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Effect of porous medium on flow of dusty incompressible visco-elastic second order oldroyd fluid through rectangular channel

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Abstract

The purpose of present study is to analyze the effect of porous medium on unsteady flow of dusty visco-elastic second order Oldroyd fluid with transient pressure gradient through a rectangular. Here we considered the second order Oldroyd fluid and investigate the same problem in this new visco-elastic model. The numerical expressions of the liquid velocity and particle velocity have been given.

Key words : Incompressible, Unsteady, Dsty visco-elastic fluid, second order Oldroyd fluid, Tansient pressure gradient, Porous medium.

Introduction

The study of the physics of flow through porous medium has become the basis of many scientific and engineering applications. This type of flow is of great importance in the petroleum engineering concerned with the movement of oil, gas and water through the reservoir of an oil or gas field to the hydrologist in the study of the migration of underground water and to the chemical engineer in the filtration process.

A model equation describing the motion of such mixed system has been given by Saffman[10], using the formulation of Saffman, Several authors Gupta and Gupta[1], Hayat *et. al.*[2], Kumar[3], Kumar[4], Kundu and Sengupta[5], Kundu and Sengupta[6], Mahdy[7], Mahdy *et. al.*[8], Rahman and Alam Sarkar[9], Sharma[11], Singh and Singh[12], Singh and Kumar[13], Singh and Singh[14], Singh *et. al.*[15], Singh and Singh[16], Varshney and Singh[17] and Varshney and Singh[18] investigated the flow models of visco-elastic fluids.

In the present paper, we have considered the problem of Kumar[3] with porous medium. Expressions for the velocities of second order visco-elastic liquid and dust particles are obtained.

Formulation of the problem :

Let us consider a rectangular Cartesian

coordinate system (x', y', z') such that the walls of the rectangular channel to the planes $x' = \pm a$ and $y' = \pm b$, where z' -axis is taken towards the direction of motion. u' (x', y', t') and w' (x', y', t') are the axial velocity of the liquid and dust particle velocity in z-direction respectively. A transient pressure gradient $-Pe^{-\omega' t'}$ varying with time is applied to the fluid.

The equation of motion of dusty visco-elastic second order liquid through porous medium

$$\left(1+\lambda'_1\frac{\partial}{\partial t'}+\lambda'_2\frac{\partial^2}{\partial t'^2}\right)\frac{\partial u'}{\partial t'}=-\frac{1}{\rho}\left(1+\lambda'_1\frac{\partial}{\partial t'}+\lambda'_2\frac{\partial^2}{\partial t'^2}\right)\frac{\partial p'}{\partial z'}+\upsilon\left(1+\mu'_1\frac{\partial}{\partial t'}+\mu'_2\frac{\partial^2}{\partial t'^2}\right)\nabla^2 u'$$
$$+\frac{KN_o}{\rho}\left(1+\lambda'_1\frac{\partial}{\partial t'}+\lambda'_2\frac{\partial^2}{\partial t'^2}\right)(w'-u')-\frac{\upsilon}{K'_1}\left(1+\lambda'_1\frac{\partial}{\partial t'}+\lambda'_2\frac{\partial^2}{\partial t'^2}\right)u' \quad (1)$$

$$m\frac{\partial w'}{\partial t'}=K(u'-w') \quad (2)$$

where p' is the pressure, λ'_1 is the stress relaxation time parameter, λ'_2 is the additional material constant, μ'_1 is the strain rate retardation time parameter, μ'_2 is the additional material constant, υ is the kinematic coefficient of viscosity, m is mass of particles, K is the stokes resistance coefficient, N_o is the number of density of the particle, ρ is the density and K_1' is the coefficient of permeability.

Introducing the non dimensional quantities

$$u=\frac{u'a}{\upsilon},\quad w=\frac{w'a}{\upsilon},\quad p=\frac{p'a^2}{\rho\upsilon^2},$$
$$t=\frac{t'\upsilon}{a^2}\quad \omega=\frac{\omega' a^2}{\upsilon},\quad (x,y,z)=\frac{1}{a}(x',y',z')$$
$$\lambda_1=\lambda'_1\frac{\upsilon}{a^2},\quad \mu_1=\mu'_1\frac{\upsilon}{a^2}\quad \lambda_2=\lambda'_2\frac{\upsilon^2}{a^4},$$
$$\mu_2=\mu'_2\frac{\upsilon^2}{a^4} \quad (3)$$

The equations (1) and (2) become

$$\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial u}{\partial t}=-\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)\frac{\partial p}{\partial z}+\left(1+\mu_1\frac{\partial}{\partial t}+\mu_2\frac{\partial^2}{\partial t^2}\right)\nabla^2 u$$
$$+\frac{\ell}{b}\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)(w-u)-\frac{1}{K_1}\left(1+\lambda_1\frac{\partial}{\partial t}+\lambda_2\frac{\partial^2}{\partial t^2}\right)u \quad (4)$$

$$b\frac{\partial w}{\partial t}=(u-w) \quad (5)$$

where

$$K_1=\frac{1}{a^2}K_1' \text{ (Porosity parameter)}$$

$$\ell=\frac{mN_o}{\rho},\quad b=\frac{m\upsilon}{Ka^2}$$

The boundary conditions are

$$u = w = 0, \text{ when } x = \pm 1, \quad -\frac{b}{a} \le y \le \frac{b}{a} \quad (6)$$

$$u = w = 0, \text{ when } y = \pm\frac{b}{a}, \quad -1 \le x \le 1 \quad (7)$$

We have considered those type of situations of the flow which is transient in nature with respect to time and periodic in nature with respect to y. From the nature of boundary conditions, we have chosen the solution of (4) and (5) as

