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A study of stratified dusty fluid in porous medium past an infinite porous vertical plate in presence of magnetic field

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Abstract

In this paper we study on stratified dusty viscous fluid through a porous medium past an infinite porous vertical plate in presence of magnetic field with time dependent suction. We consider that the free stream oscillates about a constant mean and suction velocity is exponentially decreasing function of time. The liquid velocity, particle velocity and skin-friction for liquid and dust particles have been investigated. The effect of magnetic field, Grashof number, permeability and stratification factor on velocity profiles and skin-friction have been studied with the help of graphs and tables.

Key words : Stratified dusty viscous fluid, magnetic field, porous media, stratification factor, Grashof number, permeability, infinite vertical plate.

Introduction

Stratified dusty fluid in porous medium past an infinite porous vertical plate in presence of magnetic field has received the attention of many researchers like, Liu[12] has discussed flow induced by an oscillating infinite plate in a dusty gas. Prasad and Ramacharyulu[17] presented unsteady flow of a dusty incompressible fluid between two parallel plate under an impulsive pressure gradient. Dixit[7] has studied unsteady flow of a dusty viscous fluid through rectangular ducts. Turchak and Shidlovshii[21] have discussed on the equations of motion of a stratified fluid. Dutta *et al.*[8] have founded unsteady heat transfer to pulsatile flow of a dusty viscous incompressible fluid in a channel. Mondal and Chaudhury[14] have investigated free connective flow of a stratified fluid through a porous medium bounded by a vertical plane. Datta and Dalal[5] have studied pulsatile flow and heat transfer of a dusty fluid through an infinite long

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angular pipe.

Ghosh *et al.*[9] have discussed the hydromagnetic flow of a dusty visco-elastic fluid between two infinite parallel plates. Nordsveen[15] has depicted wave and turbulence-induced secondary currents in the liquid phase in stratified duct flow. Attia[1] has presented unsteady MHD flow and heat transfer of dusty fluid between parallel plates with variable physical properties. Singh and Singh[19] have discussed hydromagnetic free convective and mass transfer flow of a viscous stratified fluid considering the variation in permeability with direction. Shapiro and Fedorovich[18] have presented unsteady convectively driver flow along a vertical plate immersed in a stably stratified fluid.

Khandelwal and Jain[10] have derived unsteady MHD flow of stratified fluid through porous medium over a moving plate in slip flow regime. Magyari *et al.*[13] have discussed unsteady free convection along an infinite vertical flat plate embedded in a stably stratified fluid saturated porous medium. Attia[2] has derived unsteady MHD Couette flow and heat transfer of dusty fluid with variable physical properties. Palani and Ganesan[16] have presented heat transfer effects on dusty gas flow past a semi-infinite inclined plate. Das *et al.*[4] have investigated magnetohydrodynamic unsteady flow of a viscous stratified fluid through a porous medium past a porous flat moving plate in the slip flow regime with heat source. Kumar and Abhilasha[11] have depicted stability of two superposed Rivlin-Ericksen visco-elastic dusty fluid in the presence of magnetic field[5].

Singh *et al.*[20] have discussed effect of thermally stratified ambient fluid on MHD convective flow along a moving non-isothermal vertical plate. Effect of heat source and variable suction on unsteady viscous stratified flow past a vertical porous flat moving plate in the slip flow regime were employed by Das *et al.*[3]. Deka and Bhattacharya[6] have presented MHD flow past an infinite vertical plate immersed in a stably stratified fluid. Three-dimensional Couette flow of a dusty fluid through a porous medium with heat transfer were presented by Vishalakshi *et al.*[22].

Formulation of the Problem :

Consider x-axis in the plane of the plate along the direction of the flow and y-axis perpendicular to the plate and passes through the x-axis. Let ρ_0 be the density and μ_0 be the viscosity of the liquid at the plate y = 0. Magnetic field B_0 is applied perpendicular to the flow region and the suction velocity v is considered to be exponentially decreasing function of time. In the analysis all the fluid properties are assumed constant except the influence of the density variation with temperature only in body force term. In the influence of density variation other terms of the momentum and variation of expansion coefficient with temperature are considered negligible. It is further assumed that U_0 is the mean free stream velocity which oscillates about a constant mean and n is the frequency of oscillation of the free stream and the magnetic Reynolds number is very small so that the influence of induced magnetic field is not considered. The free convection currents are in existence due to temperature difference $\left(T - T_\infty\right)$.

