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Effect of stratified viscous fluid on MHD free convection flow with heat and mass transfer past a vertical porous plate

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Abstract

A study of the effect of stratified viscous fluid on MHD free convection flow past a vertical porous plate with heat and mass transfer taking viscous and Darcy resistance terms into account and the constant permeability of the medium numerically and neglecting induced magnetic field in comparison to applied magnetic field is investigated. The velocity, temperature and concentration distributions are derived and discussed numerically with the helps of graphs and tables. It is observed that velocity increases with the increase in G_r (Grashof number), K (Permeability parameter) and b (Stratification parameter), but it decreases with the increase in M (Magnetic parameter).

Key words : Heat and mass transfer, Free convection, MHD flow, Porous medium, Vertical plate, Stratified viscous fluid.

Introduction

The convection problem in a porous medium has important applications in geothermal reservoirs and geothermal extractions. The process of heat and mass transfer is encountered in aeronautics, fluid fuel nuclear rector, chemical process industries and many engineering applications in which the fluid is the working medium. The wide range of technological and industrial applications has stimulated considerable amount of interest in the study of heat and mass transfer in convection flows. Free convective flow past a vertical plate has been studied extensively by Ostrach[8]. Siegel[11] investigated the transient free convection from a vertical flat plate. Cheng and Lau[3] and Cheng and Teckchandani[4] obtained numerical solutions for the convective flow in a porous medium bounded by two isothermal parallel plates in the presence of the withdrawal of the fluid. In all the above mentioned studies, the effect of porosity, permeability and the thermal resistance of the medium is ignored or treated as constant. However, porosity measurements by Benenati and Broselow[2] show that porosity is not constant but varies from the surface of the plate to its interior to which as a result permeability also varies. In case of unsteady free convective flow, Soundalgekar[13] studied the effects of viscous dissipation on the flow past an infinite vertical porous plate. The combined effect of buoyancy forces from thermal and mass diffusion on forced convection was studied by Chep. *et al*.[5]. The free convection on a horizontal plate in a saturated porous medium with prescribed heat transfer coefficient was studied by Ramanaiah and Malarvizhi[9]. Bejan and Khair[1] have investigated the vertical free convective boundary layer flow embedded in a porous medium resulting from the combined heat and mass transfer. Lin and Wu[6] analyzed the problem of simultaneous heat and mass transfer with the entire range of buoyancy ratio for most practical and chemical species in dilute and aqueous solutions. Rushi Kumar and Nagarajan[10] studied the mass transfer effects of MHD free convection flow of incompressible viscous dissipative fluid past an infinite vertical plate. Mass transfer effects on free convection flow of an incompressible viscous dissipative fluid have been studied by Manohar and Nagarajan[7]. Sivaiah *et. al*.[12] studied heat and mass transfer effects on MHD free convective flow past a vertical porous plate.

In the present section we have considered the problem of Sivaiah *et. al*.[12] by the introducing stratified viscous fluid under the same conditions taken by Sivaiah *et. al*.[12].

Mathematical analysis :

We study the two-dimensional free convection and mass transfer flow of stratified viscous fluid past an infinite vertical porous plate under the following assumptions:

- The plate temperature is constant
- Viscous and Darcy's resistance terms are taken into account with constant permeability of the medium.
- Boussinesq's approximation is valid.
- The suction velocity normal to the plate is constant and can be written as,

$$v^1 = -U_0$$

A system of rectangular co-ordinates O (x^1, y^1, z^1) is taken, such that $y^1 = 0$ on the plate and z^1 axis is along its leading edge. All the fluid properties considered constant except that the influence of the density variation with temperature is considered. The influence of the density variation in other terms of the momentum and the energy equation and the variation of the expansion coefficient with temperature is considered negligible. The variations of density, viscosity, elasticity, Stefan-Boltzman constant, thermal conductivity and heat source are supposed to be of the form

$$\rho=\rho_o e^{-b^1 y^1}, \quad \mu=\mu_o e^{-b^1 y^1}, \quad \sigma=\sigma_o e^{-b^1 y^1},$$

$$\sigma^* = \sigma_o^* e^{-b^1 y^1} \quad k_T = k_o e^{-b^1 y^1}, \quad S^1 = S_o e^{-b^1 y^1}$$

where ρ_o, μ_o, σ_o, ρ_o^* and k_o are the coefficients of density, viscosity, elasticity, Stefan-Boltzman constant, thermal conductivity and heat source respectively at $y^1 = 0$, $b^1 > 0$ represents the stratification factor.

