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Heat and mass transfer effects on the marginal stability of rotating dusty Ferro fluid flowing through a porous medium

B.K. PATRA[1], S. BISWAL[2] and A.P. MISHRA[3]

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Abstract

This paper deals with the theoretical investigation of the heat and mass transfer effects on the marginal stability of a rotating ferrofluid heated below saturating a porous medium. For a flat fluid layer contained between two free boundaries, an exact solution is obtained using a linear stability analysis theory normal mode analysis method. For the case of stationary convection, the effect of various parameters like medium permeability, dust particle coupling parameter and conduction parameter has been analyzed. The critical thermal Raleigh number for the onset of instability is also determined numerically. Results are depicted graphically. It is observed that the critical thermal Rayleigh number is reduced because the heat capacity of clean fluid is supplemented by that of the dust particles. The principle of exchange of stabilities is found to hold true for the ferrofluid with internal angular momentum saturating porous medium in the absence of dust particles, coupling parameter and micro inertia. The oscillating modes are introduced due to the presence of dust particles, coupling parameter and micro inertia, which were nonexistent in their absence. The sufficient conditions for the non existence of stability are also obtained.

Key words: Ferro fluid/Internal angular momentum/Porous medium/Heat and mass transfer.

Nomenclature

Latin Symbols:

b: Subscripts; basic state

C_v: Specific heat at constant volume (KJ/m^{3}K)

d : thickness of the ferrofluid layer(m)

$\frac{D}{Dt}$: The substantial derivative(s^{-1})

g : acceleration due to gravity(m/s^2)

$\hat{k}$: Unit vector in the Z-direction

k : the resultant wave number, $k=\sqrt{k_x^2+k_y^2}$

k_x: The wave number along x-direction (m^{-1})

k_y: The wave number along y-direction (m^{-1})

k_1: medium permeability (m^2)

K_1: thermal conductivity ($W\ m^{-1}K^{-1}$)

P : Fluid pressure (Pa)

P' : Perturbation in pressure (Pa)

q : Darcian (filter) velocity of the Ferro fluid (m/s)

q' : (u,v,w) ; the perturbation in fluid velocity $(0,0,0)$ (m/s)

q_d : Velocity in dust particles

q_1' : The perturbation in dust velocity

t : time(s) T : Temperature (K)

T_L : Constant average temperature at the bottom surface$=-\dfrac{d}{2}$ (K)

T_U : Constant average temperature at the upper surface$=+\dfrac{d}{2}$ (K)

$T_a=\dfrac{(T_L+T_U)}{2}$: Average temperature (K)

Greek letters:

α : Coefficient of thermal expansion (K^{-1})

β: a uniform temperature gradient

β^*: a uniform concentration gradient $\left(\left|\dfrac{dC}{dZ}\right|\right)$

ω: Micro rotation (1/S)

ω' : The perturbation in micro inertia (1/s)

η: Shear kinematic viscosity co-efficient (m^2/s)

ξ: Coupling viscosity coefficient vortex viscosity (m^2/s)

λ': Bulk spin viscosity coefficient (m^2/s)

η': Shear spin viscosity co-efficient (m^2/s)

δ : heat conduction co-efficient ($Wsm^{-1}K^{-1}$)

I : Moment of inertia (micro inertia constant) (kg/m^2)

ρ : Fluid density

ρ_o: density at the ambient temperature (kg/m^3)

ρ_s: density of solid (Porous matrix) material (kg/m^3)

θ: the perturbation in temperature T(K)

ρ': the perturbation in density ρ (kg/m^3)

∇ : Gradient operator (m^{-1})

ε : medium porosity ($m^3/\ m^3$)

σ: the growth rate(s^{-1})

Non-Dimensional Parameters :

Da : Medium permeability parameter (Darcy number)

N_1 : Coupling parameter (Coupling between verticity and spin effects)

N_5 : Heat conduction parameter (Coupling between spin and heat fluxes)

R_1 : Modified Rayleigh number

X_1 : Dimensionless wave number

h_1: Concentration parameter of Dust particles

T^* : Non-dimensional temperature

C^* : Non-dimensional concentration of the species

Ω^* : Non-dimensional rotation parameter.

Introduction

The literature is replete with copious examples of research work on sysnthesization of ferrofluids and characterizing them. Numerous ferrofluids have been prepared tailor-made for requisite applications in various industries. Mechanical engineering industries use them as fluids in vibration dampers, shock absorbers and vacuum seals. The most famous application of magnetic fluids is the sealing of rotating shafts. Computational industries use them as fluids stepper motors. The use of magnetic fluids as a heat transfer medium that may be magnetically held in certain position is now a day's commercially most important branch of ferrofluid manufacturing. The major application in this field is cooling of loud speakers, enabling a significant increase of the maximum acoustical power without any geometrical changes of the speaker system. Electrical and electronic industries use them to improve hi-fi loud speakers, fluids as transformer coolants and in miniaturizing inductive components that eluded electrical industries for a long time.

