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A study of unsteady MHD periodic flow of viscous incompressible fluid through a porous channel

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Abstract

An analysis of the unsteady periodic flow of a viscous incompressible fluid through a porous channel in the presence of magnetic field. The governing equations have been solved by finite difference technique. The mathematical expression for the velocity profiles of the fluid have been obtained. The numerical computation involved in the solution have been shown by tables. The effects of various parameters on the flow field have been depicted by graphs and tables.

Key words : MHD, porous channel, magnetic field, Hartmann number, finite difference technique, viscous fluid.

Introduction

The researches on the flow of fluids through porous medium are being used in almost all field of engineering and technology. In the history of fluid flow through porous channel, Navier investigated a boundary condition of fluid slip at solid surface

Such that $u = h\left(\frac{\partial u}{\partial y}\right)$

where

u - Velocity along X - axis
h - Slip coefficient

if h = 0, it indicates that there is no slip at the boundary.

Darcy's law for the fluid through porous medium is given by

$$Q = \frac{-KA(\ell_1 - \ell_2)}{\ell}$$

where Q is the total volume of the fluid percolating in unit time through a homogeneous filter bed of the height ℓ which is bounded by a plane surface A, and K is the constant depending on the properties of the fluid and of the porous medium. The heights of the upper and lower boundaries of the filter bed are ℓ_2

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and ℓ_1 respectively. The negative sign shows that the flow is increasing in the opposite direction.

Raptis *et al.*[11] have been discussed hydromegnetic free convection flow through a porous medium between two parallel plates. Johari and Singh[7] founded free convection boundary layer flow embedded in a saturated porous medium. Singh *et al.*[15] depicted heat transfer in three-dimensional MHD flow past a porous plate. Effect of applied magnetic field on transient free convective flow in a vertical channel was investigated by Jha[6].

Sreekanth *et al.*[20] founded transient MHD free convection flow of an incompressible viscous dissipative fluid. Gupta and Johri[5] discussed MHD three-dimensional flow past a porous plate. Sharma *et al.*[13] investigated the flow between angular space surrounded by coaxial cylindrical porous medium. About the physical relevance of similarity solution of the boundary layer flow equation describing mixed convection flow along a vertical plate was depicted by Steinruck[21]. Sarangi and Jose[12] have been founded unsteady MHD free convective flow and mass transfer through porous medium with constant suction and constant heat flux. Ahmed *et al.*[1] investigated free convection MHD flow and heat transfer through porous medium between two long vertical wavy walls.

Avinash and Rao[2] discussed unsteady flow of a conducting viscous fluid in a porous rectangular duct. Three-dimensional free convection flow between two parallel vertical plates moving in opposite directions were analysed by Singh and Sharma[14]. Kumar *et al.*[9] employed viscous flow coaxial cylinder in the presence of magnetic field. Singh *et al.*[17] derived computational study of hydromagnetic effect on the viscous incompressible dissipative fluid past an infinite vertical plate. Singh *et al.*[16] presented unsteady free convection flow in porous medium past a horizontal porous plate in presence of heat source, periodic free stream velocity and temperature. Chaudhary *et al.*[3] indicated free convection effects on MHD flow past an infinite vertical accelerated plate embedded in porous media with constant heat flux. Singh *et al.*[18] have been discussed a numerically study of the three-dimensional Couette MHD flow through a porous medium with heat transfer[12].

Mebine and Gumus[10] discussed on steady MHD thermally radiating and reacting thermosolutal viscous flow through a channel with porous medium. Krishna *et al.*[8] derived unsteady magnetohydrodynamic pulsatile flow of a viscous incompressible fluid through a porous medium in a flexible channel. Effect of heat source on MHD free convection flow past an oscillating porous plate in the slip flow regime were considered by Das *et al.*[4]. Recently, Singh *et al.*[19] have been investigated numerical solutions of transient MHD free and forced convective flow of viscous incompressible fluid past an infinite vertical plate.

Formulation of the Problem :

We are considering an unsteady periodic flow of viscous incompressible fluid through a porous channel in the presence of magnetic field.

The governing equations of the motion are given below :

$$\frac{\partial u'}{\partial t'} = -\frac{1}{\rho}\frac{\partial P'}{\partial x'} + \upsilon\frac{\partial^2 u'}{\partial y'^2} - \frac{\upsilon}{K'}u' - \frac{\sigma B_0^2}{\rho}u' + g\beta(T' - T_0') \tag{1}$$

$$\rho C_P \frac{\partial T'}{\partial t'} = \kappa\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q'}{\partial y'} \tag{2}$$

where

- u' - Velocity.
- ρ - Fluid density.
- P' - Pressure.
- t' - Time.
- υ - Kinematic viscosity coefficient.
- K' - Porous medium permeability coefficient.
- σ - Conductivity of the fluid.
- B_0 - Electromagnetic induction.
- g - Acceleration due to gravity.
- β - Coefficient of volume expansion due to temperature.
- T' - Fluid temperature.
- T_0' - Fluid temperature at y = 0.
- T_w' - Fluid temperature at y = a.
- C_p - Specific heat at constant pressure.
- κ - Thermal conductivity.
- q' - Radiative heat flux.

