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On the determination of optimal manpower reserve at three nodes in series

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Abstract

The welfare and prosperity of any county depends upon the natural resources available in that country. But the proper utilization of the natural resources depends upon the availability of manpower of the appropriate type. There are many industries and organizations where the skilled personnel are to be recruited and they must be given prior training before employment. In the area of Software and Information Technology the training and induction of the right type of personnel is a pressing problem. In this paper, we consider the optimal solution for the manpower to be kept as reserve inventory at two different nodes in series. This is based on the inventory model for finding the optimal size of reserve inventory at two successive stations in series.

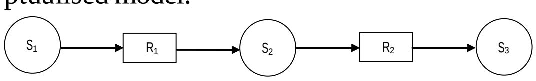
Key words : Manpower, Inventory Model, Optimal Solution.

Introduction

The welfare and prosperity of any county depends up on the natural resources available in that country. But the proper utilization of the natural resources depends up on the availability of manpower of the appropriate type. In modern days due to the advancement of civilization, progress of science and technology the need for the manpower of skilled personnel, has become imperative. But the training of the individuals according to the needs of skilled manpower of various nature has ended in the complex policies of manpower management. For a detailed study refer to Barthalomew[1], Barthalomew[2] and Forbes, McClean *et. al.*[3], Elangovan *et. al.*[4,5,6,7].

There are many industries and organizations where the skilled personnel are to be recruited and they must be given prior training before employment. In the area of Software and Information Technology the training and induction of the right type of personnel is a pressing problem. In this paper, we consider the optimal solution for the manpower to be kept as reserve inventory at two different nodes in series. This is based on the inventory model for finding the optimal size of reserve inventory at two successive stations in series. Some interesting results can also be seen in Susiganeshkumar and Elangovan[9-11].

We consider a manpower system in which the first node is the point of recruitment. This second node is the training of the selected before induction or employment. It may be noted that if there is any delay or breakdown at the first node namely the recruitment point then the training will be delayed and ultimately the shortage of trained personnel occurs. This in turn affects the running of the industry or organization. Similarly if there is any breakdown at the training node there will be again shortage of trained persons for employment. Therefore a reserve inventory of persons is to be maintained at two places namely in between first and second nodes and second and third nodes. The following configuration explains the conceptualised model.[8]



S_1, S_2 and S_3 denote the three sections or systems devoted to different activities

S_1 = Recruitment division

S_2 = Training division

S_3 = Employment induction section

R_1 = Reserve of persons at the first stage

R_2 = Reserve of persons at the second stage The reserves are interms of man hours.

In this model it is assumed that if the reserve of manpower is in excess then there is a cost of excess and similarly if the reserve of manpower is in shortage it involves the so called shortage cost. So the problem is to determine the optimal size of reserve at the two different points namely between recruitment and training and again in between training and employment. For a detailed study refer to Susiganeshkumar and Elangovan[12,13].

Assumptions :

(i) The manpower system comprises of three nodes namely the recruitment node, the training node and the employment node.

(ii) The reserve of manpower inventory is at two points namely in between recruitment and training and similarly between training and employment.

(iii) The delay in finding suitable candidates after training for employment is very costly.

Notations :

(i) The cost of excess candidates at the reserve points 1 and 2 are denoted as h_1, h_2 called holding costs.

(ii) d_1, d_2 denote the cost arising due to the shortage of candidates or manpower at the two intermediate reserve points R_1 and R_2.

(iii) The breakdown occurs at the recruitment or entry point for a random duration τ, and has the p.d.f is $g(\tau)$ and c.d.f. is $G(\tau)$.

(iv) r_1 denotes the constant rate of training of personnel at the node R_2.

(v) r_2 denotes the constant rate of employment of personnel at the node R_3.

(vi) μ_1, μ_2 the mean interarrival time of breakdowns of system R_1 and R_2 respectively.

Results

In this model it can be seen that the total expected cost due to excess of manpower inventory and also shortage is

$$E(c) = h_1R_1 + h_2R_2 + \frac{d_1}{\mu_1}\int_{R_1/r_1}^{\infty}\left(\tau - \frac{R_1}{r_1}\right)g(\tau)\,d\tau$$

$$+\frac{d_2}{\mu_1}\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}}^{\infty}\left(\tau - \frac{R_1}{r_1} + \frac{R_2}{r_2}\right)g(\tau)\,d\tau \qquad (1)$$

It may be observed that the first two terms of eqn. (1) represent the cost of excess of reserve of manpower, the third term is the cost of training section due to shortage of candidates and the fourth term is the cost due to the failure production due to non availability of manpower.