The next important field of magnetic fluids is their use in biomedical applications. For example their use as a contrast medium in X-ray examinations and for positioning tapenade for retinal detachment repair in eye surgery has been reported. Furthermore the applications of magnetic fluids for the purpose of cancer treatment either by hyperthermia; using the change of magnetization in AC field to heat the tissue or by drug targeting are obviously challenging possibilities. Experimental and theoretical physicists and engineers gave significant contributions to Ferro hydrodynamics and its applications.

Ferro hydrodynamics (FHD) deals with the mechanics of fluid motions influenced by strong forces of magnetic polarization. Ferro hydrodynamics concerns usually non conducting liquids with magnetic properties and constitutes an entire field of physics close to magneto hydrodynamics; but still different. Magnetic fluids also called ferrofluids are electrically non conducting colloidal suspense of tiny particles of solid ferromagnetic material in non electrically conducting carrier fluid like water, kerosene, hydrocarbon etc. A typical ferrofluids contains 10^{23} particles per cubic meter. These fluids behave as a homogeneous continuum and exhibit a variety of interesting phenomenon. Ferromagnetic fluids are not found in nature but are artificially synthesized. The ferromagnetic nano particles are coated with a surfactant to prevent their agglomeration. Rosenweig[1,2] in his monograph and review article, provides a detailed introduction to this subject. Chandrasekhar[3] has given a detailed account of thermal convection problems of Newtonian fluids. The theory of convective instability of ferrofluid begins with Finlayson[4] and interestingly continued by Lalas and Carmi[5], Shilomis[6], Stiles and Kagan[7], Venkatasubramanian and Caloni[8], Sidheswar[9] and Sunil and coworkers[10,11], where as the non linear stability analysis of magnetized ferrofluid be heated from below has been discussed by Sunil and Mahajan[12,13], where by introducing a generalized energy functional, a rigorous non linear stability result is derived for a thermo convective magnetized ferrofluid.

Micro polar fluids are fluids with internal structures in which coupling between the spin of each particle and the macroscopic velocity is taken into account. They represent fluids consisting of rigid, randomly oriented of spherical particles suspended in viscous medium where the deformation of fluid particles is ignored. Micro polar fluid theory was introduced by Eringen[14] in order to describe some physical systems, which do not satisfy Navier-stokes equations.

The generation of the theory including thermal effects has been developed by Kazakia and Ariman[15] and Eringen[16]. The theory of thermo – micro polar convection began with Datta and Satrry[17] and interestingly continued by Ahmadi[18], Bhattacharya and Jena[19] and Sharma and Kumar[20,21]. In the absence of applied magnetic field, the particles in the colloidal suspensions are randomly oriented and thus the fluid has no net magnetization.

Abraham[22] observed that the micro polar ferrofluid layer heated from below is more stable as compared with the classical Newtonian ferrofluid. Recently Sunil *et al.*[10] have studied the convection problems in ferrofluid with internal angular momentum saturating a porous medium.

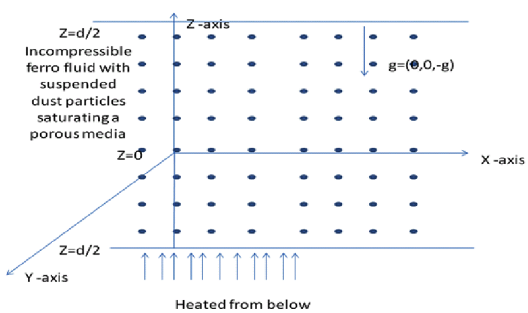
In geographical situations, the fluid is often not pure but contains suspended particles. Saffman[23] has studied the stability of laminar flow of a dusty gas. Scanlon and Segel[24] have considered the effect of suspended particles on the onset of Benard convection. The effect of dust particles on non-magnetic fluids have been studied by many authors[25-31]. Thermal convection in porous media is also of interest in geographical systems, electrochemistry, metallurgy, chemical technology, geophysics and bio-mechanics. A porous medium of very low permeability allows us to use the generalized Darcy's model including the inertial forces. This is because for a medium of very large stable particle suspension, the permeability tends to be small justifying the use of generalized Darcy's model including the inertial forces.