The boundary conditions are given by

$$u' = h\frac{\partial u'}{\partial y'}, \qquad T' = T_w' \qquad \text{at } y' = 0 \tag{3}$$

$$u' = 0, \qquad T' = T_0' \qquad \text{at } y' = a \tag{4}$$

Suppose the fluid which is optically thin with a relatively low density and its radiative heat flux is given by

$$\frac{\partial q'}{\partial y'} = 4\alpha^2(T' - T_0') \tag{5}$$

Here the dimensionless quantities are defined as

$$x = \frac{x'}{a}, \quad y = \frac{y'}{a}, \quad u = \frac{u'}{V},$$

$$\theta = \frac{T' - T_0'}{T_w' - T_0'}, \quad t = \frac{t'V}{a}$$

$$p = \frac{aP'}{V\rho\upsilon}, \quad D_a = \frac{K'}{a^2}, \quad N^2 = \frac{4\alpha^2 a^2}{\kappa},$$

$$M = \sqrt{\frac{a^2\sigma B_0^2}{\rho\upsilon}}, \quad G_r = \frac{g\beta(T_w' - T_0')a^2}{V\upsilon},$$

$$P_e = \frac{V a \rho C_p}{\kappa}, \quad R_e = \frac{Va}{\upsilon}$$

where

- V - Mean (flow) velocity.
- θ - Non-dimensional temperature.
- p - Fluid pressure.
- D_a - Darcy number.
- α - Mean radiation absorption coefficient.
- N - Radiation parameter.
- M - Hartmann number.
- G_r - Grashof number.
- P_e - Peclet number.
- R_e - Reynolds number.
- K - Porous medium shape factor.

By using the above dimensionless quantities into equation (1) to (4), we get

$$R_e\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} - (K^2 + M^2)u + G_r\theta \tag{6}$$

where $K^2 = \frac{1}{D_a} = \frac{a^2}{K'}$

$$P_e \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} - N^2 \theta \qquad (7)$$

The non-dimensional boundary conditions are

$$u = h\frac{\partial u}{\partial y}, \qquad \theta = 1 \qquad \text{at} \qquad y = 0 \qquad (8)$$

$$u = 0, \qquad \theta = 0 \qquad \text{at} \qquad y = 1 \qquad (9)$$

Suppose pressure gradient for periodic flow

$$-\frac{\partial p}{\partial x} = \lambda e^{int}$$

where

λ - Constant.

n - Frequency of oscillations.

These are coupled non-linear equations which do not have an exact solution. So we have solved these equations by implicit finite difference technique of Crank-Nicolson type because this scheme is always stable and convergent.

Then the finite difference approximations of these equation are the following.

$$R_e\left[\frac{u_{i,j+1} - u_{i,j}}{\Delta t}\right] = \frac{1}{2}\left[\frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{(\Delta y)^2} + \frac{u_{i-1,j} - 2u_{i,j} + u_{i+i,j}}{(\Delta y)^2}\right]$$

$$-\left[\frac{p_{i,j+1} - p_{i,j}}{\Delta x}\right] - \frac{(K^2 + M^2)}{2}\left[u_{i,j+1} - u_{i,j}\right] + \frac{G_r}{2}\left[\theta_{i,j+1} + \theta_{i,j}\right] \qquad (10)$$

$$P_e\left[\frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t}\right] = \frac{1}{2}\left[\frac{\theta_{i-1,j-1} - 2\theta_{i,j+1} + \theta_{i+1,j+1}}{(\Delta y)^2} + \frac{\theta_{i-1,j} - 2\theta_{i,j} + \theta_{i+1,j}}{(\Delta y)^2}\right]$$

$$-\frac{N^2}{2}\left[\theta_{i,j+1} + \theta_{i,j}\right] \qquad (11)$$

The boundary conditions take the following form of the following.

$$\left.\begin{array}{ll} u(i,0) = h\left[\dfrac{u_{i,j+1} - u_{i,j}}{\Delta y}\right], \quad \theta(i,0) = 0 & \text{for all except } i = 0 \\ u(0,j) = 0, \quad \theta(0,j) = 0 & \text{at} \quad i = 0 \\ u_{M,j} = u_{\infty}, \quad \theta_{M,j} = 0 & \end{array}\right\} \qquad (12)$$

where M corresponds to ∞. The suffix i corresponds to y and j corresponds to t.

