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Effect of heat transfer on MHD flow of visco-elastic fluid of an arbitrary inclined plane

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Abstract

The effect of heat transfer on the flow of visco-elastic conducting fluid of Rivlin – Ericksen type flowing down an inclined plane in presence of a transverse magnetic field is investigated. This type of problem finds application in many technological and engineering fields such as rocket propulsion systems, space craft re-entry aerothermodynamics, cosmical flight aerodynamics, plasma physics, Glass production and furnace engineering. Velocity of the flow has been presented for various parameters. In this study velocity of fluid increases with the increase in G_r (Grashof number), but it decreases with the increase in H (Hartmann number) and P_r (Prandtl number).

Key words : Rivlin-Ericksen fluid, Magnetic field, Heat transfer, An arbitrary inclined plane.

Introduction

Some hydromagnetic problems as investigated by Sengupta and Ghosh[1] and Sengupta and Bhattacharya[2] may be referred. Some fluids, some times exhibits various property of solids and viscous property of liquids are operation. Some problems associated with visco-elastic liquids have been considered by Sengupta and his research collabrators (Sengupta and Ghosh[3], Sengupta and Das[4], Sengupta and Kundu[5], Sengupta and Mukherjee[6]). Recently, Sultana and Ahmmed[7] have analysed MHD flow of visco-elastic fluid of an arbitrary inclined plane.

In the present paper we consider the problem Sultana and Ahmmed[7] with heat transfer. The purpose of this study is to investigate the effect of heat transfer on the flow of visco-elastic conducting fluid of Rivlin – Ericksen type flowing down an inclined plane in presence of a transverse magnetic field.

Materials and Methods

Let us consider electrical conducting visco-elastic Rivlin-Ericksen type fluid between two parallel inclined planes, the lower plane is at rest and the upper plane with heat transfer is in motion. A transverse uniform magnetic field B_o has been applied perpendicular to the time varying body force F(t′) taking the fluid initially at rest. The effect due to induced magnetic field and the perturbation of the magnetic field is neglected.

The equation of slow motion of a conducting visco-elastic Rivlin-Ericksen type fluid with heat transfer in two dimensional forms become

$$\frac{\partial u'}{\partial t'} = g'\sin\alpha + g'\beta'(T' - T_2)$$

$$-\frac{1}{\rho}\frac{\partial p'}{\partial x'} + \bar{\upsilon}\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}u' \qquad (1)$$

$$0 = g'\cos\alpha + \frac{1}{\rho}\frac{\partial p'}{\partial y'} \qquad (2)$$

the equation of heat transfer is

$$\frac{\partial T'}{\partial t'} = D_T\frac{\partial^2 T'}{\partial y'^2} \qquad (3)$$

and the equation of continuity is

$$\frac{\partial u'}{\partial x'} = 0 \qquad (4)$$

where

$$\bar{\upsilon} = \frac{\mu(1 + \lambda'\frac{\partial}{\partial t'})}{\rho}$$

ρ is the density, g' is the acceleration due to gravity, σ is the electric conductivity, B_o is the magnetic induction, α is the inclination of the plane to the horizontal and h is the height between two parallel plates, D_T is the thermal conductivity, β' is the coefficient of temperature expansion. So, $g'\sin\alpha - \frac{1}{\rho}\frac{\partial p'}{\partial x'}$ is a function of t′ alone and therefore, we can write

$$g'\sin\alpha - \frac{1}{\rho}\frac{\partial p'}{\partial x'} = -F(t') \qquad (5)$$

The value of F can be found out if the pressure is known at a given point $(x_o, 0)$. Now the equation (1) reduces to

$$\frac{\partial u'}{\partial t'} = -F(t') + g'\beta'(T' - T_2)$$

$$+ \nu(1 + \lambda'\frac{\partial}{\partial t'})\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}u' \qquad (6)$$

The boundary conditions are

(i) $u' = 0$, $T' = T_1$, $y' = 0$ for all t′

(ii) $u' = u'(t')$, $T' = T_2$, $y' = h$ for $\forall$ t′

Solution of the problem :