The viscous drag force is negligibly small in comparison with the Darcy's resistance due to the large particle suspension. Generalized Darcy's law governs the flow of ferrofluid through isotropic and homogeneous porous medium. However to be mathematically compatible and physically consistent with Navier–stokes equations. Brinkman[32] heuristically proposed the introduction of the term $\frac{\eta}{\varepsilon}\nabla^2 q$ (now known as the Brinkman term), $\left(\frac{\eta}{k_1}\right)q$, in addition to the Darcian term but the main effect is through the Darcian term; the brinkman term contributes a very little effect for flow through a porous medium. Therefore generalized Darcy's law is proposed heuristically to govern the flow of this ferrofluid through porous medium. Comprehensive review of the literature concerning convection in porous medium may be found in the book by Nield and Bejan[33]. Recently, Sunil *et al.*[10,34] have studies the thermal convection problems in ferrofluid saturating a porous medium.

In view of the recent increase in the number of non isothermal situations wherein magnetic fluids are put to use in place of classical fluids, we intend to extend our work to the problem of thermal convection in ferrofluid with internal angular momentum in the presence of dust particles saturating a porous medium. Sunil *et al.*[35] have studied the influence of dust particles and the effect of angular momentum parameters on stability in ferrofluid heated from below. But they have not considered the effects of species concentration on the flow pattern. In this paper, an attempt has been made to analyze the problem of Sunil *et al.* with varied species concentration and without magnetic field.

Mathematical formulation of the problem:

Here we consider an infinite, horizontal layer of thickness'd' of an electrically non-conducting in compressible thin ferrofluid with suspended particles heated from below saturating a porous medium. The temperature T at the bottom and top surfaces $Z = \pm\frac{1}{2}d$ are T_L and T_U, respectively and a uniform temperature gradient $\beta = \left(\frac{dT}{dZ}\right)$ is maintained. (See figure)



Both the boundaries are taken to be free and perfect conductors of heat. The gravity field g= (0, 0,-g) pervade the system.

This fluid layer assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε and medium permeability K_1, where the porosity is defined as the fraction of the total volume of the medium that is occupied by void space.

The mathematical equations governing the motion of ferrofluid with suspended dust particles saturating porous medium for the above model are as follows:

The continuing equation for an incompressible ferrofluid is

$$\vec{\nabla} \cdot \vec{q} = 0 \qquad (1)$$

The momentum and internal angular momentum equations for generalized Darcy's model including the inertial forces [Sunil *et al.*[10], Wooding[35]] are:

$$\frac{\rho_0}{\varepsilon}[\frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\vec{q} \cdot \nabla)]\vec{q} = -\nabla + \rho g + \frac{KN}{\varepsilon}(\vec{q}_d - q)$$

$$-\frac{1}{K_1}(\xi + \eta)\vec{q} + 2\xi(\nabla \times \vec{\omega}) \qquad (2)$$

$$\rho_0 I[\frac{\partial}{\partial t} + \frac{1}{\varepsilon}(\vec{q} \cdot \nabla)\vec{\omega} = 2\xi(\frac{1}{\varepsilon}\nabla \times \vec{q} - 2\vec{\omega})$$

$$+\eta'\nabla(\nabla \cdot \vec{\omega}) + \eta'\nabla^2\vec{\omega} \qquad (3)$$

Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra term equal in magnitude but opposite in sign, in the equations of motion for the particles. The buoyancy force on the particles is neglected. Inter particle reactions are also ignored, since we assume that the distances between particles are quite large compared with their diameters. The effects due to pressure, gravity, Darcian force on the particles are negligibly small and therefore ignored. If *mN* is the mass of the particles per unit volume, then the equations of motion and continuity of the dust particles, under the above assumptions are:

$$mN[\frac{\partial q_d}{\partial t}+\frac{1}{\varepsilon}(\vec{q}_d\cdot\nabla)\vec{q}_d]=KN(\vec{q}-\vec{q}_d), \quad (4)$$

$$\varepsilon\frac{\partial N}{\partial t}+\nabla\cdot(Nq_d)=0, \quad (5)$$

The temperature equation for an in compressible ferrofluid is,

$$[\rho_0 C_V]_{\frac{DT}{Dt}}+(1-\varepsilon)\rho_S C_S\frac{\partial T}{\partial t}+mNC_P(\varepsilon\frac{\partial}{\partial t}+\vec{q}_d\cdot\nabla)_T$$

$$=K_1\nabla^2 T+\delta(\nabla\times\vec{\omega})\cdot\nabla T, \quad (6)$$

$$\rho\frac{DC'}{Dt}=D'\nabla^2 C', \quad (7)$$

The density equation of state is,

$$\rho=\rho_0[1-\alpha(T-T_a)-\alpha^*(C'-C'_{a_a})], \quad (8)$$

The basic state is assumed to be quiescent state and is given by,

$$\left.\begin{array}{l}\vec{q}_b=(0,0,0),(\vec{q}_d)_b=(0,0,0),\omega_b=(0,0,0)\\ \rho=\rho_0(Z),P=P_b(Z),\beta=\dfrac{T_L-T_U}{d}\\ T_b(Z)=-\beta Z+T_a,C'_b(Z)=-\beta^* Z+C'_a\end{array}\right\}, \quad (9)$$

Where the subscript ' b' denotes the basic state.

