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Shape optimisation and power prediction of horizontal axis wind turbine blades

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Abstract

This paper presents an optimization model for rotor blades of horizontal axis wind turbines. The model refers to a design method based on blade element momentum (BEM) theory is explained for power prediction of horizontal-axis wind turbine (HAWT). The method is used to optimize the chord and twist distributions of the blades. Power predicted by this method validate with 750 KW Global Wind Power turbine. Also a computer program is written to estimate the aerodynamic performance of the existing HAWT blades and used for the performance analysis of the designed 750 KW HAWT rotor.

Key words: Blade Element Momentum Theory, Aerodynamics, Optimal Design and Aerofoil S809.

Nomenclature:

C_p : Power coefficient of wind turbine rotor
P : Power output from wind turbine rotor
m : Air mass flow rate through rotor plane
U : Free stream velocity of wind
U_{re1} : Relative wind velocity
U_R : Uniform wind velocity at rotor plane
A : Area of wind turbine rotor
R : Radius of wind turbine rotor
r : Radial coordinates at rotor plane
r_i : Blade radius for the i^{th} blade element
T : rotor thrust
Q : rotor torque
C_D : Drag coefficient of an airfoil
C_L : Lift coefficient of an airfoil
F : Tip-loss factor
Fi : Tip-loss factor for the i^{th} blade element
N : Number of blade elements
B : Number of blades of a rotor
a : Axial induction factor at rotor plane
a' : Angular induction factor
λ : Tip-speed ratio of rotor
λ_d : Design Tip- Speed ratio of rotor
λ_r : local Tip- Speed ratio of rotor
$\lambda_{r,i}$: Local tip-speed ratio for the i^{th} blade

element

C_i : Blade chord length for the i[th] blade element

ρ : Air density

Ω : Angular velocity of wind turbine rotor

α : Angle of attack

θ_i : Pitch angle for the ith blade element

φ_{opti}: Optimum relative wind angle for the i[th] blade element

σ : Solidity ratio

ν : Kinematic viscosity of air

γ : Glide ratio

Re : Reynolds number

HAWT: Horizontal-axis wind turbine

BEM: Blade element momentum

1. **Introduction**

The objectives of this study are to develop a method using BEM theory for aerodynamic design of the HAWT blades and performance analysis of the existing blades, also to build a computer program and to validate predicted power with 750KW turbine, finally to determine the aerodynamic characteristics and to create the performance curves of the designed rotor.

2. *Aerodynamic Model and Blade Design Procedure for Power prediction:*

BEM theory refers to the determination of a wind turbine blade performance by combining the equations of general momentum theory and blade element theory. A number of methods have been suggested for including effect of tip losses. An approximate method of estimating the effect of tip losses has been given by L. Prandtl and the expression obtained by Prandtl for tip-loss factor is given by the following equation[4, 5].

$$F = (2/\pi)\cos^{-1}\left[\exp\left[\frac{-(B/2)\left[1-(r/R)\right]}{(r/R)\sin\varphi}\right]\right] \quad (1)$$

The application of this equation for the losses at the blade tips is to provide an approximate correction to the equations for predicting rotor performance and blade design. Carrying the tip-loss factor through the calculations, the correct form of equations will be taken as following:

Axial momentum equation

$$dT = 4F\pi\rho U_\infty^2 a(1-a)rdr \quad (2)$$

Angular momentum equation

$$dQ = 4F\pi\rho U_\infty \Omega a'(1-a)r^3 dr \quad (3)$$

$$C_L = \frac{4F\sin\varphi}{\sigma}\frac{(\cos\varphi - \lambda_r\sin\varphi)}{(\sin\varphi + \lambda_r\cos\varphi)} \quad (4)$$

$$a = \frac{1}{\left[1 + [4F\sin^2\varphi/(\sigma C_L)\cos\varphi]\right]} \quad (5)$$

$$a' = \frac{1}{[[4F\cos\varphi/(\sigma C_l)] - 1]} \quad (6)$$

$$C_p = \frac{8}{\lambda^2}\int_{\lambda_h}^{\lambda} \mathbf{F\,Sin^2\varphi\,(\cos\varphi - \lambda_r \sin\varphi)\,(\sin\varphi + \lambda_r\cos\varphi)[1-(C_D/C_L)\cot\varphi\,\lambda_r^2\,d\lambda_r]} \quad (7)$$