To optimal values of R_1 and R_2 can be obtained by solving the equations.

$$\frac{\partial E(c)}{\partial R_1} = 0 \qquad (2)$$

$$\frac{\partial E(c)}{\partial R_2} = 0 \qquad (3)$$

On differentiating partially the above equations with respect to R_1 and R_2 and equating to zero we obtain,

$$h_1 - \frac{d_1}{\mu_1 r_1}\left[1-G\left(\frac{R_1}{r_1}\right)\right] - \frac{d_2}{\mu_1 r_1}\left[1-G\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)\right] = 0 \qquad (4)$$

and

$$h_2 - \frac{d_2}{\mu_1 r_2}\left[1-G\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)\right] = 0 \qquad (5)$$

It may be observed that Leibnitz rule for the differentiation of an integral namely

$$\int_{a(x)}^{b(x)} f(x,t)\,dt = b'(x)\,f[x, b(x)] - a'(x)f[x, a(x)]$$

$$+\int_{a(x)}^{b(x)} \frac{d}{dx} f(x,t)\,dx \qquad (6)$$

has been used, in differentiating eqn. (1) with respect to R_1 and R_2 respectively. Solving equations (4) and (5), the optimal values of R_1 and R_2 are obtained.

Model I :

Consider the case when G(.) has uniform distribution over [0, a]. The down time density of the recruitment is a constant and is independent of time, that is, the down time distribution is proportional to time.

The holding cost will arise when $a < (R_1/r_1 + R_2/r_2)$.

When $a > (R_1/r_1 + R_2/r_2)$, the equations (4) and (5) present the following,

$$h_1 - \frac{d_1}{\mu_1 r_1}\left[\frac{a-(R_1/r_1)}{a}\right]$$

$$-\frac{d_2}{\mu_1 r_1}\left[\frac{a-(R_1/r_1 + R_2/r_2)}{a}\right] = 0 \qquad (7)$$

and

$$h_2 - \frac{d_2}{\mu_1 r_2}\left[\frac{a-(R_1/r_1 + R_2/r_2)}{a}\right] = 0 \qquad (8)$$

On collecting the coefficients of R_1 and R_2, these two equations will be reduced to the two simultaneous, linear equations in R_1 and R_2 as follows.

Consider the eqn. (7) and it can be written as

$$h_1 - \frac{d_1}{\mu_1 r_1} + \frac{d_1}{\mu_1 r_1^2 a} R_1 - \frac{d_2}{\mu_1 r_1} + \frac{d_2}{\mu_1 r_1^2 a} R_1 + \frac{d_2}{\mu_1 r_1 r_2 a} R_2 = 0$$

$$\left[\frac{d_1}{\mu_1 r_1^2 a} + \frac{d_2}{\mu_1 r_1^2 a}\right] R_1 + \frac{d_2}{\mu_1 r_1 r_2 a} R_2 = \frac{d_1}{\mu_1 r_1} + \frac{d_2}{\mu_1 r_1} - h_1$$

$$\frac{d_1 + d_2}{\mu_1 r_1^2 a} R_1 + \frac{d_2}{\mu_1 r_1 r_2 a} R_2 = \frac{d_1 + d_2 - h_1 \mu_1 r_1}{\mu_1 r_1} \quad (9)$$

Consider the eqn. (8) and it can be written as,

$$h_2 - \frac{d_2}{\mu_1 r_2} + \frac{d_2}{\mu_1 r_1 r_2 a} R_1 + \frac{d_2}{\mu_1 r_2^2 a} R_2 = 0$$

$$\frac{d_2}{\mu_1 r_1 r_2 a} R_1 + \frac{d_2}{\mu_1 r_2^2 a} R_2 = \frac{d_2 - h_2 \mu_1 r_2}{\mu_1 r_2} \quad (10)$$

To solve (9) and (10), multiply eqn. (9) by $1/r_2$ and eqn. (10) by $1/r_1$. The obtained solutions are

$$\hat{R}_1 = a\, r_1 \left[1 - \frac{\mu_1}{d_1}\left(h_1 r_1 - h_2 r_2\right)\right]$$

and

$$\hat{R}_2 = \frac{a\, r_2 \mu_1}{d_1 d_2}\left[h_1 r_1 d_2 - h_2 r_2 \left(d_1 + d_2\right)\right]$$

Numerical Example :

r_1=15, r_2=20, μ = 1.2, d_1 =50, h_2 = 50 a = 1.5 are all fixed.