The perturbation equations:

We shall analyze the stability of the basic state by introducing the following perturbations:

$$\left.\begin{array}{l}\vec{q}=\vec{q}_b+\vec{q}',\vec{q}_d=(\vec{q}_d)+\vec{q}',\vec{\omega}=\vec{\omega}_b+\vec{\omega}'\\ \rho=\rho_b+\rho',P=P_b(Z)+P',T=T_b(Z)+\theta,C'=C'_b(Z)+C\end{array}\right\}, \quad (10)$$

Where $\vec{q}'=(u,v,w),q'_1=(l,r,s),\vec{\omega}'=(\omega_1,\omega_2,\omega_3)$, p',θ and C are perturbations in $\vec{q},\vec{q}_d,\vec{\omega},P,T$ and C' respectively.

The change in density ρ', caused mainly by the perturbation θ in temperature, is given by

$$\rho'=-\rho_0\alpha\theta, \quad (11)$$

$$L_1\frac{\rho_0}{\varepsilon}\frac{\partial u}{\partial t}=L_1[-\frac{\partial P'}{\partial x}-\frac{1}{K_1}(\xi+\eta)u+2\xi\Omega'_1]$$

$$-\frac{mN_0}{\varepsilon}\frac{\partial u}{\partial t}, \quad (12)$$

$$L_1\frac{\rho_0}{\varepsilon}\frac{\partial v}{\partial t}=L_1[-\frac{\partial P'}{\partial y}-\frac{1}{K_1}(\xi+\eta)v+2\xi\Omega'_2]$$

$$-\frac{mN_0}{\varepsilon}\frac{\partial v}{\partial t}, \quad (13)$$

$$L_1\frac{\rho_0}{\varepsilon}\frac{\partial w}{\partial t}=L_1[-\frac{\partial P'}{\partial z}-\frac{1}{K_1}(\xi+\eta)w+2\xi\Omega'_{31}]$$

$$+g\alpha\rho_0 L_1\theta+g\alpha^*\rho_0 L_1 C-\frac{mN_0}{\varepsilon}\frac{\partial w}{\partial t}, \quad (14)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \quad (15)$$

$$\rho_0 I \frac{\partial \omega'}{\partial t} = 2\xi[\frac{1}{\varepsilon}\nabla \times \vec{q}' - 2\vec{\omega}'] + \eta'\nabla(\nabla \cdot \omega') + \eta'\nabla^2\omega', \quad (16)$$

$$L_1[(\rho C_1 + \varepsilon m N_0 C_{pt})\frac{\partial \theta}{\partial t}] = L_1[K_1\nabla^2\theta - \delta\beta\Omega_3'] + mN_0\beta C_{pt} w, \quad (17)$$

$$\rho \frac{\partial C}{\partial t} = D'\nabla^2 C, \quad (18)$$

Where $\rho C_1 = \varepsilon \rho_0 C_v + (1-\varepsilon)\rho_s C_s$

$\rho C_2 = \rho_0 C_v, L_1 = (\frac{m}{k}\frac{\partial}{\partial t} + 1), \Omega' = (\Omega_1', \Omega_2', \Omega_3') = (\nabla \times \vec{\omega}')$

Eliminating u, v, P' between equations (12-14) u, sing equation (15), we have,

$$[L_1(\frac{\rho_0}{\varepsilon}\frac{\partial}{\partial t} + \frac{\xi + \eta}{K_1}) + \frac{mN_0}{\varepsilon}\frac{\partial}{\partial t}]\nabla^2\omega = L_1[\rho_0 g\alpha\nabla^2\theta + \rho_0 g\alpha^*\nabla^2 C + 2\xi\nabla^2\Omega_3'], \quad (19)$$

Taking curl once on and considering only 3rd component, we obtain,

$$\rho_0 I \frac{\partial \Omega_3'}{\partial t} = -2\xi(\frac{1}{\varepsilon}\nabla^2\omega + 2\Omega_3') + \eta'\nabla^2\Omega_3', \quad (20)$$

The constants involved in the above equations are omitted here in order to save space.

Normal mode analysis method :

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form:

$$(w, \theta, \Omega_3') = [W(Z,t), \theta(Z,t), \Omega_3(Z,t)]\exp\cdot i(K_x x + K_y y), \quad (20)$$

Where K_x, K_y the wave is numbers along the x and y directions respectively and $K = \sqrt{K_x^2 + K_y^2}$ is the resultant wave number.

