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MHD Free Convective Flow and Mass Transfer through Viscous Incompressible Fluid in Porous Medium in the Presence of Heat Source and Chemical Reaction

K. JAYARAMI REDDY[1] and M. JAYABHARATHA REDDY[2]

[1]Professor and HOD, Dept. of Mathematics, Priyadarsini Institute of Technology, Tirupati, Chittoor Dist.- 517 561 A.P. (INDIA)
[2]Research Scholar, Dept. of Mathematics, SKIT, Sri Kalahasthi, Chittoor Dist. AP-517 644 (INDIA)

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Abstract

In this paper, unsteady magnetohydrodynamic free convective flow and mass transfer through viscous incompressible fluid past a heated vertical porous plate immersed in porous medium in the presence of heat source and chemical reaction of the uniform transverse magnetic field, oscillating free stream and heat source when viscous dissipation effect is also taken into account. The velocity, temperature and concentration distributions are derived, discussed numerically and their profiles for various values of physical parameters are shown through graphs. The coefficient of skin-friction, Nusselt number and Sherwood number at the plate are derived, discussed numerically and their numerical values for various values of physical parameters are presented through graphically.

Key words: Unsteady, MHD, chemical reaction, mass transfer, porous medium, and heat source.

1. Introduction

The study of flow and mass transfer for an electrically conducting micropolar fluid past a porous plate under the influence of a magnetic field has attracted the interest of many investigators in view of its applications in many engineering problems such as Magneto Hydro Dynamic (MHD) generator, Oil exploration, Plasma studies, and Geothermal energy extractions. Soundalgekar and Takhar[11]. Micropolar fluids are fluids with micro structure belonging to a class of fluids with non-symmetrical stress tensor. Physically, they represent fluids consisting of randomly oriented particles suspended in viscous medium Aero *et al.*[11], Dep[3] and Lukaszewicz[8]. Takhar and Agarwal[12] studied the mixed convective flow of a steady, incompressible micropolar fluid over a stretching sheet. Kim[5] studied the unsteady two-dimensional laminar flow of a viscous incompressible micropolar fluid past a semi-infinite porous plate embedded in a porous medium. Uwanta[13] studied micropolar fluid flow in a channel with Poiseuille effects. Kim[6] investigated transient mixed radiative convection flow of a micropolar fluid past a moving, semi infinite vertical porous plate. Hassanien and Essawy[4] studied the natural convection flow of micropolar fluid from a permeable uniform heat flux surface in porous media. Makinde and Mhone[9] have investigated heat transfer to MHD oscillatory flow in a channel filled with porous medium. Lok *et al.*[7] investigated unsteady mixed convection flow of a micropolar fluid near the stagnation point on a vertical surface. Uwanta[14] studied the effects of mass transfer on laminar convective hydromagnetic flow of radiating gas in a vertical infinite channel. Mostafa[10] studied thermal radiation effect on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity. Ahmad[2] has studied the effects of thermophoresis on natural convection boundary layer flow of a micropolar fluid.

Aim of the paper is to investigate influence of chemical reaction and radiation on an unsteady MHD free convective flow and mass transfer through a viscous incompressible, electrically conducting fluid past an infinite vertical heated porous plate with suction, embedded in porous medium in the presence of a uniform transverse magnetic field, oscillating free stream and heat source by taking into account the viscous dissipation.

2. *Formulation of the problem :*

We consider the unsteady two-dimensional flow of an incompressible viscous fluid past an infinite vertical porous plate, through which suction occurs with constant velocity. The x'-axis is along the plate in the upward direction and the y'-axis is normal to it. All the fluid properties are considered constant except the influence of the density variations with temperature and concentration. The radiation to the x' -direction is considered negligible as compared to the y' - direction. The equations governing the problem are:

Continuity equation

$$\frac{\partial v'}{\partial y'} = 0 \tag{1}$$

Momentum equation

$$\rho\left(\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right) = -\frac{\partial p'}{\partial x'} - \rho g_{x'} + \nu\rho\frac{\partial^2 u'}{\partial y'^2} - \sigma B_0^2 u' - \frac{\mu}{K}u' \qquad \text{d(2)}$$

Energy equation

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} + \frac{Q_0}{\rho c_p}\left(T' - T'_\infty\right) \tag{3}$$

Diffusion equation

$$\frac{\partial C'}{\partial t'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - Kr\left(C' - C'_\infty\right) \tag{4}$$

where u' and v' are the components of the velocity parallel and perpendicular to the plate, t'- the time, p' - the pressure, ρ - the fluid density, g_x'- the acceleration due to gravity, T'- the fluid temperature, ν -the kinematic viscosity, c_p- the specific heat at constant pressure, k- the thermal conductivity, q_r - the radiative heat flux in the y'- direction, C'- the concentration, and D- the diffusion coefficient.