Table 1. The Variation in $\hat{R}_1$ consequent to the changes in h_1

h_1	40	50	60	70	80	90	100
$\hat{R}_1$	7320	9150	10980	12810	14640	16470	18300

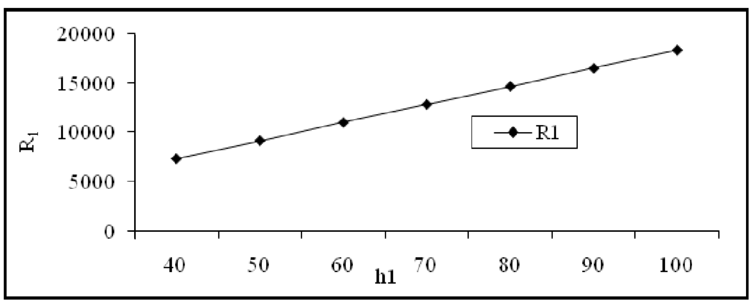


Fig.1. The Variation in $\hat{R}_1$ consequent to the changes in h_1

Table 2. The Variation in $\hat{R}_1$ consequent to the changes in d_1

d_1	50	55	60	65	70	75	80
$\hat{R}_1$	7320	7336	7350	7362	7371	7380	7388

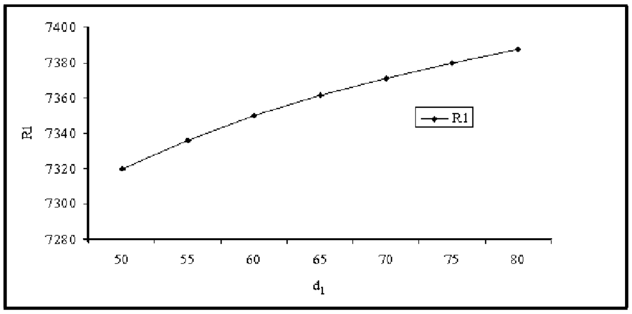


Fig. 2. The Variation in $\widehat{\boldsymbol{R}}_1$ consequent to the changes in d_1

Table 3. The Variation in $\widehat{\boldsymbol{R}}_2$ consequent to the changes in μ_1

μ_1	1.2	1.6	2.0	2.4	2.8	3.2	3.6
$\widehat{\boldsymbol{R}}_2$	144	192	240	288	336	384	432

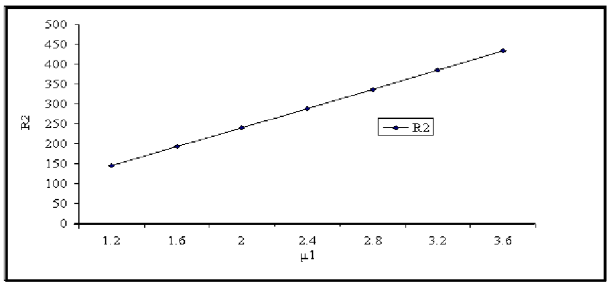


Fig. 3. The Variation in $\widehat{\boldsymbol{R}}_2$ consequent to the changes in μ_1

Table 4. The Variation in $\widehat{\boldsymbol{R}}_2$ consequent to the changes in h_2

h_2	30	31	32	33	34	35	36
$\widehat{\boldsymbol{R}}_2$	144	125	106	86	67	48	29

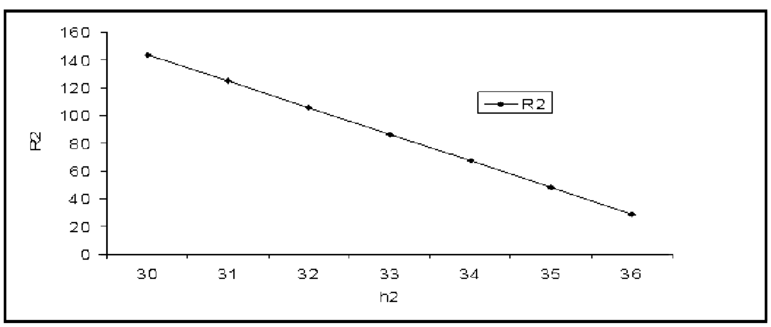


Fig. 4. The Variation in $\widehat{\boldsymbol{R}}_2$ consequent to the changes in h_2

Table 5. The Variation in $\hat{\boldsymbol{R}}_2$ consequent to the changes in d_2

d_2	150	160	170	180	190	200	210
$\hat{\boldsymbol{R}}_2$	144	153	161	168	174	180	185