Also $\Delta t = t_{j-1} - t_j$ and $\Delta y = y_{i+1} - y_i$

$$[\Delta t = 0.02 \text{ and } \Delta y = 0.1]$$

Equation (6) and (7) can be written in the form

$$A_1 u_{i-1,j+1} + B_1 u_{i,j+1} + D_1 u_{i+1,j+1} = E_1 \quad (13)$$

$$A_2 \theta_{i-1,j+1} + B_2 \theta_{i,j+1} + D_2 \theta_{i+1,j+1} = E_2 \quad (14)$$

where

$$RT = \frac{\Delta t}{2(\Delta y)^2}, \quad pRT = \frac{\Delta t}{2p(\Delta y)^2},$$

$$A_1 = RT, \quad B_1 = -2RT - 1, \quad D_1 = RT,$$

$$E_1 = -RTu_{i-1,j} + (2RT - 1)u_{i,j} - RTu_{i,j} - \frac{G_r}{2}[\theta_{i,j+1} + \theta_{i,j}] - \frac{(K^2 + M^2)}{2}[u_{i,j+1} - u_{i,j}]$$

$$A_2 = pRT, \quad B_2 = -2pRT - 1, \quad D_2 = pRT$$

$$E_2 = -pRT\theta_{i-1,j} + (2pRT - 1)\theta_{i,j} - pRT\theta_{i+1,j} - E\frac{\Delta t}{(\Delta y)^2}$$

Knowing the values u and θ at a time t, we calculate the values at a time $(t + \Delta t)$ as following, substitute i = 1, 2....... (M - 1) in equation (13) which results in a diagonal system of equations in unknown values of θ.

Using the initial and boundary condition, the system can be solved by Gauss - elimination method. Thus θ is known at all values of y at time $(t + \Delta t)$, then applying the same procedure is continued to obtain the solution till desired time t. In order to check the accuracy of this scheme. We have compared numerical results with exact solution of equation (6) and (7) and we observe that they agree well. For gaining physical insight into this problem, numerical calculations are carried out u and θ for permeability parameter K' and time t.

Table 1. Shear stress for various values of parameter, $G_r = 1$, $R_e = 1$, M = 1, N = 1, $P_e = 0.7$, n = 1, $\lambda = 1$, K = 1.

t	h = 0.0	h = 0.5	h = 1.0	h = 1.5	h = 2.0	h = 2.5
0	34.434	20.279	14.205	10.829	8.680	7.191
1	34.429	20.276	14.203	10.827	8.678	7.190
2	34.412	20.266	14.196	10.822	8.674	7.186
3	34.386	20.250	14.185	10.813	8.666	7.180
4	34.348	20.228	14.168	10.800	8.656	7.171
5	34.299	20.198	14.148	10.784	8.643	7.160

Table 2. Velocity profiles for various values of parameter, $G_r = 1$, $R_e = 1$, M = 1, N = 1, Pe = 0.7, n = 1, λ = 1, K = 1, h = 1.

y	t = 0	t = $\pi/4$	t = $\pi/2$	t = $3\pi/4$	t = π	t = $5\pi/4$
0.0	14.205	9.752	0	-9.752	-14.205	-9.752
0.2	10.546	7.164	0	-7.164	-10.546	-7.164
0.4	7.331	4.891	0	-4.891	-7.331	-4.891
0.6	4.420	2.833	0	-2.833	-4.420	-2.833
0.8	1.684	0.898	0	-0.898	-1.684	-0.898
1.0	0	0	0	0	0	0

Table 3. Velocity profiles for various values of parameter, $G_r = 1$, $R_e = 1$, N = 1, $P_e = 0.7$, n = 1, λ = 1, K = 1, h = 1, t = 0.

y	M = 0.0	M = 0.5	M = 1.0	M = 1.5	M = 2.0	M = 2.5
0.0	21.404	19.190	14.205	9.367	5.894	3.642
0.2	16.397	14.592	10.546	6.656	3.908	2.167
0.4	11.766	10.394	7.331	4.410	2.375	1.110
0.6	7.395	6.474	4.420	2.474	1.132	0.310
0.8	3.176	2.713	1.684	0.712	0.046	0.002
1.0	0	0	0	0	0	0

Table 4. Velocity profiles for various values parameters, $G_r = 1$, $R_e = 1$, M = 1, N = 1, n = 1, λ = 1, h = 1, t = 0.

y	$P_e = 0.0$	$P_e = 0.5$	$P_e = 1.0$	$P_e = 1.5$	$P_e = 2.0$	$P_e = 2.5$
0.0	14.737	14.448	13.769	12.964	12.075	11.419
0.2	10.946	10.728	10.216	9.609	8.914	8.443
0.4	7.618	7.462	7.095	6.660	6.130	5.824
0.6	4.606	4.505	4.268	3.987	3.600	3.447
0.8	1.775	1.725	1.609	1.470	1.211	1.205
1.0	0	0	0	0	0	0