$$u = U(x)\cos my\ e^{-\omega t} \quad (8)$$

$$w = W(x)\cos my \quad (9)$$

Condition (7) will be satisfied if $\cos m\frac{b}{a} = 0$

or $$m = (2n+1)\frac{\pi a}{2b}, \quad n = 0, 1, 2, 3\ldots \quad (10)$$

Now the boundary conditions (6) become

$$U = W = 0 \text{ when } x = \pm 1 \quad (11)$$

We construct the solution as the sum of all possible solutions for each value of n

$$u = \sum_{n=0}^{\infty} U(x)\cos my\ e^{-\omega t} \quad (12)$$

$$w = \sum_{n=0}^{\infty} W(x)\cos my\ e^{-\omega t} \quad (13)$$

Using equation (12) and (13) in equation (5), we get

$$\sum_{n=0}^{\infty} W(x)\cos my = \frac{1}{(1-b\omega)}\sum_{n=0}^{\infty} U(x)\cos my \quad (14)$$

By putting $\frac{\partial p}{\partial z} = -Pe^{-\omega t} (\omega > 0)$ in (4) and using (12), (13) and (14), we get

$$\sum_{n=0}^{\infty}\frac{d^2U(x)}{dx^2}\cos my - \sum_{n=0}^{\infty}\left\{m^2 - \frac{\left(1-\lambda_1\omega+\lambda_2\omega^2\right)\left(\omega - \frac{\ell\omega}{1-b\omega} - \frac{1}{K_1}\right)}{\left(1-\mu_1\omega+\mu_2\omega^2\right)}\right\}U(x)\cos my$$

$$+\frac{\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{\left(1-\mu_1\omega+\mu_2\omega^2\right)}P = 0 \quad (15)$$

Equating the coefficient of cos my equal to zero, we get

$$\frac{d^2U}{dx^2} - \frac{K_2^2}{a^2}U + A_n = 0 \quad (16)$$

where $K_2^2 = \left\{m^2 - \frac{\left(1-\lambda_1\omega+\lambda_2\omega^2\right)\left(\omega - \frac{\ell\omega}{1-b\omega} - \frac{1}{K_1}\right)}{\left(1-\mu_1\omega+\mu_2\omega^2\right)}\right\}a^2$

and $$A_n = \frac{(-1)^n 4P\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{(2n+1)\pi\left(1-\mu_1\omega+\mu_2\omega^2\right)} \quad (17)$$

Solving equation (16) and using boundary condition (11), we get

$$U(x) = \frac{(-1)^{n+1} 4P\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{(2n+1)\pi\left(1-\mu_1\omega+\mu_2\omega^2\right)K_2^2}\left\{1 - \frac{\cosh\frac{K_2}{a}x}{\cosh\frac{K_2}{a}}\right\}$$

Thus the velocity of liquid is given by

$$u(x,y,t)=\sum_{n=0}^{\infty}\left[\frac{(-1)^{n+1}4P\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{(2n+1)\pi\left(1-\mu_1\omega+\mu_2\omega^2\right)K_2^2}\left\{1-\frac{\cosh\frac{K_2}{a}x}{\cosh\frac{K_2}{a}}\right\}\right]e^{-\omega t}\cos(2n+1)\frac{\pi ay}{2b} \quad (18)$$

and the velocity of dust particles is given by

$$w(x,y,t)=\sum_{n=0}^{\infty}\left[\frac{(-1)^{n+1}4P\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{(2n+1)\pi\left(1-\mu_1\omega+\mu_2\omega^2\right)(1-b\omega)K_2^2}\left\{1-\frac{\cosh\frac{K_2}{a}x}{\cosh\frac{K_2}{a}}\right\}\right]e^{-\omega t}\cos(2n+1)\frac{\pi ay}{2b} \quad (19)$$

Deduction :

Case I : If we put $\mu_1,\mu_2=0$ in equation (18) we shall obtain the Maxwell fluid which is given below

$$u(x,y,t)=\sum_{n=0}^{\infty}\left[\frac{(-1)^{n+1}}{(2n+1)}\frac{4P\left(1-\lambda_1\omega+\lambda_2\omega^2\right)}{\pi K_2^2}\left\{1-\frac{\cosh\frac{K_2}{a}x}{\cosh\frac{K_2}{a}}\right\}\right]e^{-\omega t}\cos(2n+1)\frac{\pi ay}{2b}$$

where $K_2^2=\left[m^2-\left(1-\lambda_1\omega+\lambda_2\omega^2\right)\left(\omega-\frac{\ell\omega}{1-b\omega}-\frac{1}{K_1}\right)\right]a^2$

Case II : Putting $\mu_1,\mu_2=0$ and $\lambda_1,\lambda_2=0$ in equation (18) we shall obtain the purely viscous fluid which is given below

$$u(x,y,t)=\sum_{n=0}^{\infty}\left[\frac{(-1)^{n+1}}{(2n+1)}\frac{4P}{\pi K_2^2}\left\{1-\frac{\cosh\frac{K_2}{a}x}{\cosh\frac{K_2}{a}}\right\}\right]e^{-\omega t}(2n+1)\frac{\pi ay}{2b}$$

where $K_2^2=\left\{m^2-\left(\omega-\frac{\ell\omega}{1-b\omega}-\frac{1}{K_1}\right)\right\}a^2$

Case III : Putting in equation (18) we shall obtain the same result of Kumar[3].

Discussion

The expression for velocity (in the presence of porous medium) of second order Oldroyd visco-elastic fluid is given by equation (18) and velocity of dust particles is given by equation (19).

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