On applying (20), equations (19), (20), and (17) in dimensionless form becomes,

$$[(\frac{1}{\varepsilon}\frac{\partial}{\partial t^*} + \frac{1+N_1}{K_1^*} + \frac{f}{\varepsilon}\frac{\partial}{\partial t^*}](D^2 - a^2)W^* = L_1^*[-aR^{1/2}T^* + 2N_1(D^2 - a^2)\Omega_3^*, \quad (21)$$

$$I'\frac{\partial \Omega_3'}{\partial t^*} = -2N_1[\frac{1}{\varepsilon}(D^2 - a^2)W^* + 2\Omega_3^*] + N_3(D^2 - a^2)\Omega_3^*, \quad (22)$$

$$L_1^*[\varepsilon h P_r \frac{\partial T^*}{\partial t^*}] = L_1^*[(D^2 - a^2)T^* + aR^{1/2}W^*] + aR^{1/2}hW^*, \quad (23)$$

and

$$S_c \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial Z^{*2}}, \quad (24)$$

where the following non dimensional parameters and non dimensional quantities are introduced.

$$t^* = \frac{\nu t}{d^2}, W^* = \frac{Wd}{\nu}, R = \frac{g\alpha\beta d^4 \rho C_2}{\nu K_1}, T^* = \frac{K_1 a R^{1/2}}{\rho C_2 \beta \nu d}\Theta, a = Kd, Z^* = \frac{Z}{d}, D = \frac{\partial}{\partial Z^*}, K_1^* = \frac{K_1}{d^2}$$

$$P_r = \frac{\nu \rho C_1}{K_1}, S_C^{-1} = \frac{D'}{\rho}, N_1 = \frac{\xi}{\eta}, N_3 = \frac{\xi}{\eta d^2},$$

$$I' = \frac{I}{d^2}, \Omega_3' = \frac{\Omega_3 d_3}{\nu}, \tau = \frac{m\nu}{Kd^2},$$

$$L_1^* = (\tau \frac{\partial}{\partial t^*} + 1), f = \frac{mN_0}{\rho_0}, h = \frac{mN_0 C_{pt}}{\rho C_2}$$

The conditions at the free boundaries are:

$$W^* = D^2 W^* = T^* = \Omega_3^* = C^* = 0 \text{ at } Z = \pm\frac{1}{2}, \tag{25}$$

Thus the exact solutions of the system (25-28) subject to these boundary conditions are:

$$W^* = A_1 e^{\sigma t^*} \cos \Pi Z^*, T^* = B_1 e^{\sigma t^*} \cos \Pi Z^*, \Omega_3' = D_1 e^{\sigma t^*} \cos \Pi Z^*, C^* = B_2 e^{\sigma t^*} \cos \pi z^*, \tag{26}$$

Where A_1, B_1, B_2, D_1 constants and σ are is the growth rate which is in general, a complex constant.

Substituting solution (26)in equations (21-23)(dropping asterisks for convenience), we get, the equations involving coefficients of $A_1, B_1, B_2 and D_1$.

For the existence of non trivial solutions, the determinant of the coefficients of A_1, B_1, B_2 & D_1 must vanish.

This determinant on simplification yields,

$$iX_5\sigma_i^5 + X_4\sigma_i^4 - iX_3\sigma_i^3 - X_2\sigma_i^2 + iX_1\sigma_i + X_0 = 0, \tag{27}$$

With $X_0, \ldots\ldots\ldots, X_5$ are written in appendix.

Discussion of Results

When the instability sets in as stationary convection, the marginal state will be characterized by $\sigma_i = 0$ [Chandrasekhar[3]], then the Rayleigh number is given by,

$$R_1 = \frac{(1+X_1)^2\left[\frac{(1+N_1)(4N_1+bN_3)}{Da} - \frac{4N_1^2 b}{\varepsilon}\right]}{X_1\left[4N_1h_1 + b\left(h_1N_3 - \frac{2N_1N_5}{\varepsilon}\right)\right]}, \tag{28}$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless numbers X_1,Da,h_1,N_1,N_3, and N_5.

The classical results in respect of Newtonian fluids can be obtained as the limiting cases of present study.

Limiting cases:

Setting N_1=0, in the absence of dust particles(h_1=0) and keeping N_3 arbitrary in (28),we get,

$$R_1 = \frac{(1+X_1)^2}{X_1 Da}, \tag{28a}$$

(Which is exactly the equation obtained by Sunil *et al.*[10])

which is the classical Rayleigh Benard result in porous medium for the Newtonian fluid case.

To investigate the effect of dust particles, medium permeability, coupling parameter and heat conduction parameter, we examine the behavior of $\frac{dR_1}{dh_1}, \frac{dR_1}{dDa}, \frac{dR_1}{dN_1}, \frac{dR_1}{dN_3}, and$ $\frac{dR_1}{dN_5}$ analytically.