Table 5. Velocity profiles for various values of parameter, $G_r = 1$, M = 1, N = 1, $P_e = 0.7$, n = 1, λ = 1, h = 1, t = 0.

y	$R_e = 0.0$	$R_e = 0.5$	$R_e = 1.0$	$R_e = 1.5$	$R_e = 2.0$	$R_e = 2.5$
0.0	14.892	14.708	14.205	13.498	12.694	11.872
0.2	11.101	10.952	10.546	9.974	9.326	8.663
0.4	7.750	7.638	7.331	6.900	6.412	5.914
0.6	4.701	4.625	4.420	4.132	3.806	3.474
0.8	1.824	1.786	1.684	1.539	1.376	1.211
1.0	0	0	0	0	0	0

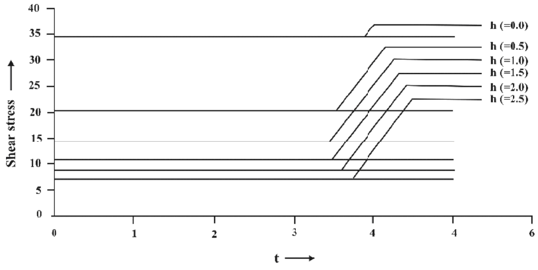

Figure 1 : Shear stress for various values of wall-slips parameter (h).

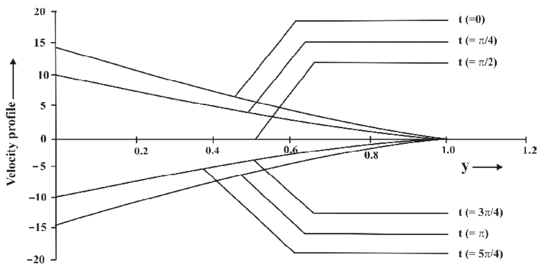

Figure 2 : Velocity profiles for various values of time (t).

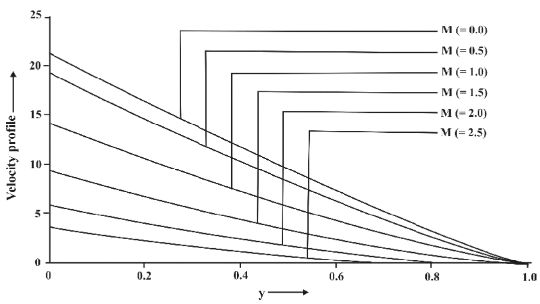

Figure 3 : Velocity profiles for various values of Hartmann number (M).

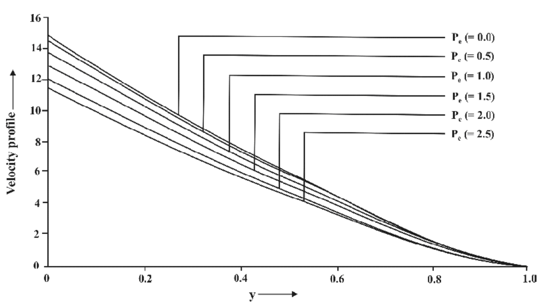

Figure 4 : Velocity profiles for various values of Peclet number (P_e).

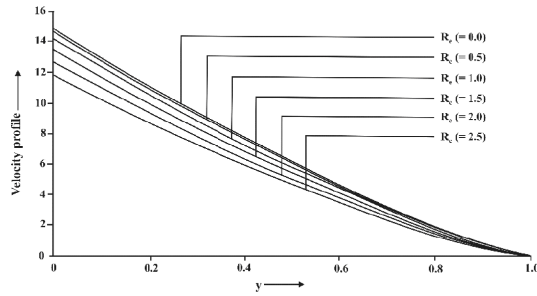

Figure 5 : Velocity profiles for various values of Reynolds number (R_e).

Results and Discussion

The effects of unsteady MHD periodic flow of viscous incompressible fluid through a porous channel of various parameters like Hartmann number (M), Peclet number (P_e), Reynolds number (R_e), time (t) and wall-slip parameter (h) on velocity and temperature profiles have been shown in the tables and graphs.

Figure (1) depicts the tendency shear stress distribution corresponding to different values of h. Hence for every smaller values of h the value of shear stress is seen much more than that of larger one. Figure (2) depicts that the velocity profile of the fluid is periodic with period π. Figure (3) depicts that the velocity profiles for various values of magnetic parameter M. It shows that the flow decelerates on the increase of magnetic parameter M. Figure (4) depicts that the fluid velocity decreases as the value of peclet number increases corresponding to the same value of y. Figure (5) depicts that the fluid velocity decreases as the value of Reynolds number increases corresponding to the same value of y.

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