The boundary conditions are

$$u' = 0, \quad T' = T'_w, \; C' = C'_w \qquad \text{at} \quad y' = 0$$
$$u' = U'_\infty = U_0\left(1 + \varepsilon e^{\omega' t'}\right), \; T' \to T'_\infty, \; C' \to C'_\infty \qquad \text{as} \quad y' \to \infty \tag{5}$$

where T' - the fluid temperature far away from the plate, C'_w - the species concentration at the plate, C'_∞ - the species concentration far away from the plate, U_0 - the mean free stream velocity, ω'- the frequency of vibration of the fluid.

$$\rho\left(\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right) = \rho\frac{dU'}{\partial t'} + g_{x'}\left(\rho_\infty - \rho\right)$$
$$+ \nu\rho\frac{\partial^2 u'}{\partial y'^2} - \sigma B_0^2 u' - \frac{\mu}{K}u' \tag{6}$$

The state equation is

$$gx'\left(\rho_\infty - \rho\right) = g_{x'}\rho\beta\left(T' - T'_\infty\right) + g_{x'}\rho\beta^*\left(C' - C'_\infty\right) \tag{7}$$

where β is the coefficient of thermal expansion and β^*- the coefficient of concentration expansion.

In the case of an optically thin gray fluid the local radiant absorption is expressed as:

$$-\frac{\partial q_r}{\partial y'} = 4d\sigma^*\left(T'^4_\infty - T'^4\right) \tag{8}$$

where d is the absorption coefficient and σ^*- the Stefan-Boltzmann constant.

We assume that the temperature differences within the flow are sufficiently small such that may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about and neglecting higher T'_∞ -order terms, thus:

$$T'^4 = 4T'^3_\infty T' - 3T'^4_\infty \tag{9}$$

Equation (9) through (10) takes the form

$$-\frac{\partial q_r}{\partial y'} = 16d\sigma^* T'^3_\infty\left(T'_\infty - T'\right) \tag{10}$$

Equation (1) gives:

$$v' = -v_0\left(v_0 > 0\right) \tag{11}$$

On substituting eqs. (7), (8), (10) and (11) in eqs. (3), (4) and (6) we take

$$\frac{\partial u'}{\partial t'} - v_0\frac{\partial u'}{\partial y'} = \frac{dU'}{\partial t'} + g_{x'}\rho\beta\left(T' - T'_\infty\right)$$
$$+ g_{x'}\rho\beta^*\left(C' - C'_\infty\right) - \sigma B_0^2 u' - \frac{\mu}{K}u' \tag{12}$$

$$\frac{\partial T'}{\partial t'} - v_0\frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y'} + \frac{Q_0}{\rho c_p}\left(T' - T'_\infty\right) \tag{13}$$

$$\frac{\partial C'}{\partial t'} - v_0\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} - Kr\left(C' - C'_\infty\right) \tag{14}$$

On introducing the following non-dimensional quantities:

$$u = \frac{u'}{U_0}, v = \frac{v'}{v_0}, y = \frac{v_0 y'}{\nu}, U_\infty = \frac{U'_\infty}{U_0}, U_p = \frac{u'_p}{U_0}, t = \frac{t' v_0^2}{\nu},$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty}, C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \omega = \frac{\omega'\nu}{v_0^2},$$

$$K = \frac{K' v_0^2}{v^2}, \Pr = \frac{\nu \rho c_p}{k} \quad (15)$$

$$Sc = \frac{\nu}{D}, M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, Gr = \frac{\nu \beta g \left(T_w' - T_\infty'\right)}{U_0 v_0^2},$$

$$Gc = \frac{\nu \beta^* g \left(C_w' - C_\infty'\right)}{U_0 v_0^2},$$

$$Q = \frac{\nu Q_0}{\rho c_p V_0^2}, N = \frac{16 d \sigma^* T_\infty'^3}{k v_0^2},$$

Equations (12), (13), (14) are reduced to the following non-dimensional form

$$\frac{1}{4}\frac{\partial u}{\partial t} - \left(1 + \varepsilon A e^{\omega t}\right)\frac{\partial u}{\partial y} = \frac{1}{4}\frac{dU}{dt} + \frac{\partial^2 u}{\partial y^2} + G_r T + G_m C + \left(M + \frac{1}{K}\right)(U - u) \quad (15a)$$

$$\frac{1}{4}\frac{\partial T}{\partial t} - \left(1 + \varepsilon A e^{\omega t}\right)\frac{\partial T}{\partial y} = \frac{1}{\Pr}\frac{\partial^2 T}{\partial y^2} - \left(N - Q\right)T \quad (15b)$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - \left(1 + \varepsilon A e^{\omega t}\right)\frac{\partial C}{\partial y} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} - K_r C \quad (15c)$$