Under these conditions, the problem

is governed by the following system of Equations:

Equation of continuity:

$$\frac{\partial v^1}{\partial y^1} = 0 \tag{1}$$

Equation of Momentum:

$$\rho\left(\frac{\partial u^1}{\partial t^1} + v^1\frac{\partial u^1}{\partial y^1}\right) = \rho g\beta(T^1 - T^1_\infty) + \rho g\beta^1(C^1 - C^1_\infty)$$

$$+\frac{\partial}{\partial y^1}\left(\mu\frac{\partial u^1}{\partial y^1}\right) - \left(\sigma B_0^2 + \frac{\mu}{K^1}\right)u^1 \tag{2}$$

Equation of Energy:

$$\frac{\partial T^1}{\partial t^1} + v^1\frac{\partial T^1}{\partial y^1} = \frac{1}{\rho C_p}\frac{\partial}{\partial y^1}\left(k_T\frac{\partial T^1}{\partial y^1}\right) \tag{3}$$

Equation of Concentration:

$$\frac{\partial C^1}{\partial t^1} + v^1\frac{\partial C^1}{\partial y^1} = D\left(\frac{\partial^2 C^1}{\partial {y^1}^2}\right) \tag{4}$$

where u^1, v^1 are the velocity components. T^1, C^1 are the temperature and concentration components, ν is the kinematic viscosity, ρ is the density, σ is the electric conductivity, B_0 is the magnetic induction, K_T is the thermal conductivity and D is the concentration diffusivity, C_p is the specific heat at constant pressure.

The boundary conditions for the velocity, temperature and concentration fields are:

$$u^1 = 0, T^1 = T^1_w, C^1 = C^1_w \text{ at } y^1 = 0$$

$$u^1 = 0, T^1 = T_\infty, C^1 = C_\infty \text{ at } y^1 \to \infty \tag{5}$$

Let us introduce the non-dimensional variables

$$u = \frac{u^1}{U_0}, \quad t = \frac{t^1 U_0^2}{\nu_o}, \quad y = \frac{y^1 U_0}{\nu_o},$$

$$\theta = \frac{T^1 - T^1_\infty}{T^1_w - T^1_\infty}, \quad C = \frac{C^1 - C^1_\infty}{C^1_w - C^1_\infty}$$

$$K = \frac{K^1 U_0^2}{{\nu_o}^2}, \qquad P_r = \frac{\mu_o C_p}{k_o},$$

$$S_c = \frac{\nu_o}{D}, \qquad M = \frac{\sigma_o B_0^2 \nu_o}{\rho_o U_0^2},$$

$$N_0 = \frac{\beta^1(C^1_w - C^1_\infty)}{\beta(T^1_w - T^1_\infty)}, G_r = \frac{\nu_o g\beta(T^1_w - T^1_\infty)}{U_0^3},$$

$$b = \frac{b^1 \nu_o}{U_o}$$

where P_r is the Prandtl number, G_r is the Grashof number, N_0 is the buoyancy ratio, S_c is the Schmidt number, M is the magnetic parameter, K is the permeability parameter, b is the stratification parameter. Other physical variables have their usual meaning.

Introducing the non-dimensional quantities describes above, the governing equations reduce to

$$\frac{\partial u}{\partial t}-(1-b)\frac{\partial u}{\partial y}=G_r(\theta+N_0C)+\frac{\partial^2 u}{\partial y^2}-\left(M+\frac{1}{K}\right)u \tag{6}$$

$$P_r\frac{\partial \theta}{\partial t}-(P_r-b)\frac{\partial \theta}{\partial y}=\frac{\partial^2 \theta}{\partial y^2} \tag{7}$$

$$\frac{\partial C}{\partial t}-\frac{\partial C}{\partial y}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} \tag{8}$$

and the corresponding boundary conditions are

$$u=0,\theta=1,C=1 \text{ at } y=0$$
$$u=0,\theta=0,C=0 \text{ at } y\to\infty \tag{9}$$

Method of solution :

We assume the solution of eq. (6), (7), (8) as

$$u(y,t)=u_0(y)e^{-nt},$$
$$\theta(y,t)=\theta_0(y)e^{-nt},$$
$$C(y,t)=C_0(y)e^{-nt} \tag{10}$$

