases while temperature and concentration increases with increase in the permeability parameter.
- It is observed that the velocity decreases with an increase in the magnetic parameter where as temperature and concentration increases with an increase in the magnetic parameter.

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