We are now going to put the equation in a non-dimensional form by setting

$$u = \frac{u'}{U_0},\ p = \frac{p'}{\rho U_0^2}\ g = \frac{hg'}{U_0^2}\ t = \frac{t'U_o}{h},$$

$$y = \frac{y'}{h},\ x = \frac{x'}{h}\lambda = \frac{\lambda' U_0}{h}, \theta = \frac{T' - T_2}{T_1 - T_2}$$

$$H = B_o h\sqrt{\frac{\sigma}{\rho}} \quad \text{(Hartmann number)}$$

$$G_r = g\beta'(T_1 - T_2) \quad \text{(Grashof number)}$$

$$R = \frac{hU_0}{\nu} \quad \text{(Reynolds number)}$$

$$P_r = \frac{\nu}{D_T} \quad \text{(Prandtl number)}$$

Thus the governing equation in non-dimensional form is

$$\frac{\partial u}{\partial t} = -F(t) + \frac{1}{R}\left(1 + \lambda \frac{\partial}{\partial t}\right)\frac{\partial^2 u}{\partial y^2} - \frac{H^2}{R}u + G_r\theta \tag{7}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{RP_r}\frac{\partial^2 \theta}{\partial y^2} \tag{8}$$

Now, the boundary conditions are

(i) $u = 0$, $\theta = 1$, $y = 0$ for all t

(ii) $u = u(t)$, $\theta = 0$, $y = 1$ for t

Fluid motion due to transient body force:

Firstly, we suppose that a transient force $F_0 e^{-\frac{\omega U_0 t}{h}}$ is applied to the fluid and the velocity u and temperature θ of the fluid are considered as

$$u = \frac{\bar{u}}{U_o} e^{-\frac{\omega U_0 t}{h}}, \qquad \theta = \frac{\bar{\theta}}{U_o} e^{-\frac{\omega U_0 t}{h}}$$

Putting $u = \frac{\bar{u}}{U_o} e^{-\frac{\omega U_0 t}{h}}$, $\theta = \frac{\bar{\theta}}{U_o} e^{-\frac{\omega U_0 t}{h}}$ and $F(t) = F_o e^{-\frac{\omega U_0 t}{h}}$, the equations (7) and (8) take the form

$$\frac{d^2\bar{u}}{dy^2} + a_1^2 \bar{u} = \beta + \frac{\beta G_m}{F_o}\bar{\theta} \tag{9}$$

$$\frac{d^2\bar{\theta}}{dy^2} + a_2^2 \bar{\theta} = 0 \tag{10}$$

where

$$a_1^2 = \frac{N}{\left(1 - \frac{\lambda \omega U_o}{h}\right)}$$

$$a_2^2 = \frac{\omega R P_r U_o}{h}$$

$$\beta = \frac{F_o U_o R}{\left(1 - \frac{\lambda \omega U_o}{h}\right)}$$

$$N = \frac{R\omega U_o}{h} - H^2$$

The solution of the equations (9) and (10) are

$$u = \frac{1}{U_o}\left[A\cos a_1 y + B\sin a_1 y + \frac{\beta}{a_1^2} + \frac{G_r\beta}{F_o(a_1^2 - a_2^2)}(\cos a_2 y - \cos a_2 . \sin a_2 y)\right] e^{-\frac{\omega U_0 t}{h}} \tag{11}$$

$$\theta = [\cos a_2 y - \cos a_2 . \sin a_2 y] e^{-\frac{\omega U_0 t}{h}} \tag{12}$$

Applying boundary conditions

(i) u = 0, y = 0 for all t

(ii) $u = \frac{F_o U_o}{h} e^{-\frac{\omega U_0 t}{h}}$, y = 1, $\forall t$

We get

$$A = -\frac{\beta}{a_1^2} - \frac{G_r \beta}{F_o(a_1^2 - a_2^2)}$$

and $B = \frac{1}{\sin a_1}\left[\frac{F_o U_o^2}{h} + \frac{G_r \beta \cos a_1}{F_o(a_1^2 - a_2^2)} - \frac{\beta}{a_1^2}\right]$

Results and Discussion

Fluid Velocity Profiles are tabulated in Table 1, 2 & 3 and plotted in Fig. 1, 2 & 3 for $U_o = 0.2$, $F_o = 1$, R = 15, $\omega = 25$, h = 0.5, y = 0.5, $\lambda = 0.05$ and different values of H (Hartmann number), G_r (Grashof number) and P_r (Prandtl number).