Using eq.(10) in eq. (6), (7), (8) and we get

$$u_0''+(1-b)u_0'-\left[\left(M+\frac{1}{K}-n\right)\right]u_0$$

$$=-G_r\theta_0-G_rN_0C_0 \tag{11}$$

$$\theta_0''+(P_r-b)\theta_0'+nP_r\theta_0=0 \tag{12}$$

$$C_0''+S_cC_0'+S_cnC_0=0 \tag{13}$$

Now the corresponding boundary conditions are

$$u_0=0,\theta_0=1,C_0=1 \text{ at } y=0$$
$$u_0=0,\theta_0=0,C_0=0 \text{ at } y\to\infty \tag{14}$$

Equations (11) to (13) are ordinary linear differential equations, now u_0,θ_0 and C_0 with boundary conditions (14) are

$$u_0=(A_1+A_2)e^{-m_3y}-A_1e^{-m_1y}-A_2e^{-m_2y} \tag{15}$$

$$\theta_0=e^{-m_1y} \tag{16}$$

$$C_0=e^{-m_2y} \tag{17}$$

where

$$m_1=\frac{(P_r-b)+\sqrt{(P_r-b)^2-4nP_r}}{2}$$

$$m_2=\frac{S_c+\sqrt{S_c^2-4S_cn}}{2}$$

$$m_3=\frac{(1-b)+\sqrt{(1-b)^2+4\left(M+\frac{1}{K}-n\right)}}{2}$$

$$A_1=\frac{G_r}{\left[m_1^2-(1-b)m_1-\left(M+\frac{1}{K}-n\right)\right]}$$

$$A_2 = \frac{G_r N_0}{\left[m_2^2 - (1-b)m_2 - \left(M + \frac{1}{K} - n\right)\right]}$$

Hence, The equations for u, θ and C will be as follows

$$u(y,t) = \left[(A_1 + A_2)e^{-m_3 y} - A_1 e^{-m_1 y} - A_3 e^{-m_2 y}\right]e^{-nt} \quad (18)$$

$$\theta(y,t) = e^{-m_1 y}e^{-nt} \quad (19)$$

$$C(y,t) = e^{-m_2 y}e^{-nt} \quad (20)$$

Skin Friction:

The skin friction coefficient at y = 0 is given by

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \left[-m_3(A_1 + A_2) + m_1 A_1 + m_2 A_2\right]e^{-nt} \quad (21)$$

Result and Discussion

Fluid velocity distribution of fluid flow is tabulated in Table 1 and plotted in Fig. 1 having six graphs at $P_r = 0.71$, $S_c=0.4$, $n = 0.1$, $t = 0.1$, $N_0 = 1.5$ for following different value of G_r, M, K and b.

	G_r	M	K	b
For Graph-1	2	0.02	100	0
For Graph-2	2	0.02	100	0.05
For Graph-3	4	0.02	100	0.05
For Graph-4	2	0.04	100	0.05
For Graph-5	2	0.02	1000	0.05
For Graph-6	2	0.02	100	0.10

