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A Method for Medical Diagnosis using Intuitionistic Fuzzy Sets

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Abstract

Intuitionistic Fuzzy Theory was proposed by K.T. Atanassov in 1986. The theory has found wide applications in diverse fields including medical diagnosis. Sanchez proposed an approach for medical diagnosis using Fuzzy Set theory. De *et al.* developed this method for Intuitionistic Fuzzy Sets using composition of Intuitionistic Fuzzy Relations. In this paper we propose an alternate method by replacing the max-min composition of De *et al*. with a max-prod composition which seems more effective in decision making, especially in the field of medical diagnosis.

Key words: Fuzzy Sets, Intuitionistic Fuzzy Sets, Hesitancy Grade, Similarity Measure, Singleton IFS, Derived Singleton IFS, Null IFS, Pseudo-null IFS, IF Relations, Medical Diagnosis.

Introduction

The classical theory of sets proposed by Cantor was generalized by Zadeh[8] into Fuzzy Sets (FS) and this was further generalized to Intuitionistic Fuzzy Sets (IFS) by Atanassov[2]. Both FS and IFS theories were proposed to treat real world vagueness and medical knowledge is an area where vagueness abound.

Medical knowledge is often fuzzy as a disease may have several symptoms in varying degrees and the same symptoms may be indicative of several diseases. Hence, FS and IFS methods may be more suitable than Bayesian methods in diagnostic decision process[5]. In Esogbue and Elder discuss the use of FSs in information gathering and diagnosis. FS theory was applied to medical diagnosis by several authors including Sanchez, Addlassnig and Vile and Delgado.

De *et al.*,[4] proposed a method for applying IFS theory in diagnosis by generalizing the method of Sanchez. Szmidt and Kacprzyk[7] also considered the use of similarity measures.

Own[6] uses a switch between IFSs and Type II Fuzzy sets.

In this paper, we introduce a new method which is simple to compute and improves upon the method due to De *et al*.

2. *Preliminary concepts and definitions :*

The concept of FSs is useful in dealing with real world vagueness. However, ordinary FSs consider only the membership grade in a given set. Since the membership is not total, we need to consider also non-membership. In addition, there is also an aspect of indeterminacy. The notion of IFSs given below takes into consideration these aspects also.

2.1 Definition[2] An *Intuitionistic Fuzzy Set (IFS) A* in a universe *X* is an object $A=\{<x, \mu_A(x), \nu_A(x)>: x \in X\}$, with $0 \le \mu_A(x) \le 1$, $0 \le \nu_A(x) \le 1$, and $\mu_A(x) + \nu_A(x) \le 1$, for each $x \in X$.

Both μ_A and ν_A are functions from X to [0, 1]. For each $x \in X$, $\mu_A(x)$ and $\nu_A(x)$ are respectively called *membership degree* and *non-membership degree* of *x* in *A*.

2.2 Definition[2] Let A be an IFS on X and $x \in X$. Then, $h_A(x) = 1- (\mu_A(x) + \nu_A(x))$ is called the *hesitancy grade* or simply, the *hesitancy* of *x*. It is the degree of uncertainty of *x* as a member of the set A. The hesitancy may also be called *indeterminacy.*

2.3 Notation: In this paper, we adopt the following notations.
$\mu_A(x)=A^+(x)$, $\nu_A(x)=A^-(x)$ and $h_A(x)=A^0(x)$
Also, the collection of all IFSs on X will be denoted by IFS(X)

2.4 Definition[2] Let A, B $\in$ IFS(X). We say that A is a *subset* of B, denoted by $A \subseteq B$, if and only if $A^+(x) \le B^+(x)$ and $A^-(x) \ge B^-(x)$ $\forall$ $x \in X$.

2.5 Definition[2] The *complement* of an IFS A, denoted by $\bar{A}$, is given by $\bar{A} = \{<x, A^-(x), A^+(x)>: x \in X\}$.