Designing blade shape from a known aerofoil type means determining the geometric parameters such as chord length distribution and twist distribution along the blade length for a certain tip-speed ratio at which the power coefficient of the rotor is maximum. For this reason firstly the change of the power coefficient of the rotor with respect to tip-speed ratio should be calculated in order to determine the design tip speed ratio, λ_d. Corresponding to which the rotor has a maximum power coefficient[3]. The blade design parameters will then be according to this design tip-speed ratio. The overall power coefficient, C_p depends on relative wind angle (φ), local tip speed ratio (λ_r), the glide ratio and tip loss factor (F). To get maximum C_p value from this equation is only possible to make the elemental power coefficient maximum for each blade element. In other words, the term in the integral of the mentioned equation (7) should be maximum for each blade element in order to get maximum overall power coefficient from the summation of each. When the optimum relative wind angle values are plotted with respect to the corresponding local tip-speed ratio values at which the elemental power coefficient is maximum for a wide range of glide ratios, the relationship as shown in Figure-2.1 can be found to be nearly independent of glide ratio and tip-loss factor[5].

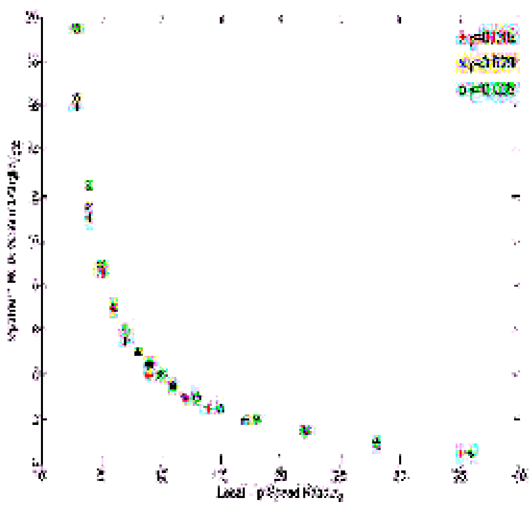

Fig. 2.1

Hence the condition given before can be rearranged as following;

$\varphi_{opt} \sim \text{MAX}\{\sin^2\varphi\ (\cos\varphi - \lambda_r\ \sin\varphi)\ (\sin\varphi + \lambda_r\cos\varphi)\}$

Therefore a general relationship can be obtained between optimum relative wind angle and local tip-speed ratio which will be applicable for any airfoil type[3].

$$\frac{\partial}{\partial\varphi}\left\{\sin^2\varphi\left(\cos\varphi - \lambda_r\sin\varphi\right)\left(\sin\varphi + \lambda_r\cos\varphi\right)\right\} = 0$$

$$\varphi_{opt} = (2/3)\tan^{-1}(1/\lambda_r) \qquad (8)$$

To find out the maximum power coefficient for a selected airfoil type, dividing the blade length into N elements, the local tip speed ratio for each blade element can then be calculated with the use of following equation[5]:

$$\Lambda_{r,i} = \lambda\ (r_i/R) \qquad (9)$$

Optimum Relative wind for each blade element

$$\varphi_{opt,i} = (2/3)\ \tan^{-1}\ (1/\lambda_{r,i}) \qquad (10)$$

Tip- loss factor for each blade element

$$F_i = (2/\pi)\cos^{-1}\left[\exp\left[\frac{-(B/2)\left[1-(r_i/R)\right]}{(r_i/R)\sin\varphi_{opt,i}}\right]\right] \qquad (11)$$

Twist distribution:

$$\theta_i = \varphi_{opt,i} - \alpha_{design} \qquad (12)$$

Chord length distribution:

$$c_i = \frac{8\pi r_i F_i \sin\varphi_{opt,i}}{BC_{L,design}}\frac{(\cos\varphi_{opt,i} - \lambda_{r,i}\sin\varphi_{opt,i})}{(\sin\varphi_{opt,i} + \lambda_{r,i}\cos\varphi_{opt,i})} \qquad (13)$$