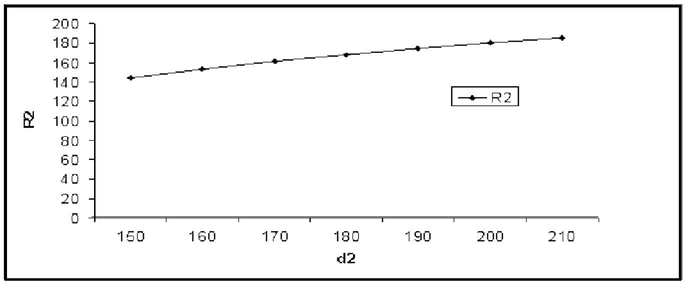


Fig. 5. The Variation in $\hat{\boldsymbol{R}}_2$ consequent to the changes in d_2

On the basis of the model I discussed the following conclusions are drawn using the numerical illustration.

(i) As the cost of manpower reserve between stages S_1 and S_2 namely h_1 increases a small reserve R_1 is suggested as an optimal one. This is indicated in Table 1 and Fig. 1.

(ii) If the cost of shortages at the first node namely d_1 increases, it is suggested that a larger manpower at R_1 is desirable as indicated in Table 2 and Fig. 2.

(iii) If the cost of manpower reserve at node 2 is higher namely as h_2 increases then a decrease in the value of R_2 is suggested as indicated in Table 4 and Fig. 4.

(iv) If d_2, the cost of shortages at the second node increases a larger reserve namely R_2 is desirable as indicated in Table 5 and Fig. 5.

(v) If μ_1 the parameter of the interarrival times between the breakdowns of S_1 increases then the interarrival times will be shorter since $E(\tau)=1/\mu_1$, decreases. Hence a larger reserve at R_2 is suggested as indicated in Table 3 and Fig. 3.

Model II :

In the above model an additional assumption that the down time of R_1 is a random variable which satisfies the so called Setting the Clock Back to Zero (SCBZ) property is introduced. This property has been due to Raja Rao and Talwalker[8]. Under this property the p.d.f. of the down time which is namely $g(\tau)$ is such that

$$g(\tau) = \theta_1 e^{-\theta_1 \tau} \text{ if } \tau \le \tau_0$$
$$= \theta_2 e^{-\theta_2 \tau} e^{\tau_0} (\theta_2 - \theta_1) \text{ if } \tau > \tau_0$$

when τ_0 is called the truncation point.

It can be shown that $\int_0^\infty g(\tau) d\tau = 1$,

and τ_0 is called the truncation point which itself is a random variable that follows exponential distribution with parameter λ.

Now the cost function under this assumption is given as,

$$E(c) = h_1R_1 + h_2R_2 + \frac{d_1}{\mu_1}\left[\int_{R_1/r_1}^{\tau_0}\left(\tau - \frac{R_1}{r_1}\right)\theta_1 e^{-\theta_1\tau}\, d\tau\ p[\tau \le \tau_0] + e^{\tau_0(\theta_2-\theta_1)}\int_{\tau_0}^{\infty}\left(\tau - \frac{R_1}{r_1}\right)\theta_2 e^{-\theta_2\tau}\, d\tau\ p[\tau > \tau_0]\right]$$
$$+ \frac{d_2}{\mu_2}\left[\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}}^{\tau_0}\left(\tau - \frac{R_1}{r_1} - \frac{R_2}{r_2}\right)\theta_1 e^{-\theta_1\tau}\, d\tau\ p[\tau \le \tau_0] + e^{\tau_0(\theta_2-\theta_1)}\int_{\tau_0}^{\infty}\left(\tau_1 - \frac{R_1}{r_1} - \frac{R_2}{r_2}\right)\theta_2 e^{-\theta_2\tau}\, d\tau\ p[\tau > \tau_0]\right]$$