Equation (28) gives,

$$\frac{dR_1}{dh_1}=\frac{b^2L_0[\frac{(1+N_1)b}{Da}+\frac{4N_1}{Da}+4N_1^2(\frac{1}{Da}-\frac{b}{\varepsilon})][4N_1+b]}{X_1L_3[4N_1h_1+b(h_1-\frac{2N_1N_5}{\varepsilon})]^2}, \qquad (29)$$

Which is always negative if $\frac{1}{Da}>\frac{b}{\varepsilon}$, (30)

This shows that the dust parameters have a destabilizing effect when condition (30) holds. Here we observe that, In non-porous medium, $\frac{dR_1}{dh_1}$ is always negative, implying there by the destabilizing effect of dust particles.

From (28), we also find that,

$$\frac{dR_1}{dDa}=\frac{b^2L_0(1+N_1)[b+4N_1]}{X_1D_a^2L_3[4N_1h_1+b(h_1-\frac{2N_1N_5}{\varepsilon})]^2}, \qquad (31)$$

Which is always negative if $N_3>\frac{2N_1N_5}{\varepsilon}$, (32)

In the absence of coupling parameter N_1, the medium permeability always has a destabilizing effect on the system. Thus in ferrofluid heated from below saturating a porous medium, there is a competition between the destabilizing role of medium permeability and stabilizing role of coupling parameter, but there is a complete destabilization with the above inequality given by (32). Thus, the destabilizing behavior of medium permeability is virtually unaffected by magnetization parameters but is significantly affected by internal angular momentum parameters i.e. $N_1, N_3 and N_5$.

$$\frac{dR_1}{dN_1}=\frac{b^2L_0[\frac{b^2N_3}{Da}(\frac{2N_5}{\varepsilon}+N_3h_1)+8N_1(\frac{1}{Da}-\frac{b}{\varepsilon})\{2N_1h_1+b(N_3h_1-\frac{N_1N_5}{\varepsilon})\}]}{X_1L_3[4N_1h_1+b(N_3h_1-\frac{2N_1N_5}{\varepsilon})]^2}, \qquad (33)$$

Which is always negative if $\frac{1}{Da}>\frac{b}{\varepsilon}$ and

$$N_3>\frac{2N_1N_5}{\varepsilon}, \qquad (34)$$

This shows that coupling parameter has a stabilizing effect when conditions (34) hold. In a non-porous medium, (34) yields that $\frac{dR_1}{dN_1}$ is always positive, implying thereby the stabilizing effect of coupling parameter. Thus the medium permeability and porosity have also a significant role in developing condition for the stabilizing behavior of coupling parameter.

In the absence of external magnetic field, the variation of Rayleigh number with the change of heat conduction parameter is obtained from equation (28) as,

$$\frac{dR_1}{dN_5}=\frac{(1+X_1)^2\left[\frac{(1+N_1)N_3b}{Da}+4N_1^2(\frac{1}{Da}-\frac{b}{\varepsilon})\right]}{X_1^3N_1\left[4N_1h_1+b(N_3h_1-\frac{2N_1N_5}{\varepsilon})\right]^2}\frac{2b}{\varepsilon}, \qquad (35)$$

Which is always positive if $\frac{1}{Da} > \frac{b}{\varepsilon}$, (36)

This shows that the heat conduction parameter has a stabilizing effect when condition (36) holds. Here we also mark that in a non-porous medium, $\frac{dR_1}{dN_5}$ is always positive, implying thereby the stabilizing effect of heat conduction parameter.

Figure-1 shows the effect of Da on $\frac{dR_1}{dh_1}$. It is observed that the increase in Darcy's number reduces the modified Raleigh number. In general $\frac{dR_1}{dh_1}$ falls with the increase of h_1.

Figure-2 shows the effect of species concentrations (h_1) on $\frac{dR_1}{dDa}$. From the profiles of $\frac{dR_1}{dDa}$ ~Da, it is marked that the increase in h_1 reduces the variation of Rayleigh number with respect to Darcy's number. In general $\frac{dR_1}{dDa}$ falls with the increase of Da for N_1=0.0.

In Figure-3, the profiles of $\frac{dR_1}{dDa}$ ~ Da are plotted, varying the concentration of dust particles. The profiles exhibit that the variation of Rayleigh number with respect to the Darcy's number falls as h_1 rises taking N_1=0.2. But the nature of the curves (I-V) of figure-3 remains same as that of figure-2. However when N_1=0.0, the fall of $\frac{dR_1}{dDa}$ ~ Da, is slower than that of the case for N_1=0.2.

Figure-4 illustrates the effect of species concentrations (h_1) on modified Rayleigh number. The profiles of $\frac{dR_1}{dDa}$ ~ Da, reveal that $\frac{dR_1}{dDa}$ reduces quickly when h_1 varies from 1 to 2. But the curve (III-V) becomes close as h_1 takes higher values. The fall of modified Rayleigh number with respect to Darcy's number is similar for variation of dust particles concentration taking N_1=0.4.