Power coefficient is determined using a sum approximating the integral in equation (7)

$$C_p = \sum_{i=1}^{N}\left[\begin{array}{l}\left(\frac{8\Delta\lambda_r}{\lambda^2}\right)F_i\sin^2\varphi_{opt,i}\left(\cos\varphi_{opt,i} - \lambda_{r,i}\sin\varphi_{opt,i}\right)\ldots \\ \left(\sin\varphi_{opt,i} + \lambda_{r,i}\cos\varphi_{opt,i}\right)\left[1-(C_D/C_L)\cot\varphi_{opt,i}\right]\lambda_{r,i}^2\end{array}\right] \qquad (14)$$

3. *Calculation and validation:*

Lift & Drag Coefficient for S-809 Aerofoil given in Table 1

Table 1

Angle of Attack (deg)	S809 Published	S809 Published	Glide ratio γ=Cd/CL
-20.1	-0.56	0.3027	-0.54054
-18.1	-0.67	0.3069	-0.45806
-16.1	-0.79	0.1928	-0.24405
-14.2	-0.84	0.0898	-0.1069
-12.2	-0.7	0.0553	-0.079
-10.1	-0.63	0.039	-0.0619
-8.2	-0.56	0.0233	-0.04161
-6.1	-0.64	0.0131	-0.02047
-4.1	-0.42	0.0134	-0.0319
-2.1	-0.21	0.0119	-0.05667
0.1	0.05	0.0122	0.244
2	0.3	0.0116	0.038667
4.1	0.54	0.0144	0.026667
6.2	0.79	0.0146	0.018481
8.1	**0.9**	**0.0162**	**0.018**
10.2	0.93	0.0274	0.029462
11.3	0.92	0.0303	0.032935
12.1	0.95	0.0369	0.038842
13.2	0.99	0.0509	0.051414
14.2	1.01	0.0648	0.064158
15.3	1.02	0.0776	0.076078
16.3	1	0.0917	0.0917
17.1	0.94	0.0994	0.105745
18.1	0.85	0.2306	0.271294
19.1	0.7	0.3142	0.448857
20.1	0.66	0.3186	0.482727
30	0.705	0.4784	0.678582
40	0.729	0.6743	0.924966
50	0.694	0.8799	1.267867
60	0.593	1.0684	1.801686
70	0.432	1.2148	2.812037
80	0.227	1.2989	5.722026
90	0	1.308	
		Min glide Ratio=	0.018

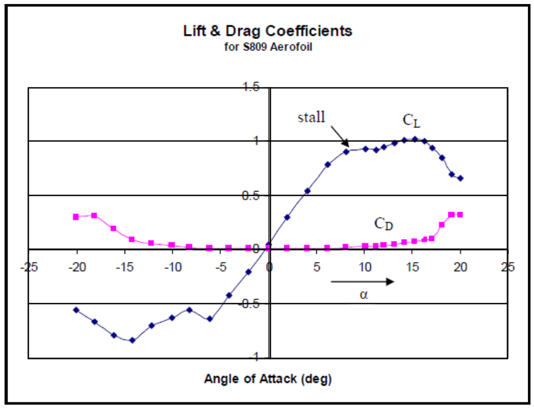

Fig. 3.1 Angle of attack vs. Lift & Drag Coefficient

Lift and Drag coefficients for S809 aerofoil are shown in Figure 3.1. This plot shows that for low values of angle of attack the aerofoil successfully produces a large amount of lift with little drag. At around α=15 a phenomenon known as stall occurs where there is a massive increase in drag and sharp reduction in lift. Also the variation of glide ratio of S809 airfoil with different angle of attacks is illustrated in Figure 3.2. It clearly benefits the blade designer to use or design airfoils with low glide ratio[1].