$$E(c) = h_1R_1 + h_2R_2 + \frac{d_1}{\mu_1}\left[\theta_1\int_{R_1/r_1}^{\tau_0}\left(\tau - \frac{R_1}{r_1}\right)e^{-\theta_1\tau}e^{-\lambda\tau_0}\, d\tau + e^{\tau_0(\theta_2-\theta_1)}\theta_2\int_{\tau_0}^{\infty}\left(\tau - \frac{R_1}{r_1}\right)e^{-\theta_2\tau}\left[1 - e^{-\lambda\tau_0}\right]d\tau\right]$$
$$+ \frac{d_2}{\mu_2}\left[\theta_1\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}}^{\tau_0}\left(\tau - \frac{R_1}{r_1} - \frac{R_2}{r_2}\right)e^{-\theta_1\tau}\, e^{-\lambda\tau_0}\, d\tau + e^{\tau_0(\theta_2-\theta_1)}\theta_2\int_{\tau_0}^{\infty}\left(\tau_1 - \frac{R_1}{r_1} - \frac{R_2}{r_2}\right)e^{-\theta_2\tau}\left[1 - e^{-\lambda\tau_0}\right]d\tau\right]$$

$$\frac{\partial E(c)}{\partial R_1} = h_1 + \frac{d_1}{\mu_1}\left\{0 - \frac{1}{r_1}(0) + \theta_1\int_{\frac{R_1}{r_1}}^{\tau_0}\left(-\frac{1}{r_1}\right)e^{-\theta_1\tau}e^{-\lambda\tau_0}d\tau\right\} +$$
$$\frac{d_1}{\mu_1}e^{\tau_0(\theta_2-\theta_1)}\theta_2\int_{\tau_0}^{\infty}\left(-\frac{1}{r_1}\right)e^{-\theta_2\tau}\left[1 - e^{-\lambda\tau_0}\right]d\tau +$$
$$\frac{d_2}{\mu_2}\left\{0 - \frac{1}{r_1}(0) + \theta_1\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}}^{\tau_0}\left(-\frac{1}{r_1}\right)e^{-\theta_1\tau}e^{-\lambda\tau_0}d\tau\right\} +$$
$$e^{\tau_0(\theta_2-\theta_1)}\frac{d_2}{\mu_2}\left[\theta_2\int_{\tau_0}^{\infty}\left(-\frac{1}{r_1}\right)e^{-\theta_2\tau}\left[1 - e^{-\lambda\tau_0}\right]d\tau\right]$$
$$= I_1 + I_2 + I_3 + I_4$$

$$I_1 = h_1 + \frac{d_1}{\mu_1}\left[\theta_1\int_{\frac{R_1}{r_1}}^{\tau_0}\left(-\frac{1}{r_1}\right)\right]e^{-\theta_1\tau}e^{-\lambda\tau_0}d\tau$$

$$= h_1 - \frac{d_1\theta_1}{\mu_1 r_1}\left[e^{-\lambda\tau_0}\int_{\frac{R_1}{r_1}}^{\tau_0} e^{-\theta_1\tau}d\tau\right]$$

$$= h_1 - \frac{d_1\theta_1}{\mu_1 r_1}e^{-\lambda\tau_0}\left[\frac{e^{-\theta_1\tau}}{-\theta_1}\right]_{\frac{R_1}{r_1}}^{\tau_0}$$

$$= h_1 + \frac{d_1}{\mu_1 r_1}e^{-\lambda\tau_0}\left[e^{-\theta_1\tau_0} - e^{-\theta_1\frac{R_1}{r_1}}\right]$$

$$= h_1 + \frac{d_1}{\mu_1 r_1}\left[e^{-(\lambda+\theta_1)\tau_0} - e^{-\theta_1\frac{R_1}{r_1}-\lambda\tau_0}\right]$$

$$I_2 = \frac{d_1}{\mu_1}e^{(\theta_2-\theta_1)\tau_0}\,\theta_2\int_{\tau_0}^{\infty}\left(-\frac{1}{r_1}\right)e^{-\theta_2\tau}\left[1-e^{-\lambda\tau_0}\right]d\tau$$

$$= -\frac{d_1\,\theta_2}{\mu_1 r_1}e^{(\theta_2-\theta_1)\tau_0}\left[1-e^{-\lambda\tau_0}\right]\int_{\tau_0}^{\infty}e^{-\theta_2\tau}\,d\tau$$

$$= -\frac{d_1\,\theta_2}{\mu_1 r_1}e^{(\theta_2-\theta_1)\tau_0}\left[1-e^{-\lambda\tau_0}\right]\left[\frac{e^{-\theta_2\tau}}{-\theta_2}\right]_{\tau_0}^{\infty}$$

$$= \frac{d_1}{\mu_1 r_1}e^{(\theta_2-\theta_1)\tau_0}\left[1-e^{-\lambda\tau_0}\right]\left(-e^{-\theta_2\tau_0}\right)$$

$$= -\frac{d_1}{\mu_1 r_1}\left[1-e^{-\lambda\tau_0}\right]\left(e^{-\theta_1\tau_0}\right)$$