In Figure-5, the effect of dust particle concentration (h_1) on $\frac{dR_1}{dDa}$ have been exhibited taking coupling parameter (N_1=0.6). It is noticed that $\frac{dR_1}{dDa}$ falls rapidly with Da for h_1=(Curve-I). But, increase in the value of h_1 from 2 to 6, produces no effect on

$\frac{dR_1}{dDa}$ ~Da.

In Figure-6, the effect of heat conduction parameter N_5 on $\frac{dR_1}{dN_5}$ *i.e.* it deals with the variation of Rayleigh number with the change

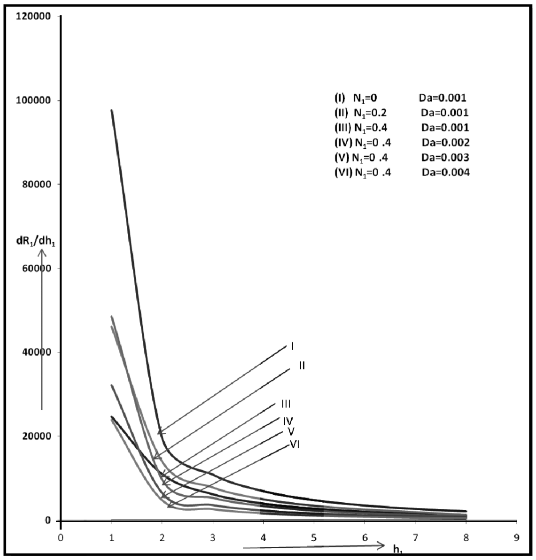


Figure 1. Graph between $\frac{dR_1}{dh_1} \sim h_1$

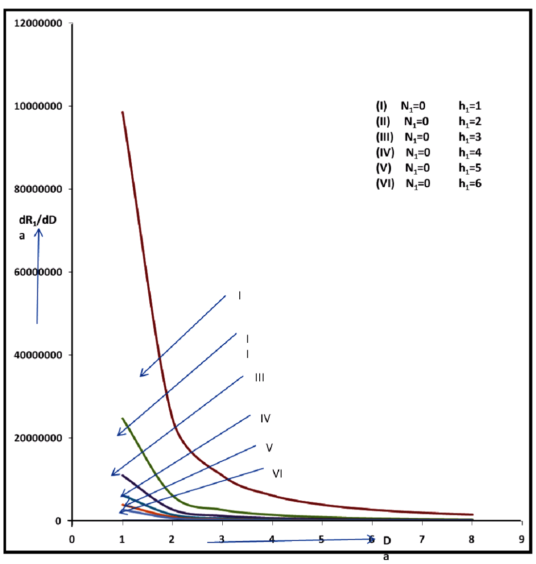


Figure 2. Graph between $\frac{dR_1}{dDa} \sim Da$ for N_1=0

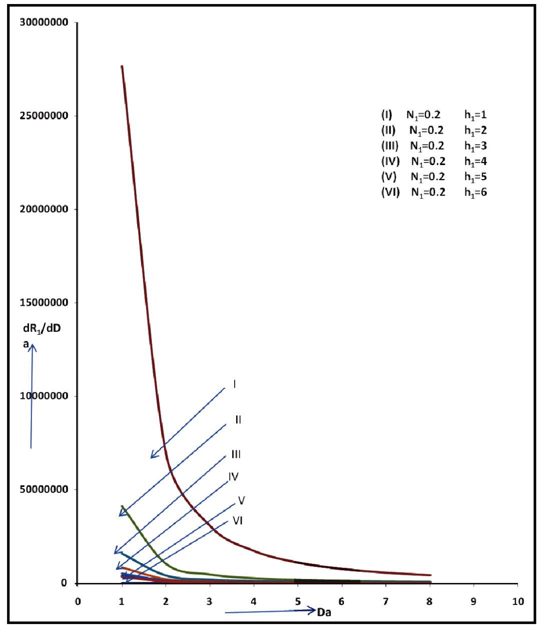


Figure 3. Graph between $\frac{dR_1}{dDa} \sim Da$ for N_1=0.2

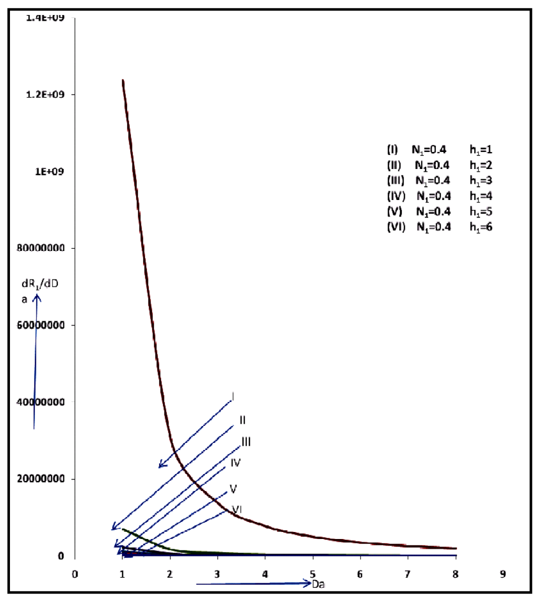


Figure 4. Graph between $\frac{dR_1}{dDa} \sim Da$ for N_1=0.4

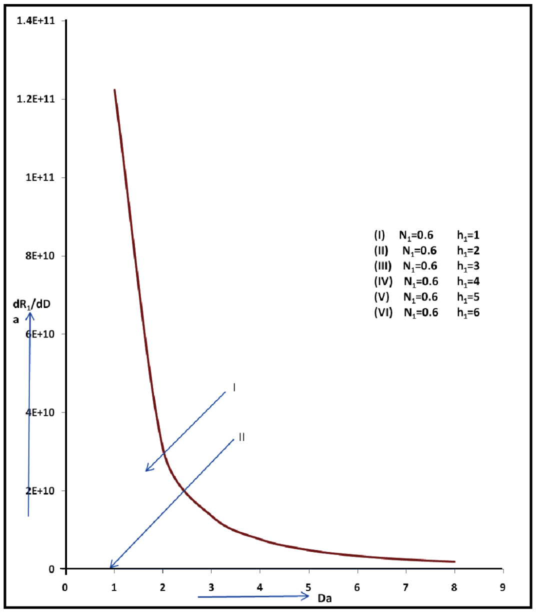


Figure 5. Graph between $\dfrac{dR_1}{dDa}$ ~Da for N_1=0.6

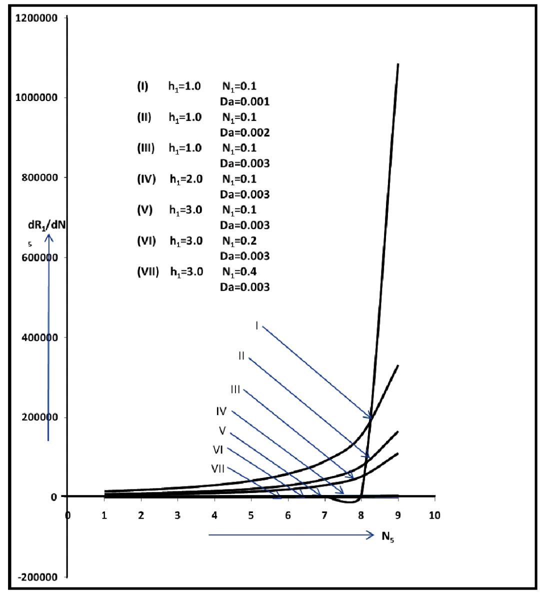


Figure 6. Graph between $\dfrac{dR_1}{dN_5}$ ~N_5 for N_3=2.0

in heat conduction parameter for various values of h_1= Concentration parameter of dust particles, N_1=Coupling Parameter, Da= Medium Permeability Parameter (Darcy's number) taking spin diffusion parameter N_3=2.0. It is observed that the increase in Darcy's number reduces the value of $\dfrac{dR_1}{dN_5}$.