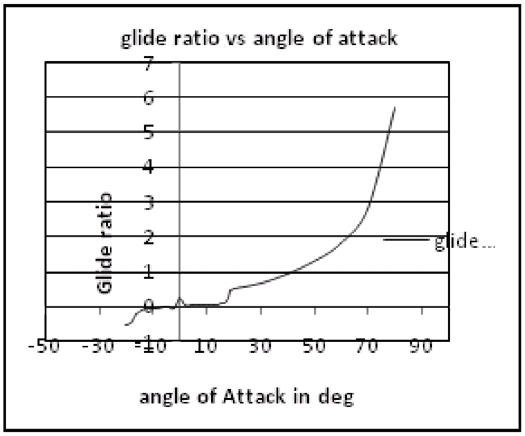

Fig. 3.2. Glide ratio vs Angle of attack

Table 2.

r/R	λ–r	ϕ	F–1	c/R	C_p	θ
0.1	0.7	36.67	1	0.147	0.004	28.6
0.125	0.875	32.54	1	0.146	0.006	24.4
0.15	1.05	29.07	1	0.141	0.007	21
0.2	1.4	23.69	1	0.125	0.01	15.6
0.25	1.75	19.83	1	0.11	0.013	11.7
0.3	2.1	16.98	1	0.097	0.015	8.88
0.35	2.45	14.8	1	0.086	0.018	6.7
0.4	2.8	13.1	1	0.078	0.02	5
0.45	3.15	11.74	1	0.07	0.023	3.64
0.5	3.5	10.63	1	0.064	0.025	2.53
0.55	3.85	9.707	1	0.059	0.027	1.61
0.6	4.2	8.928	1	0.054	0.03	0.83
0.67	4.69	8.024	1	0.049	0.033	-0.1
0.7	4.9	7.69	0.99	0.047	0.034	-0.4
0.75	5.25	7.19	0.99	0.043	0.036	-0.9
0.8	5.6	6.75	0.97	0.04	0.037	-1.4
0.85	5.95	6.36	0.94	0.037	0.038	-1.7
0.9	6.3	6.013	0.87	0.032	0.036	-2.1
0.95	6.65	5.701	0.7	0.025	0.031	-2.4
1	7	5.42	0	0	0	-2.7

Calculated values for designed blade of local tip speed ratio, optimum blade angle, twist angle, tip loss factor and power coefficient using equation from 8 to 14 is shown in table-2.

In Fig. 3.3 Twist distribution with respect to radial location and in Fig. 3.4 chord ratio variation of each blade element both of which are normalized with blade radius is shown. Finally the power coefficient variation with respect to tip-speed ratio is given graphically in Figure 3.5. From this figure it can be seen that the design tip-speed ratio should be taken to as 7 where the power coefficient is at maximum value for the designed blade[3].

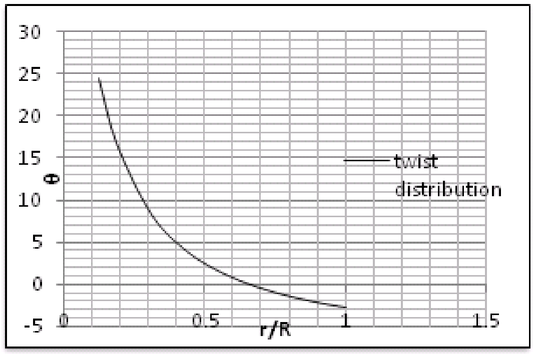

Fig. 3.3 Twist Distribution for designed blade

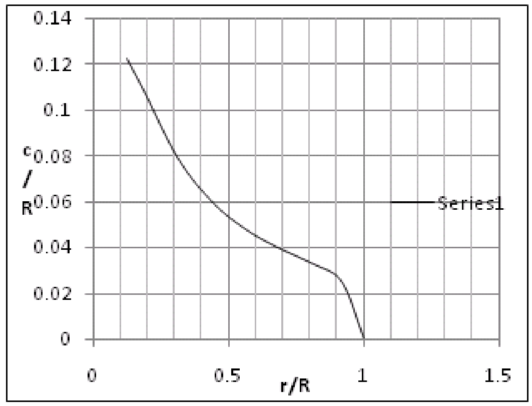

Fig. 3.4 chord ratio variation with radial location

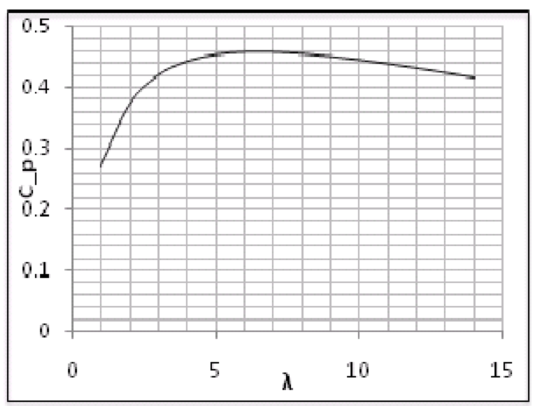

Fig. 3.5 tip speed Ratio vs Cp

Modification of blade geometry :