$$I_1 + I_2 = h_1 + \frac{d_1}{\mu_1 r_1}\left[-e^{-\theta_1\frac{R_1}{r_1}-\lambda\tau_0} - e^{-\theta_1\tau_0}\right]$$

$$= h_1 - \frac{d_1}{\mu_1 r_1}\left[e^{-\theta_1\frac{R_1}{r_1}-\lambda\tau_0} + e^{-\theta_1\tau_0}\right] \quad (11)$$

$$I_3 = \frac{d_2}{\mu_2}\left[\theta_1\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}}^{\tau_0}\left(-\frac{1}{r_1}\right)e^{-\theta_1\tau}e^{-\lambda\tau_0}d\tau\right]$$

$$= -\frac{d_2\theta_1}{\mu_2 r_1}e^{-\lambda\tau_0}\left[\int_{\frac{R_1}{r_1}+\frac{R_2}{r_2}}^{\tau_0}e^{-\theta_1\tau}\,d\tau\right]$$

$$= -\frac{d_2\theta_1}{\mu_2 r_1}e^{-\lambda\tau_0}\left[\frac{e^{-\theta_1\tau}}{-\theta_1}\right]_{\frac{R_1}{r_1}+\frac{R_2}{r_2}}^{\tau_0}$$

$$= \frac{d_2}{\mu_2 r_1}e^{-\lambda\tau_0}\left[e^{-\theta_1\tau_0} - e^{-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)}\right]$$

$$I_4 = \frac{d_2}{\mu_2}e^{(\theta_2-\theta_1)\tau_0}\,\theta_2\int_{\tau_0}^{\infty}\left(-\frac{1}{r_1}\right)e^{-\theta_2\tau}\left[1-e^{-\lambda\tau_0}\right]d\tau$$

$$= -\frac{d_2\,\theta_2}{\mu_2 r_1}e^{(\theta_2-\theta_1)\tau_0}\left[1-e^{-\lambda\tau_0}\right]\int_{\tau_0}^{\infty}e^{-\theta_2\tau}\,d\tau$$

$$= -\frac{d_2\,\theta_2}{\mu_2 r_1}e^{(\theta_2-\theta_1)\tau_0}\left[1-e^{-\lambda\tau_0}\right]\left[\frac{e^{-\theta_2\tau}}{-\theta_2}\right]_{\tau_0}^{\infty}$$

$$= \frac{d_2}{\mu_2 r_1}e^{(\theta_2-\theta_1)\tau_0}\left[1-e^{-\lambda\tau_0}\right]\left(-e^{-\theta_2\tau_0}\right)$$

$$= -\frac{d_2}{\mu_2 r_1}\left[1-e^{-\lambda\tau_0}\right]\left(e^{-\theta_1\tau_0}\right)$$

$$I_3 + I_4 = \frac{d_2}{\mu_2 r_1}e^{-\lambda\tau_0}\left[e^{-\theta_1\tau_0} - e^{-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)}\right] - \frac{d_2}{\mu_2 r_1}\left[1-e^{-\lambda\tau_0}\right]\left(e^{-\theta_1\tau_0}\right)$$

$$= \frac{d_2}{\mu_2 r_1}\left[e^{-(\lambda+\theta_1)\tau_0} - e^{-\lambda\tau_0-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)} - e^{-\theta_1\tau_0} + e^{-(\lambda+\theta_1)\tau_0}\right] \quad (12)$$

$$\frac{\partial E(c)}{\partial R_1} = h_1 - \frac{d_1}{\mu_1 r_1}\left[e^{-\theta_1\frac{R_1}{r_1}-\lambda\tau_0} + e^{-\theta_1\tau_0}\right] + \frac{d_2}{\mu_2 r_1}\left[e^{-(\lambda+\theta_1)\tau_0} - e^{-\lambda\tau_0-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)} - e^{-\theta_1\tau_0} + e^{-(\lambda+\theta_1)\tau_0}\right] \quad (13)$$

$$\frac{\partial E(c)}{\partial R_2} = h_2 + \frac{d_2}{\mu_2}\left[\frac{-\theta_1}{r_2}\, e^{-\lambda\tau_0}\right]\left[\frac{e^{-\theta_1\tau}}{-\theta_1}\right]_{\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)}^{\tau_0} + \frac{d_2}{\mu_2} e^{\tau_0(\theta_2-\theta_1)}\left[\frac{-\theta_2}{r_2}\right]\left[1 - e^{-\lambda\tau_0}\right]\left[\frac{e^{-\theta_2\tau}}{-\theta_2}\right]_{\tau_0}^{\infty}$$