The governing equations of the motion are given by

$$\rho\left[\frac{\partial u_1}{\partial t}+v^*\frac{\partial u_1}{\partial y}\right]=-\frac{\partial p}{\partial x}+\frac{\partial}{\partial y}\left(\mu\frac{\partial u_1}{\partial y}\right)$$

$$-\frac{\mu}{k}u_1-\sigma B^2u_1-\rho g_x-KN(v_1-u_1) \quad (1)$$

$$\frac{\partial}{\partial y}(\rho v)=0 \quad (2)$$

$$m\left(\frac{\partial v_1}{\partial t}\right)=K(u_1-v_1) \quad (3)$$

where $\rho=\rho_0e^{-\beta y}$, $\mu=\mu_0e^{-\beta y}$, $B=B_0e^{-\frac{\beta y}{2}}$ and $N=N_0e^{-\beta y}$

For flow of free stream

$$\rho\left[\frac{\partial U_1}{\partial t}\right]=-\frac{\partial p}{\partial x}-\frac{\mu}{k}U_1-\sigma B^2U_1-\rho_\infty g_x$$

$$+KN(V_1-U_1) \quad (4)$$

and $$m\frac{\partial V_1}{\partial t}=K(U_1-V_1) \quad (5)$$

The equation (2) reveals that suction velocity is a function of time t only, we assume

$$v^*=-v(1+\in e^{-nt}) \quad (6)$$

From equation of state, we have

$$g_x(\rho_\infty-\rho)=g_x\beta^*\rho(T-T_\infty) \quad (7)$$

where

v	-	Non-zero suction velocity.
T	-	Wall temperature.
T_∞	-	Free stream temperature.

Eliminating $\frac{\partial p}{\partial x}$ from equation (4) and (1) and using equation (6) and (7), we get

$$\frac{\partial u_1}{\partial t}-v(1+\in e^{-nt})\frac{\partial u_1}{\partial y}=\frac{\partial U_1}{\partial t}-\upsilon_0\frac{\partial^2u_1}{\partial y^2}+g_x\beta^*(T-T_\infty)$$

$$+\frac{\upsilon_0}{k}(U_1-u_1)-\frac{\sigma B_0^2}{\rho}(U_1-u_1)$$

$$-\frac{KN_0}{\rho}[(V_1-v_1)-(U_1-u_1)] \quad (8)$$

Here the dimensionless quantities are defined as

$$u_1^*=\frac{u_1}{U_0},\ U_1^*=\frac{U_1}{U_0},\ v_1^*=\frac{v_1}{U_0},\ V_1^*=\frac{V_1}{U_0},$$

$$y^*=\frac{yu_0}{\upsilon_0},\quad t^*=\frac{tu_0^2}{\upsilon_0}\quad n^*=\frac{n\upsilon_0}{U_0} \text{ and}$$

$$k^*=\frac{kU_0^2}{\nu_0^2}$$

By using the above dimensionless quantities in equations (8), (5) and (3), we get

$$\frac{\partial u_1}{\partial t}-v(1+\in e^{-nt})\frac{\partial u_1}{\partial y}=\frac{dU_1}{dt}+\frac{\partial^2u_1}{\partial y^2}$$

$$-a\frac{\partial u_1}{\partial y}+G_r+\left(M^2+\frac{1}{k}\right)(U_1-u_1)$$

$$-\frac{\ell}{b}[(V_1-v_1)-(U_1-u_1)] \quad (9)$$

$$b\frac{\partial V_1}{\partial t}=U_1-V_1 \quad (10)$$

$$b\frac{\partial v_1}{\partial t} = u_1 - v_1 \tag{11}$$

Here

$$a = \frac{\upsilon_0\beta}{\mu_0},\quad G_r = \frac{\upsilon_0 g_x \beta^*(T - T_\infty)}{U_0 u_0^2},$$

$$M = \frac{B_0}{\upsilon_0}\sqrt{\frac{\sigma\upsilon_0}{\rho_0}},$$

$$\ell = \frac{mN_0}{\rho},\quad b = \frac{mu_0^2}{\upsilon_0 K}$$

where

a - Stratification factor.

Gr - Grashof number.

M - Hartmann number.

ℓ - Mass concentration of dust particles.

b - Relaxation time parameter of dust particles.