When the twist distribution of a designed blade shape is analyzed from the Figure 3.3, it can be seen that a designed blade has to be twisted strongly, especially near the root again as it occurs in the chord-length variation of the same blade. For that reason twist distribution can be modified considering the ease of fabrication. This modification has been performed such that twist distribution of the designed blade has been linearized by using the least-square method. Also at some distance from blade root the twist becomes negative which generally causes the blade elements from this distance to blade tip to be stalled. To prevent these blade elements from being in stall region, their twist values are set to zero as shown in Figure 3.6. The chord-length variation of a designed blade shape is not linear and near the root the blade chord-length is increasing steeply as it is shown in Figure 3.4. Here such type of designed blade shape will be modified such that its shape becomes linear tapered[5] as shown in Fig. 3.7.

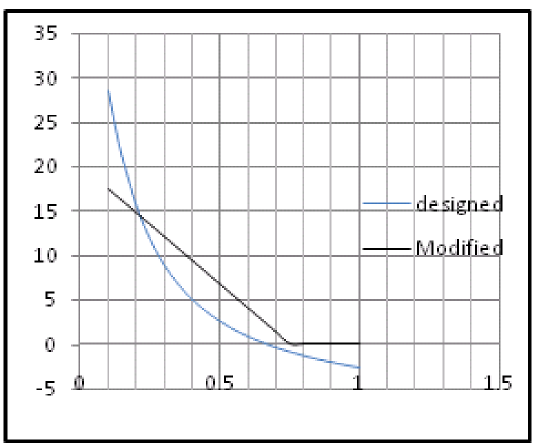

Fig. 3.6 Twist Distribution for Modified blade

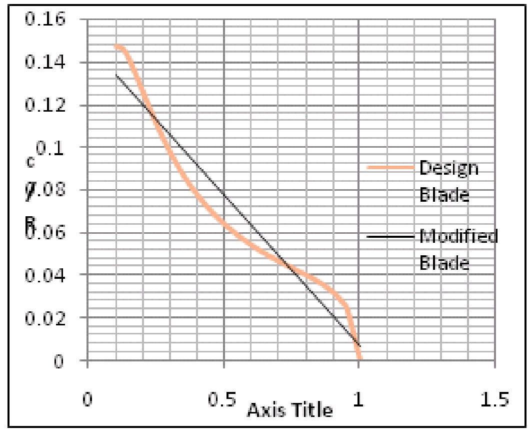

Fig. 3.7 chord ratio variation with radial location for Modified blade

4. *Power prediction of modified blade shape:* In this case lift coefficient and angle of attack have to be determined from the known blade geometry parameters. This requires an iterative solution. The computer program written for this iteration procedure to calculates the induction factors according to the flow chart[4] given in Figure 4.1.