$$= h_2 + \frac{d_2}{\mu_2 r_2}\, e^{-\lambda\tau_0}\left[e^{-\theta_1\tau_0} e^{-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)}\right] + \frac{d_2}{\mu_2 r_2} e^{\tau_0(\theta_2-\theta_1)}\left[1 - e^{-\lambda\tau_0}\right]\left[-e^{-\theta_2\tau_0}\right]$$

$$= h_2 + \frac{d_2}{\mu_2 r_2}\left[e^{-(\lambda+\theta_1)\tau_0} - e^{-\lambda\tau_0-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)} - e^{-\theta_1\tau_0} + e^{-(\lambda+\theta_1)\tau_0}\right] \quad (14)$$

From eqn. (13)

$$h_1 = \frac{d_1}{\mu_1 r_1}\left[e^{-\lambda\tau_0} + e^{-\theta_1\tau_0}\right] + \frac{d_1}{\mu_1 r_1}\left[e^{-\theta_1\frac{R_1}{r_1}}\right] + \frac{d_2}{\mu_2 r_1}\left[e^{-(\lambda+\theta_1)\tau_0} - e^{-\lambda\tau_0-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)} - e^{-\theta_1\tau_0} + e^{-(\lambda+\theta_1)\tau_0}\right] + \frac{d_2}{\mu_2 r_1}\left[-e^{-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)}\right]$$

$$h_1 = A + \frac{d_1}{\mu_1 r_1}\left[e^{-\theta_1\frac{R_1}{r_1}}\right] + B + \frac{d_2}{\mu_2 r_1}\left[-e^{-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)}\right]$$

$$\frac{d_1}{\mu_1 r_1}\left[e^{-\theta_1\frac{R_1}{r_1}}\right] - \frac{d_2}{\mu_2 r_1}\left[e^{-\theta_1\frac{R_1}{r_1}}\right] - \frac{d_2}{\mu_2 r_1}\left[e^{-\theta_1\frac{R_2}{r_2}}\right] = h_1 - A - B$$

Where,

$$A = \frac{d_1}{\mu_1 r_1}\left[e^{-\lambda\tau_0} + e^{-\theta_1\tau_0}\right], \quad B = \frac{d_2}{\mu_2 r_1}\left[e^{-(\lambda+\theta_1)\tau_0} - e^{-\lambda\tau_0} - e^{-\theta_1\tau_0} + e^{-(\lambda+\theta_1)\tau_0}\right]$$

Therefore

$$\left[\frac{d_1}{\mu_1 r_1} - \frac{d_2}{\mu_2 r_1}\right]\left[e^{-\theta_1\frac{R_1}{r_1}}\right] - \frac{d_2}{\mu_2 r_1}\left[e^{-\theta_1\frac{R_2}{r_2}}\right] = h_1 - A - B$$

Where,

$$C = \left[\frac{d_1}{\mu_1 r_1} - \frac{d_2}{\mu_2 r_1}\right], \quad D = \frac{d_2}{\mu_2 r_1}, \quad E = h_1 - A - B$$

$$C\left[e^{-\theta_1\frac{R_1}{r_1}}\right] - D\left[e^{-\theta_1\frac{R_2}{r_2}}\right] = E \quad (15)$$

From eqn. (14)

$$-h_2 = \frac{d_2}{\mu_2 r_2}\left[e^{-(\lambda+\theta_1)\tau_0} - e^{-\lambda\tau_0} - e^{-\theta_1\tau_0} + e^{-(\lambda+\theta_1)\tau_0}\right] + \frac{d_2}{\mu_2 r_2}\left[e^{-\theta_1\left(\frac{R_1}{r_1}+\frac{R_2}{r_2}\right)}\right]$$

$$-h_2 = \frac{r_1}{r_2}\,B - \frac{d_2}{\mu_2 r_2}\left[e^{-\theta_1\frac{R_1}{r_1}}\right] - \frac{d_2}{\mu_2 r_2}\left[e^{-\theta_1\frac{R_2}{r_2}}\right]$$

$$-h_2 = \frac{r_1}{r_2}B - \frac{r_1}{r_2}D\left[e^{-\theta_1\frac{R_1}{r_1}}\right] - \frac{r_1}{r_2}D\left[e^{-\theta_1\frac{R_2}{r_2}}\right]$$

$$\frac{r_1}{r_2}D\left[e^{-\theta_1\frac{R_1}{r_1}}\right] + \frac{r_1}{r_2}D\left[e^{-\theta_1\frac{R_2}{r_2}}\right] = h_2 + \frac{r_1}{r_2}B$$

$$D\left[e^{-\theta_1\frac{R_1}{r_1}}\right] + D\left[e^{-\theta_1\frac{R_2}{r_2}}\right] = \frac{r_2}{r_1}\left\{h_2 + \frac{r_1}{r_2}B\right\} \tag{16}$$