The non-dimensional boundary condition are

$$\left.\begin{array}{lll} t > 0;\ u_1 = v_1 = 0 & \text{at} & y = 0 \\ \quad u_1 = v_1 = 1 + \in e^{-nt} & \text{as} & y \to \infty \end{array}\right\} \tag{12}$$

Solution of the Problem :

In the neighbourhood of the plate, we assume

$$\left.\begin{array}{l} u_1(y,t) = f_1(y) + \in f_2(y)e^{-nt} \\ v_1(y,t) = g_1(y) + \in g_2(y)e^{-nt} \end{array}\right\} \tag{13}$$

$$\left.\begin{array}{l} U_1(t) = 1 + \in C_1 e^{-nt} \\ V_1(t) = 1 + \in C_2 e^{-nt} \end{array}\right\} \tag{14}$$

where C_1 and C_2 are real constants.

Substituting equation (13) and (14) in the equation (10) and (11), and compare the similar terms, we have

$$f_1(y) = g_1(y) \tag{15}$$

$$f_2(y) = (1 - nb)g_2(y) \tag{16}$$

and $$C_2 = \frac{C_1}{1 - nb} \tag{17}$$

Using equation (13) to (17) in the equation (9), we obtain

$$f_1'' + (v - a)f_1' - \left(M^2 + \frac{1}{k}\right)f_1 = -G_r - \left(M^2 + \frac{1}{k}\right) \tag{18}$$

and

$$f_2'' + (v - a)f_2' - \left(M^2 + \frac{1}{k} - n - \frac{\ell n}{1 - bn}\right)f_1 = -vf_1'(y)$$
$$- \left(M^2 + \frac{1}{k} - n - \frac{\ell n}{1 - bn}\right)C_1 \tag{19}$$

The boundary condition are transformed to

$$\left.\begin{array}{lll} f_1 = f_2 = g_1 = g_2 = 0 & \text{at} & y = 0 \\ f_1 = g_1 = 1 = f_2 = g_2 & \text{as} & y \to \infty \end{array}\right\} \tag{20}$$

On solving the equation (18) and (19) under the boundary conditions equation (20), we obtain $f_1(y)$ and $f_2(y)$. Using these in equation (15) and (16), we have

$$u_1(y,t) = a_0 - a_0 e^{-H_2 y} + \in \left[1 - a_1 e^{-H_2 y} + (a_1 - 1)e^{-H_4 y}\right]e^{-nt} \tag{21}$$

$$v_1(y,t) = a_0 - a_0 e^{-H_2 y} + \in \left[1 - a_2 e^{-H_2 y} + (a_2 - 1)e^{-H_4 y}\right]e^{-nt} \tag{22}$$

where $C_1 = 1,\qquad C_2 = \frac{1}{1-bn},\qquad a_0 = \frac{G_r}{\left(M^2 + \frac{1}{k}\right)},$

$$a_1 = \frac{a_0 H_2}{H_2^2 - (v-a)H_2 - \left(M^2 + \frac{1}{k} - n - \frac{\ell n}{1-bn}\right)},\quad a_2 = \frac{a_1}{1-bn},$$

$$H_2 = \frac{1}{2}\left[(v-a) + \left\{(v-a)^2 + 4\left(M^2 + \frac{1}{k}\right)\right\}^{1/2}\right]$$

$$H_4 = \frac{1}{2}\left[(v-a) + \left\{(v-a)^2 + 4\left(M^2 + \frac{1}{k} - n - \frac{\ell n}{1-bn}\right)\right\}^{1/2}\right]$$

Non-dimensional Skin-friction

Skin-friction for liquid

$$\tau_1 = \left(\frac{\partial u_1}{\partial y}\right)_{y=0} = H_2 + \in\left[a_1 H_2 - (a_1 - 1)H_4\right]e^{-nt} \tag{23}$$

Skin-friction for dust particles

$$\tau_2 = \left(\frac{\partial v_1}{\partial t}\right)_{y=0} = H_2 + \in\left[a_2 H_2 - (a_2 - 1)H_4\right]e^{-nt} \tag{24}$$

Table 1. Effect of magnetic field on skin-friction, ℓ = 0.3, b = 0.2, n = 1.0, $\in$ = 0.05, G_r = 1.0, v = 0.5, k = 0.25 and a = 0.05.