As an obvious consequence of the definitions we have, $\forall$ $x \in X$:

(i) $0 \le \bar{A}^+(x) \le 1$, $0 \le \bar{A}^-(x) \le 1$, and $\bar{A}^+(x) + \bar{A}^-(x) \le 1$

(ii) $\bar{A}^0(x) = A^0(x)$, and

(iii) $\bar{\bar{A}} = A$

2.6 Definition An IFS *A* is called a *Null IFS* if and only if $A^-(x) = 1$ $\forall$ $x \in X$. It may be denoted by φ.

2.7 Definition An IFS *A* is called a Pseudo *Null IFS* if and only if $A^0(x) = 1$ $\forall$ $x \in X$. It may be denoted by φ^*.

2.8 Definition Let X be a universe consisting of a single element and let $A \in$ IFS(X). Then we say that A is a *singleton IFS.*

2.9 Example Let $X = \{x_1\}$ and $A = \{<x_1, 0.4, 0.5>\}$. Then A is a singleton IFS. Here $A^+(x_1)=0.4$, $A^-(x_1) = 0.5$ and $A^0(x_1)= 0.1$. Its complement is $\bar{A} = \{< x_1, 0.5, 0.4>\}$.

Note that $\bar{A}^+(x_1)=0.5=A^-(x_1)$, $\bar{A}^-(x_1) = 0.4 = A^+(x_1)$ and $\bar{A}^0(x_1) = 0.1 = A^0(x_1)$

2.10 Definition Let $A \in IFS(X)$ where $X = \{x_1, x_2, \ldots, x_n\}$ is a finite universe. Then, we say that A is a *finite IFS.*

2.11 Example Let $X=\{x_1, x_2, x_3\}$ and $A = \{<x_1, 0.4, 0.6>, <x_2, 0.3, 0.5>, <x_3, 0.8, 0.1>\}$. Then A is a finite IFS.

2.12 Definition[3] Let X and Y be two sets. An *Intuitionistic Fuzzy Relation* (IFR) from X to Y is an IFS on X×Y, and is denoted by $R(X \rightarrow Y)$, or simply by R. That is, $R=\{<(x,y), R^+(x,y), R^-(x,y)>/ x \in X, y \in Y\}$. Thus, an IFR from X to Y is an IFS on X×Y.

In what follows we consider only finite IFSs. Hence, unless otherwise specified, all our universes will be finite.

3. The Method due to De, Biswas and Roy[4]:

Sanchez proposed a method for medical diagnosis for fuzzy sets. De *et al*. generalized this method for IFSs which is based on a max-min and min-max composition of IFRs. They obtained medical knowledge as an IFR from a set of symptoms to a set of diagnoses. This involves three steps:

1. Determination of symptoms
2. Formulation of medical knowledge based on IFRs.
3. Diagnosis on the basis of composition of IFRs.

Let P be a set of patients with symptoms from a set S and let D be a set of diseases. Then, in the method due to De *et al*., the diagnosis is an IFR T from P to D which is obtained by the following membership grades:

$$T^+(p_i, d_k) = \bigvee_{s \in S} [Q^+(p_i, s) \wedge R^+(s, d_k)], \quad (1)$$

and

$$T^-(p_i, d_k) = \bigwedge_{s \in S} [Q^-(p_i, s) \vee R\text{-}(s, d_k)], \quad (2)$$

where $\vee$ = max and $\wedge$ = min.

Remark 3.1: As observed by Szmidt and Kacprzyk[7], this method considers only dominating symptoms. Since the membership grade is computed using a min operation, even the values that may come close to the minimum are discarded. Hence, we propose a new method which takes into consideration all the membership values.

4. The proposed New Method :

In this method we replace the max-min composition of De *et al*., by a max-prod composition and the min-max composition by a min-sum composition. Here, the respective membership values are computed as follows:

$$T^+(p_i, d_k) = \max_{s \in S} (Q^+(p_i, s) \otimes R^+(s, d_k)), \quad (3)$$

And,

$$T^-(p_i, d_k) = \min_{s \in S} (Q^-(p_i, s) \otimes \vee R^-(s, d_k)) \quad (4)$$

where $\otimes$ denotes the product operation and $\otimes$ denotes the sum operation.