It is interesting to record here that the rise in the value of h_1 rapidly decreases the value of $\dfrac{dR_1}{dN_5}$, where the profile becomes parallel to the axis of N_5 for h_1=3.0.

Further it is marked that the increase in N_1, decreases $\dfrac{dR_1}{dN_5}$ to a great extent, where $\dfrac{dR_1}{dN_5}$ attains negative value as N_5 for increases from 7 to 8 for N_1=0.4. At N_5=8, the profile sharply rises in positive direction of $\dfrac{dR_1}{dN_5}$.

Conclusion

Following conclusions are drawn from the above analysis of the marginal stability of rotating dusty ferrofluid flowing through porous medium.

(i) Increase in Darcy's number reduces

the modified Raleigh number.

(ii) Increase in the concentration parameter of dust particles (h_1), reduces the variation of Raleigh number with respect to Darcy's number.

(iii) The variation of Raleigh number with respect to the Darcy's number falls as concentration parameter of dust particles (h_1) rises taking coupling parameter N_1=0.2.

(iv) The fall of modified Raleigh number with respect to the Darcy's number is similar for variation of dust particle concentration taking N_1=0.4.

(v) Increase in the value of concentration parameter of dust particles (h_1) from 2 to 6 produces no effect on $\frac{dR_1}{dDa}$~*Da*, where '*Da*' is the Darcy's number.

(vi) The variation of modified Raleigh number with the heat conduction parameter falls with Darcy's number.

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