M / t	M = 1.0		M = 2.0	
	τ_1	τ_2	τ_1	τ_2
1.0	2.510	2.511	2.724	2.723
2.0	2.485	2.486	2.697	2.696
3.0	2.475	2.476	2.687	2.686
4.0	2.471	2.472	2.684	2.683
5.0	2.470	2.471	2.683	2.682

Table 2. Effect of permeability on skin-friction, ℓ = 0.3, b = 0.2, n = 1.0, $\in$ = 0.05, G_r = 1.0, v = 0.5, M = 1.0 and t = 1.0.

t \ k	k = 0.20		k = 0.25	
	τ_1	τ_2	τ_1	τ_2
0.05	2.728	2.729	2.510	2.511
0.10	2.700	2.701	2.482	2.483
0.15	2.673	2.674	2.456	2.457
0.20	2.646	2.647	2.428	2.429
0.25	2.619	2.620	2.400	2.401

Table 3 : Effect of Grashof number on skin-friction, ℓ = 0.3, b = 0.2, n = 1.0, $\in$ =0.05, M = 1.0, v = 0.5, k = 0.25 and a = 0.05.

t \ G_r	G_r = 1.0		G_r = 2.0	
	τ_1	τ_2	τ_1	τ_2
1.0	2.692	2.692	2.510	2.511
2.0	2.483	2.483	2.485	2.486
3.0	2.474	2.474	2.475	2.476
4.0	2.470	2.470	2.471	2.472
5.0	2.468	2.468	2.470	2.471

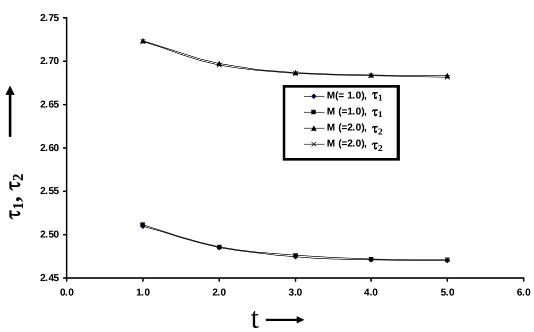

Figure 1. Effect of magnetic field on skin-friction.

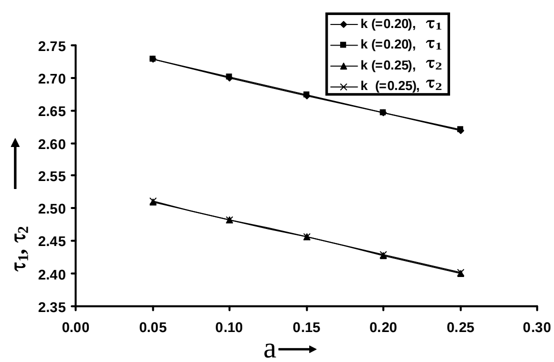

Figure 2. Effect of permeability on skin-friction.

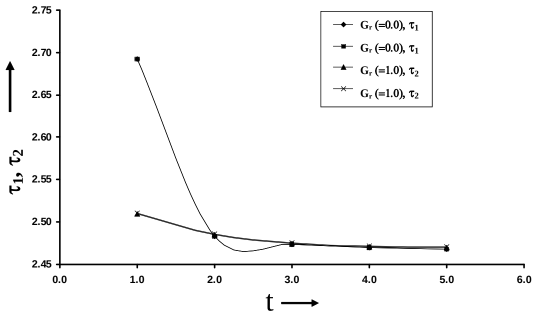

Figure 3. Effect of Grashof number on skin-friction.

Results and Discussion

Table (1) and figure (1) depicts the effect of magnetic field on skin-friction for liquid and dust particles at $\ell = 0.3$, b = 0.2, n = 1.0, $\in = 0.05$, $G_r = 1.0$, v = 0.5, k = 0.25 and a = 0.05. It is clear from the table (1) and figure (1) that when the intensity of magnetic field increases the skin-friction increases for liquid and dust particles both for all values of time. Besides, when the time increases the skin-friction decreases for liquid and dust particles for each given magnitude of magnetic intensity.

Table (2) and figure (2) depicts the effect of permeability on the skin-friction for liquid and dust particles at $\ell = 0.3$, b = 0.2, n = 1.0, $\in = 0.05$, $G_r = 1.0$, v = 0.5, M = 1.0 and t = 1.0. It is clear from the table (2) and figure (2) that for all values of stratification factor a, the skin-friction decreases for liquid and dust particles. Besides, for all values of a, the skin-friction decreases for liquid and particle velocity for each given value of permeability parameter.

Table (3) and figure (3) depicts the effect of Grashof number on the skin-friction for liquid and dust particles at $\ell = 0.3$, b = 0.2, n = 1.0, $\in = 0.05$, M = 1.0, v = 0.5, k = 0.25 and a = 0.05. It is clear from the table (3) and figure (3) that skin-friction decreases for liquid and dust particles for all values of time. Besides, when the time increases the skin-friction decreases for liquid and dust particles for each given value of Grashof number. It is also clear that when Grashof number is zero, the skin-friction for liquid and dust particles is equal for all values of time.

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