Adding to equation (15) and equation (16),

$$(D + C)\left[e^{-\theta_1\frac{R_1}{r_1}}\right] = E + \frac{r_2}{r_1}h_2 + B$$

$$\left[e^{-\theta_1\frac{R_1}{r_1}}\right] = \frac{E + \frac{r_2}{r_1}h_2 + B}{(D + C)}$$

$$-\theta_1\frac{R_1}{r_1} = log\left\{\frac{E + \frac{r_2}{r_1}h_2 + B}{(D + C)}\right\}$$

$$\hat{R}_1 = -\frac{r_1}{\theta_1}log\left\{\frac{E + \frac{r_2}{r_1}h_2 + B}{(D + C)}\right\}$$

From equation (16),

$$D\left[e^{-\theta_1\frac{R_2}{r_2}}\right] = \frac{r_2}{r_1}h_2 + B - D\left[e^{-\theta_1\frac{R_1}{r_1}}\right]$$

$$D\left[e^{-\theta_1\frac{R_2}{r_2}}\right] = \frac{r_2}{r_1}h_2 + B - D\left[\frac{E + \frac{r_2}{r_1}h_2 + B}{(D + C)}\right]$$

$$e^{-\theta_1\frac{R_2}{r_2}} = \frac{1}{D}\left\{\frac{r_2}{r_1}h_2 + B - D\left[\frac{E + \frac{r_2}{r_1}h_2 + B}{(D + C)}\right]\right\}$$

$$-\theta_1\frac{R_2}{r_2} = \log\left\{\frac{1}{D}\left\{\frac{r_2}{r_1}h_2 + B - D\left[\frac{E + \frac{r_2}{r_1}h_2 + B}{(D + C)}\right]\right\}\right\}$$

$$\hat{R}_2 = -\frac{r_2}{\theta_1}log\left\{\frac{r_2}{r_1}h_2 + B - D\left[\frac{E + \frac{r_2}{r_1}h_2 + B}{(D + C)}\right]\right\}$$

Given the values of the parameters and the costs the expressions for $\hat{R}_1$ and $\hat{R}_2$ can be evaluated numerically.

Numerical Illustration :

For Example if r_1=10, r_2=15, $\mu_1 = 1.2$, $\mu_2 = 1.5$, d_1 =50, d_2 =100, h_1 = 12, h_2 = 20, $\theta_1 = 1.5$, $\tau_0 = 20$, $\lambda = 1.5$ are all fixed we find the optimal values of R_1 and R_2 as follows. $\hat{R}_1$= 7.116286 and $\hat{R}_2$=18.21213.

The simultaneous estimation of the reserve of manpower namely $\hat{R}_1$ and $\hat{R}_2$ for the fixed values of all the other parameters has been found out. The simultaneous variations in $\hat{R}_1$ and $\hat{R}_2$ consequent to the changes in each one of the parameter keeping the others fixed is also possible.

Discussion

The manpower planning models

developed in this paper will be used to facilitate the adequate provision of manpower for the software industry at different levels of human resource management. As such, it should provide an invaluable tool for the economic development of the province. The models also enable as to estimate the "manpower gap" in future years and thus facilitating the development by the Tamilnadu software industries by providing appropriate training strategies for different levels of management personnel to reduce the gap. The models developed in this paper also provides a tool for assessing the manpower profile and predicting future manpower development on an industry wide basis.

Suggestions for Future Research :

These are many areas of an organisation or industry in which the application of suitable Manpower planning models is quite necessary. It would be very much useful in every sector of human activity. First of all it is imperative to identify those areas of human activity where the demand for manpower and supply are at disequilibrium. Especially in the area of specialist skill, it becomes necessary to identify where the disequilibrium exists and also were there is interruption in the work schedule due to shortage of manpower. The identification of such areas, the type of problems involved and the conversion of a real life situation into a suitable model are essential to develop Human Resource Management which will yield profits not only to the management but also to the society itself.

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