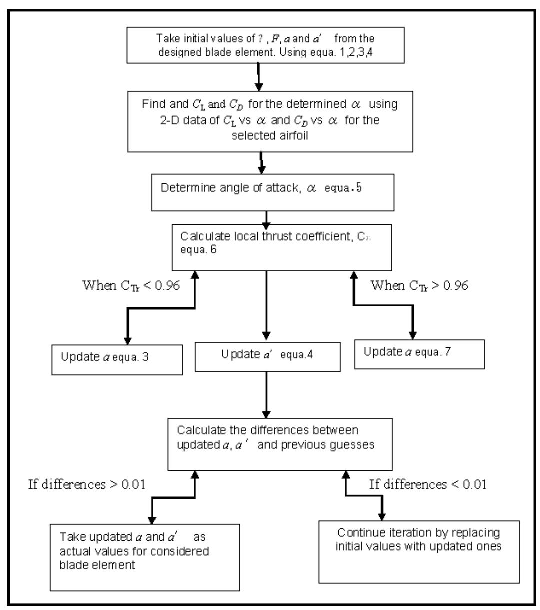

Fig. 4.1

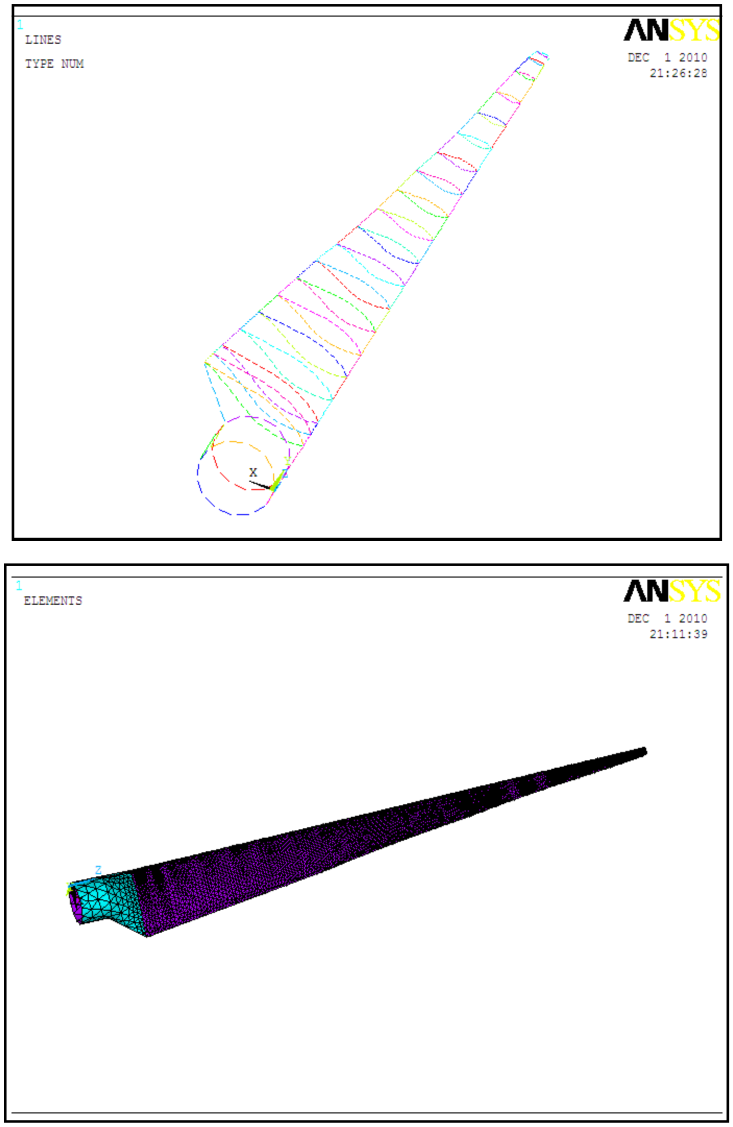

Fig. 4.2 3D View of Modified blade Shape

Model turbine vs GWP-750 KW: The GWP-750 KW wind turbine has a 47 meter rotor, 750KW generators, and operates between 9 and 14.9 RPM. Figure-4.3 shows the power output of the GWP turbine compared to that of the model developed for the optimization study. The power output of the GWP machine was from GWP literature. The power output from the model was calculated using a 45 meter rotor, a 750 KW generator.

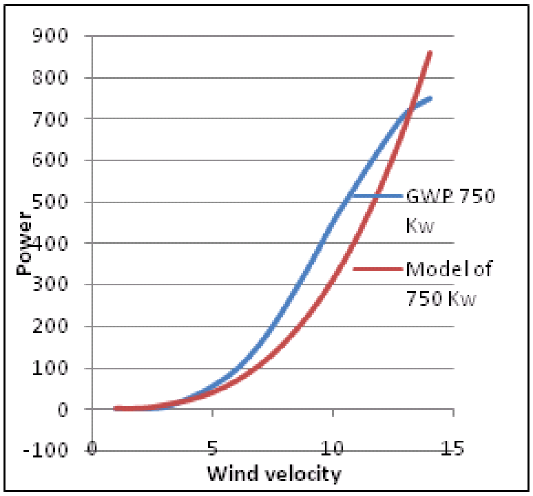

Fig. 4.3 Power variation with wind Velocity

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