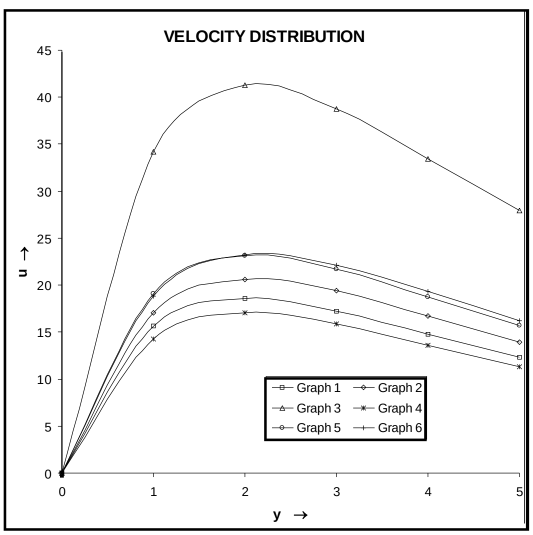

Fig. 1

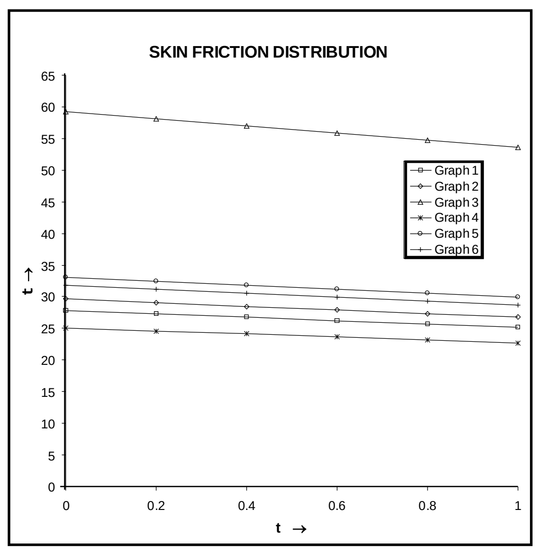

Fig. 2

It is observed from Fig. 1 that all velocity graphs are increasing sharply up to y = 1.2 after that velocity in each graph begins to decrease and tends to zero with the increasing in y. It is also observed from Fig. 1 that velocity increases with the increase in Gr, K and b, but it decreases with the increase in M.

The skin friction distribution is tabulated in Table 2 and plotted in Fig. 2 having six graphs. It is observed from Fig. 2 that skin friction increases with the increase in G_r, K and b, but it decreases with the increase in M.

The temperature distribution is tabulated in Table 3 and plotted in Fig. 3 having three

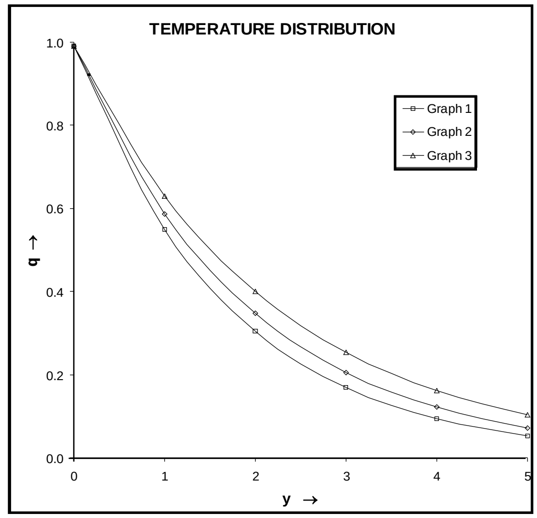

Fig. 3

Table 1. Value of velocity u for Fig. 1 at P_r = 0.71, S_c = 0.4, n = 0.1, t = 0.1, N_0 = 1.5 different values of G_r, M, K and b.

y	Graph 1	Graph 2	Graph 3	Graph 4	Graph 5	Graph 6
0	0	0	0	0	0	0
1	15.65877	17.08198	34.16396	14.26324	19.10093	18.81693
2	18.58670	20.63465	41.26929	17.03716	23.10199	23.21675
3	17.25280	19.37940	38.75881	15.85997	21.72204	22.13424
4	14.79087	16.72977	33.45953	13.60035	18.77185	19.29418
5	12.29422	13.95348	27.90696	11.28817	15.67086	16.18080

Table 2. Value of skin friction τ for Fig. 2 at P_r = 0.71, S_c = 0.4, n = 0.1, N_0 = 1.5 different values of G_r, M, K and b.

t	Graph 1	Graph 2	Graph 3	Graph 4	Graph 5	Graph 6
0	27.83550	29.64376	59.28752	25.09350	33.10652	31.76169
0.2	27.28432	29.05677	58.11355	24.59662	32.45097	31.13276
0.4	26.74405	28.48141	56.96282	24.10957	31.80839	30.51629
0.6	26.21449	27.91744	55.83488	23.63217	31.17855	29.91203
0.8	25.69541	27.36464	54.72928	23.16422	30.56117	29.31973
1	25.18660	26.82278	53.64557	22.70554	29.95602	28.73916

Table 3. Value of temperature θ for Fig. 3 at P_r = 0.71, n = 0.1, t = 0.1 different values of b

y	Graph 1	Graph 2	Graph 3
0	0.99005	0.99005	0.99005
1	0.54881	0.58626	0.62939
2	0.30422	0.34715	0.40012
3	0.16864	0.20556	0.25436
4	0.09348	0.12172	0.16170
5	0.05182	0.07208	0.10280

graphs. It is observed from Fig. 3 that temperature increases with the increase b. The concentration does not change with the change in above parameters taken for velocity.

Particular case :

When b is equal to zero, this problem reduces to the problem of Sivaiah *et. al*. (2009).

Conclusion

1. The velocity increases with the increase in b Stratification parameter).
2. The skin friction increases with the increase in b.
3. The temperature also increases with the increase in b.

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