where θ is the dimensionless temperature, t-the dimensionless time, u-the dimensionless velocity along x - axis, T- the dimensionless temperature of the fluid, C- the dimensionless concentration of the fluid, ω– the frequency, Gr– the Grashof number for heat transfer, Gc– the Grashof number for mass transfer, M– the magnetic parameter, K– the permeability parameter, Pr– the Prandtl number, N– the radiation parameter, Q–the heat source/sink parameter, Sc–the Schmidt number, G_r the chemical reaction parameter, $\varepsilon > 0, A << 1$ and $0 < \varepsilon A < 1$

The boundary conditions in dimensionless form are given by

$$u = 0, \quad \theta = 1, \quad \phi = 1 \quad \text{at } y = 0$$
$$u \to U(t) = 1 + \varepsilon e^{\omega t}, \; T \to 0, \quad C \to 0 \quad y \to \infty$$

3. Solution of the problem :

To reduce the above system of partial differential equations to a system of ordinary differential equations in a dimensionless form, we may represent the translational velocity, temperature and concentration as

$$\begin{aligned} u(y) &= u_0(y) + \varepsilon u_1(y) + o(\varepsilon^2) \\ \theta(y) &= \theta_0(y) + \varepsilon \theta_1(y) + o(\varepsilon^2) \\ \phi(y) &= \phi_0(y) + \varepsilon \theta_1(y) + o(\varepsilon^2) \end{aligned} \quad (16)$$

And the free stream velocity becomes

$$U(t) = 1 + \varepsilon e^{\omega t} \quad (17)$$

Using equation (16) and (17) into the equations (15a, 15b, 15c) and equating the harmonic and non-harmonic terms, and neglecting the higher order terms of o(ε^2), we obtain

$$u_0'' + u_0' - (M + 1/K)u_0 = -(M + 1/K) - GrT_0 - GmC_0$$
$$T_0'' + \Pr T_0' - (Q - N)\Pr T_0 = 0$$
$$C_0'' + ScC_0' - k_r C_0 = 0$$
$$u_1'' + u_1' - (M + 1/K)u_1 = -(M + 1/K) - Au_0' - GrT_1 - GmC_1$$
$$T_1'' + \Pr T_1' - (Q - N)\Pr T_1 = -\Pr AT_0'$$
$$C_1'' + ScC_1' - ScK_r C_1 = -AScC_0'$$

The corresponding boundary conditions are

$$u_0 = 0,\; u_1 = 0, T_0 = 1,\; T_1 = 0, C_0 = 1,\; C_1 = 0 \qquad \text{at} \quad y = 0$$
$$u_0 \to 0,\; u_1 \to 1,\; T_0 \to 0,\; T_1 \to 0, C_0 \to 0,\; C_1 \to 0 \quad \text{as} \; y \to \infty$$

Result and Discussion

Fig. 1 illustrates the effect of chemical reaction parameter on the velocity field. It is noticed that as the chemical reaction parameter increases the velocity field decreases. The effect of Hartmann number on the velocity field has been illustrated in Fig. 2. It is observed that as the Hartmann number increases the velocity field decreases. Fig. 3. illustrates the effect of Grashof number on the velocity field. It is noticed that as the Grashof number increases the velocity field increases. Fig. 4. illustrates the effect of heat source parameter on the velocity profiles. It is noticed that as the heat source parameter increases the velocity field increases. The effect of permeability parameter on the velocity field is shown in Fig. 5. It is observed that as the permeability parameter increases the velocity field increases. The effect of Schmidt number on the velocity field is shown

Fig. 1. Effects of chemical reaction parameter (Kr) on velocity profiles.

Fig. 2. Effects of magnetic parameter (M) on velocity profiles.

Fig.3. Effects of Grashof number for heat transfer (Gr) on velocity profiles.

Fig. 4. Effects of heat source/ sink parameter (Q) on velocity profiles.

Fig. 5. Effects of permeability parameter (K) on velocity profiles

Fig. 6. Effects of Schmidt number (Sc) on velocity profiles

Fig.7. Effects of Prandtl number (Pr) on temperature profiles

Fig.8. Effects of heat source/sin parameter (Q) on temperature profiles.

in Fig. 6. It is observed that as the Schmidt number increases the velocity field decreases.

The effect of Prandtl number on the temperature field is shown in Fig. 7. It is observed that, an increase in the Prandtl number contributes to decrease in the temperature. Fig. 8. illustrates the effect of heat source parameter on the temperature field. It is noticed that as the heat source parameter increases the temperature field to be an increases

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