It is observed from Fig. 1, 2 & 3 that all velocity Graphs are decreasing sharply up to t = 0.4, then after velocity in each Graphs begins to decrease and tends to zero with the increase in t. It is also observed from Fig. 1, 2 & 3 that velocity increases with the increase in G_r, but it decreases with the increase in H and P_r.

Conclusion

The velocity of fluid increases with the increase in G_r (Grashof number), but it decreases with the increase in P_r (Prandtl number).

Particular case

When G_r (Grashof number) and P_r (Prandtl number) are equal to zero, this problem reduces to the problem of Sultana and Ahmmed[7].

Table 1. Values of velocity at $U_o = 0.2$, $F_o = 1$, R = 15, $\omega = 25$, h = 0.5, y = 0.5, $\lambda = 0.05$, $G_r = 5$, $P_r = 0.3$ and different values of H.

t	Graph-1 (H = 6)	Graph-2 (H = 7)	Graph-3 (H = 8)
0	0.490000	0.343000	0.177000
0.2	0.066314	0.046420	0.023954
0.4	0.008975	0.006282	0.003242
0.6	0.001215	0.000850	0.000439
0.8	0.000164	0.000115	0.000059
1	0.490000	0.343000	0.177000

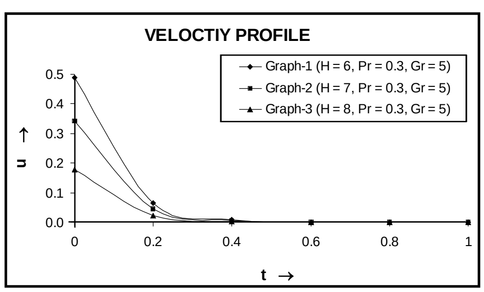

Fig. 1

Table 2. Values of velocity at $U_o = 0.2$, $F_o = 1$, $R = 15$, $\omega = 25$, $h = 0.5$, $y = 0.5$, $\lambda = 0.05$, $H = 6$, $P_r = 0.3$ and different values of G_r

t	Graph-1 (G_r = 5)	Graph-2 (G_r = 10)	Graph-3 (G_r = 15)
0	0.490000	0.630000	0.775000
0.2	0.066314	0.085261	0.104885
0.4	0.008975	0.011539	0.014195
0.6	0.001215	0.001562	0.001921
0.8	0.000164	0.000211	0.000260
1	0.000022	0.000029	0.000035

Table 3. Values of velocity at $U_o = 0.2$, $F_o = 1$, $R = 15$, $\omega = 25$, $h = 0.5$, $y = 0.5$, $\lambda = 0.05$, $H = 6$, $G_r = 5$ and different values of P_r

t	Graph-1 (P_r = 0.3)	Graph-2 (P_r = 0.4)	Graph-3 (P_r = 0.5)
0	0.490000	0.430000	0.345000
0.2	0.066314	0.058194	0.046691
0.4	0.008975	0.007876	0.006319
0.6	0.001215	0.001066	0.000855
0.8	0.000164	0.000144	0.000116
1	0.000022	0.000020	0.000016

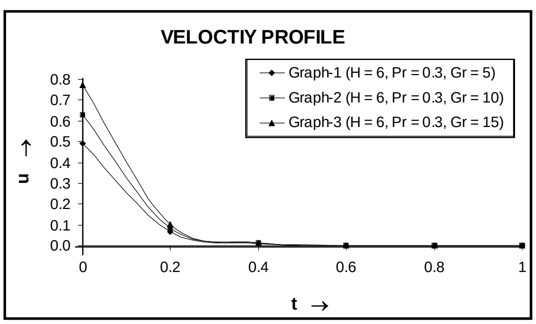

Fig. 2

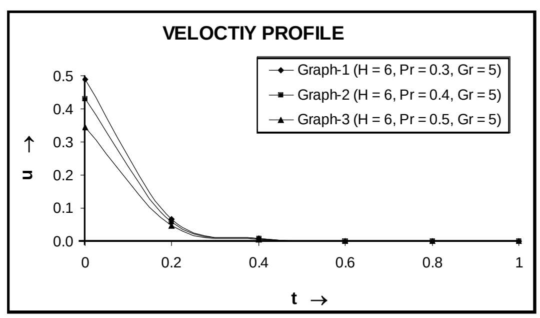

Fig. 3

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