4.1 Remark: Here T is again an IFR and any diagnosis should be a crisp result. We have discussed methods of fuzzification of IFSs in[1] and using any of these methods, we can obtain a FS which may further be converted to a crisp set by employing any of the standard defuzzifcation methods.

4.2 Example: We illustrate the method by an example. For convenience, we rely on the same hypothetical data as in De et al., and Szmidt and Kacprzyk.

Consider P ={Allen, Bobby, Chris, Dan}, a set of four patients, S = {Temperature, Headache, Stomach pain, Cough, Chest Pain}, a set of five symptoms and D = {Fever, Malaria, Typhoid, Stomach Problem, Chest Problem}, possible diagnoses for these symptoms. We obtain the IFR Q P×S, by observation.

Table 4.1. Observed data about patients and symptoms

Q	Temperature	Headache	Stomach Pain	Cough	Chest Pain
Allen	(0.8, 0.1)	(0.6, 0.1)	(0.2, 0.8)	(0.6, 0.1)	(0.1, 0.6)
Bobby	(0.0, 0.8)	(0.4, 0.4)	(0.6, 0.1)	(0.1, 0.7)	(0.1, 0.8)
Chris	(0.8, 0.1)	(0.8, 0.1)	(0.0, 0.6)	(0.2, 0.7)	(0.0, 0.5)
Dan	(0.6, 0.1)	(0.5, 0.4)	(0.3, 0.4)	(0.7, 0.2)	(0.3, 0.4)

For the above observed data, we can obtain the following IFR R S×D showing the relation between symptoms and diseases. The relation may be obtained from doctor's experience or from a database.

Table 4.2. Relation between symptoms and diseases

R	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Temperature	(0.4, 0.0)	(0.7, 0.0)	(0.3, 0.3)	(0.1, 0.7)	(0.1, 0.8)
Headache	(0.3, 0.5)	(0.2, 0.6)	(0.6, 0.1)	(0.2, 0.4)	(0.0, 0.8)
Stomach Pain	(0.1, 0.7)	(0.0, 0.9)	(0.2, 0.7)	(0.8, 0.0)	(0.2, 0.8)
Cough	(0.4, 0.3)	(0.7, 0.0)	(0.2, 0.6)	(0.2, 0.7)	(0.2, 0.8)
Chest Pain	(0.1, 0.7)	(0.1, 0.8)	(0.1, 0.9)	(0.2, 0.7)	(0.8, 0.1)

Finally, the relation T between the patients and the diseases given by $T = R \circ Q$, where the composition is obtained using the formulas in equations (3) and (4). The IFS T gives the diagnosis. Also, we observe that $T \subseteq P \times D$. We give in the table below the composition for the above data.

Table 4.3. Relation between patients and diseases

$T=R \circ Q$	Viral Fever	Malaria	Typhoid	Stomach Problem	Chest Problem
Allen	(0.32, 0.1)	(0.36, 0.1)	(0.36, 0.19)	(0.16, 0.46)	(0.12, 0.74)
Bobby	(0.12, 0.7)	(0.08, 0.7)	(0.24, 0.46)	(0.48, 0.1)	(0.12, 0.82)
Chris	(0.32, 0.1)	(0.56, 0.1)	(0.48, 0.37)	(0.16, 0.46)	(0.08, 0.55)
Dan	(0.28, 0.1)	(0.49, 0.1)	(0.18, 0.37)	(0.24, 0.4)	(0.24, 0.46)

The final diagnosis may be obtained by considering the maximum of membership values in each row. When the maximum membership is not unique, the non-membership also is considered and in this case, the minimum value is chosen. Thus, for the above data we get the following diagnosis:

Allen - Malaria, Bobby - Stomach Problem, Chris - Malaria and Dan - Malaria

Conclusion

The method proposed by De *et al*. considers only the prominent values of membership grades and hence the weight of the symptoms is not taken care of. In the present method, we take the product and this considers the weight of all the symptoms as well. Thus, the present method gives a more effective method for medical diagnosis.

Medical decision making being a complex process, the theoretical methods proposed alone may not be sufficient and the personal intervention of a medical expert is always necessary. However, such methods can assist the expert and make the diagnostic process easier and faster.

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